p. 130: 63-68, 80-82

63. $\lim_{x\to\pm\infty}\frac{5+4x}{x+3}=\lim_{x\to\pm\infty}\frac{\frac{5}{x}+4}{1+\frac{3}{x}}=\frac{0+4}{1+0}=4$, so $y=4$ is a horizontal asymptote. $y=f(x)=\frac{5+4x}{x+3}$, so $\lim_{x\to-3^+}f(x)=-\infty$ since $5+4x\to-7$ and $x+3\to0^+$ as $x\to-3^+$. Thus, $x=-3$ is a vertical asymptote. The graph confirms our work.

64. $\lim_{x\to\pm\infty}\frac{2x^2+1}{3x^2+2x-1}=\lim_{x\to\pm\infty}\frac{2+\frac{1}{x^2}}{3+\frac{2}{x}-\frac{1}{x^2}}=\frac{2}{3}$, so $y=\frac{2}{3}$ is a horizontal asymptote. $y=f(x)=\frac{2x^2+1}{3x^2+2x-1}=\frac{2x^2+1}{(3x-1)(x+1)}$. The denominator is zero when $x=\frac{1}{3}$ and $x=-1$ but the numerator is nonzero, so $x=\frac{1}{3}$ and $x=-1$ are vertical asymptotes. The graph confirms our work.

65. $\lim_{x\to\pm\infty}\frac{2x^2+x-1}{x^2+x-2}=\lim_{x\to\pm\infty}\frac{1+\frac{1}{x}-\frac{1}{x^2}}{1+\frac{1}{x}-\frac{2}{x^2}}=\frac{2+0+0}{1+0-0}=2$, so $y=2$ is a horizontal asymptote. $y=f(x)=\frac{2x^2+x-1}{x^2+x-2}=\frac{(2x-1)(x+1)}{(x+2)(x-1)}$, so $\lim_{x\to-2^-}f(x)=-\infty$, $\lim_{x\to1^-}f(x)=-\infty$, $\lim_{x\to1^+}f(x)=\infty$. Thus, $x=-2$ and $x=1$ are vertical asymptotes. The graph confirms our work.

66. $\lim_{x\to\pm\infty}\frac{1+x^4}{x^2-x^4}=\lim_{x\to\pm\infty}\frac{\frac{1}{x^4}+1}{\frac{1}{x^2}-1}=\frac{0+1}{0-1}=-1$, so $y=-1$ is a horizontal asymptote. $y=f(x)=\frac{1+x^4}{x^2-x^4}=\frac{1+x^4}{x^2(1-x^2)}=\frac{1+x^4}{x^2(1-x)(1+x)}$.

The denominator is zero when $x=0,-1$ and 1, so these are vertical asymptotes. Notice that as $x\to0$, the numerator and denominator are both positive, so $\lim_{x\to0}f(x)=\infty$. The graph confirms our work.

67. $y = f(x) = \dfrac{x^3 - x}{x^2 - 6x + 5} = \dfrac{x\left(x^2 - 1\right)}{(x-1)(x+5)} = \dfrac{x(x-1)(x+1)}{(x-1)(x+5)} = \dfrac{x(x+1)}{x-5}$

for $= g(x)$ for $x \neq 1$. The graph of g is the same as the graph of f with the exception of a hole in the graph of f at $x = 1$. By long division, $g(x) = \dfrac{x^2 + x}{x-5} = x + 6 + \dfrac{30}{x-5}$. As $x \to \pm\infty, g(x) \to \pm\infty$, so there is no horizontal asymptote. The denominator of g is zero when $x = 5$, and $\lim_{x \to 5^-} g(x) = -\infty$ and $\lim_{x \to 5^+} g(x) = \infty$, so $x = 5$ is a vertical asymptote. The graph confirms our work.

68. $\lim_{x \to \infty} \dfrac{2e^x}{e^x - 5} = \lim_{x \to \infty} \dfrac{2e^x}{e^x - 5} \cdot \dfrac{1/e^x}{1/e^x} = \lim_{x \to \infty} \dfrac{2}{1 - \left(5/e^x\right)} = \dfrac{2}{1-0} = 2$, so $y = 2$ is

a horizontal asymptote. $\lim_{x \to -\infty} \dfrac{2e^x}{e^x - 5} = \dfrac{2}{0-5} = 0$, so is a horizontal asymptote. The denominator is zero (and the numerator isn't) when $e^x - 5 = 0 \Rightarrow e^x = 5 \Rightarrow x = \ln 5$. Thus, $x = \ln 5$ is a vertical asymptote. The graph confirms our work.

80. If the graph of $y = \dfrac{mx^3 + x + a}{x^3 - 2}$ crosses it horizontal asymptote at the point (6, –5) then

$\lim_{x \to \infty} \dfrac{mx^3 + x + a}{x^3 - 2} = \lim_{x \to \infty} \dfrac{m + \frac{1}{x^2} + \frac{a}{x^3}}{1 - \frac{2}{x^3}} = -5$, so $m = -5$.

But $y(6) = -5 = \dfrac{m \cdot 6^3 + 6 + a}{6^3 - 2} = \dfrac{-5 \cdot 6^3 + 6 + a}{6^3 - 2} = \dfrac{-1074 + a}{214} \Rightarrow -5(214) = -1074 + a \Rightarrow 4 = a.$

Thus, the value of $m + a$ is (**B**) –1.

81. To find the horizontal asymptotes, we must evaluate $\lim_{x \to \pm\infty} \dfrac{mx + m \cdot 6^{-x}}{4x + 6^{-x}}$. If $m = 48$ then

$\lim_{x \to \infty} \dfrac{48x + 48 \cdot 6^{-x}}{4x + 6^{-x}} = \lim_{x \to \infty} \dfrac{48x + \frac{48}{6^x}}{4x + \frac{1}{6^x}} = \lim_{x \to \infty} \dfrac{48 + \frac{48}{x \cdot 6^x}}{4 + \frac{1}{x \cdot 6^x}} = \dfrac{48 + 0}{4 + 0} = \dfrac{48}{4} = 12$, so in this case, $y = 12$ is a

horizontal asymptote. However, if $m = 12$, $\lim_{x \to -\infty} \dfrac{12x + 12 \cdot 6^{-x}}{4x + 6^{-x}} = \lim_{x \to -\infty} \dfrac{12 \cdot 6^{-x}}{6^{-x}} = \lim_{x \to -\infty} 12 = 12$. Therefore the line $y = 12$ will be a horizontal asymptote for (**C**) both $m = 48$ and $m = 12$.

82. Let $f(x)=\dfrac{3+4^{\frac{1}{x}}}{5+4^{\frac{1}{x}}}$, and $t=\dfrac{1}{x}$. Then as $x\to 0^-, t\to -\infty$, and $\lim\limits_{x\to 0^-} f(x)=\lim\limits_{t\to -\infty}\dfrac{3+4^t}{5+4^t} = \lim\limits_{x\to 0^+}\dfrac{3+0}{5+0}=\dfrac{3}{5}$.

Also, as $x\to 0^+, t\to\infty$, so $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{t\to\infty}\dfrac{3+4^t}{5+4^t}=\lim\limits_{x\to\infty}\dfrac{4^t}{4^t}=1$. Finally,

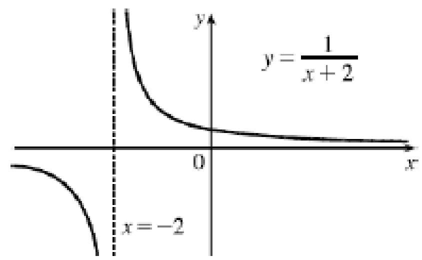
$\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\dfrac{3+4^{\frac{1}{x}}}{5+4^{\frac{1}{x}}}=\dfrac{3+1}{5+1}=\dfrac{4}{6}=\dfrac{2}{3}$. Therefore $\lim\limits_{x\to 0^-} f(x)+\lim\limits_{x\to 0^+} f(x)+\lim\limits_{x\to\infty} f(x)=\dfrac{3}{5}+1+\dfrac{2}{3}$, which is **(A)** $\dfrac{34}{15}$.

p. 143: 17-18, 27-32, 35 (use calculator), 44-45, 50-51, 57-60

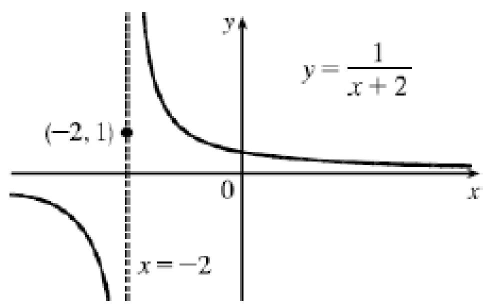
17. The function $f(x)=\begin{cases}2 & \text{if } x\neq 3\\ 4 & \text{if } x=3\end{cases}$ satisfies $\lim\limits_{x\to 3^+} f(x)=\lim\limits_{x\to 3^-} f(x)=2\neq f(3)=4$.

18. The function $f(x)=\dfrac{2(x-2)}{(x+2)}$ has a vertical asymptote at $x=-2$ and a horizontal asymptote at $x=2$.

27. $f(x)=\dfrac{1}{x+2}$ is discontinuous at $a=-2$ because $f(-2)$ is undefined.

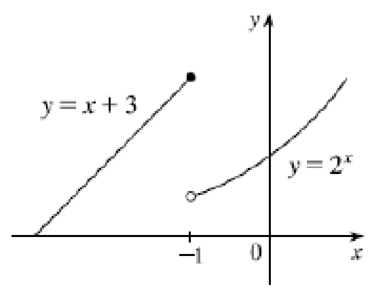


28. $f(x)=\begin{cases}\dfrac{1}{x+2} & \text{if } x\neq -2\\ 1 & \text{if } x=-2\end{cases}$. For this function, $f(-2)=1$, but $\lim\limits_{x\to -2^-} f(x)=-\infty$ and $\lim\limits_{x\to -2^+} f(x)=\infty$, so $\lim\limits_{x\to -2} f(x)$ does not exist and f is discontinuous at $x=-2$.

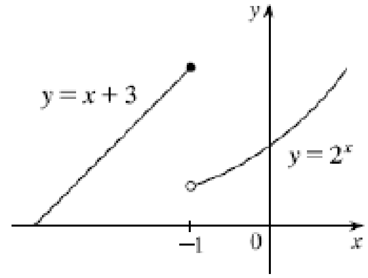


29. $f(x)=\begin{cases}x+3 & \text{if } x\le -1\\ 2^x & \text{if } x>-1\end{cases}$

$\lim\limits_{x\to -1^-} f(x)=\lim\limits_{x\to -1^-}(x+3)=-1+3=2$ and $\lim\limits_{x\to -1^+} f(x)=\lim\limits_{x\to -1^+} 2^x=2^{-1}=\dfrac{1}{2}$.
Since the left-hand and right-hand limits of f at -1 are not equal, $\lim\limits_{x\to -1} f(x)$ does not exist, and f is discontinuous at -1.



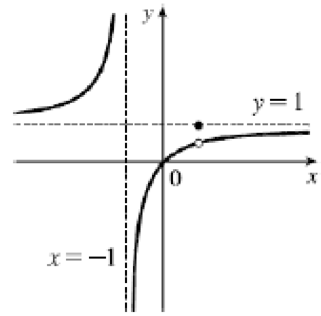
29. $f(x)=\begin{cases} x+3 & \text{if } x\le -1 \\ 2^x & \text{if } x>-1 \end{cases}$



$\lim\limits_{x\to -1^-} f(x)=\lim\limits_{x\to -1^-}(x+3)=-1+3=2$ and $\lim\limits_{x\to -1^+} f(x)=\lim\limits_{x\to -1^+} 2^x=2^{-1}=\dfrac{1}{2}$.

Since the left-hand and right-hand limits of f at -1 are not equal, $\lim\limits_{x\to -1} f(x)$ does not exist, and f is discontinuous at -1.

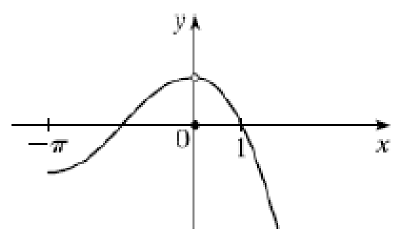
30. $f(x)=\begin{cases} \dfrac{x^2-x}{x^2-1} & \text{if } x\ne 1 \\ 1 & \text{if } x=1 \end{cases}$



$\lim\limits_{x\to 1} f(x)=\lim\limits_{x\to 1}\dfrac{x^2-x}{x^2-1}=\lim\limits_{x\to 1}\dfrac{x(x-1)}{(x+1)(x-1)}=\lim\limits_{x\to 1}\dfrac{x}{x-1}=\dfrac{1}{2}$, but

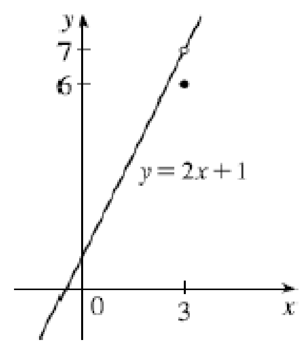
$f(1)=1\ne\dfrac{1}{2}$ so f is discontinuous at 1.

31. $f(x)=\begin{cases} \cos x & \text{if } x<0 \\ 0 & \text{if } x=0 \\ 1-x^2 & \text{if } x>0 \end{cases}$



$\lim\limits_{x\to 0} f(x)=1$, but $f(0)=0\ne 1$, so f is not continuous at 0.

32. $f(x)=\begin{cases} \dfrac{2x^2-5x-3}{x-3} & \text{if } x\ne 3 \\ 6 & \text{if } x=3 \end{cases}$



$\lim\limits_{x\to 3} f(x)=\lim\limits_{x\to 3}\dfrac{2x^2-5x-3}{x-3}=\lim\limits_{x\to 3}\dfrac{(2x+1)(x-3)}{x-3}=\lim\limits_{x\to 3}2x+1=7$, but

$f(3)=6\ne 7$, so f is discontinuous at 3.

35. Let $f(x)=\begin{cases}\dfrac{4-\sqrt{x^2+x+10}}{x-2} & \text{if } x\neq 2\\ -\dfrac{5}{8} & \text{if } x=2\end{cases}$.

Then $\lim\limits_{x\to 2} f(x)=\lim\limits_{x\to 2}\dfrac{4-\sqrt{x^2+x+10}}{x-2}=\lim\limits_{x\to 2}\dfrac{4-\sqrt{x^2+x+10}}{x-2}\cdot\dfrac{4+\sqrt{x^2+x+10}}{4+\sqrt{x^2+x+10}}$

$$=\lim_{x\to 2}\frac{16-\left(x^2+x+10\right)}{(x-2)\left(4+\sqrt{x^2+x+10}\right)}=\lim_{x\to 2}\frac{-\left(x^2+x-6\right)}{(x-2)\left(4+\sqrt{x^2+x+10}\right)}$$

$$=\lim_{x\to 2}\frac{-(x+3)(x-2)}{(x-2)\left(4+\sqrt{x^2+x+10}\right)}=\lim_{x\to 2}\frac{-(x+3)}{\left(4+\sqrt{x^2+x+10}\right)}$$

$=\dfrac{-5}{4+\sqrt{4+2+10}}=\dfrac{-5}{4+\sqrt{16}}=-\dfrac{5}{8}$ and so f is continuous at $x=2$.

44. The function $y=\dfrac{1}{1+e^{1/x}}$ is discontinuous at $x=0$ because the left- and right-hand limits at $x=0$ are different.

45. The function $y=\tan^2 x$ is discontinuous at $x=\frac{\pi}{2}+2\pi k$, where k is any integer. The function is discontinuous where $\tan^2$ is 0, that is at $x=\pi k$. So $y=\ln\left(\tan^2 x\right)$ is discontinuous at $x=\frac{\pi}{2}n$, n any integer.

50. $f(x)=\begin{cases}1-x^2 & \text{if } x\le 1\\ \ln x & \text{if } x>1\end{cases}$

By Theorem 3, since $f(x)$ equals the polynomial $1-x^2$ on $(-\infty,1]$, f is continuous on $(-\infty,1]$. By Theorem 3, since $f(x)$ equals the logarithmic function $\ln x$ on $(1,\infty)$, f is continuous on $(1,\infty)$.

51. $f(x)=\begin{cases}1-x^2 & \text{if } x\le 1\\ \ln x & \text{if } x>1\end{cases}$

By Theorem 3, the trigonometric functions are continuous. Since $f(x)=\sin x$ on $(-\infty,\pi/4)$ and $f(x)=\cos x$ on $(\pi/4,\infty)$, f is continuous on $(-\infty,\pi/4)\cup(\pi/4,\infty)$. In addition, $\lim\limits_{x\to(\pi/4)^-} f(x)=\lim\limits_{x\to(\pi/4)^-}\sin x=\sin\frac{\pi}{4}=1/\sqrt{2}$ since the sine function is continuous at $\pi/4$. Similarly, $\lim\limits_{x\to(\pi/4)^+} f(x)=\lim\limits_{x\to(\pi/4)^+}\cos x=\cos\frac{\pi}{4}=1/\sqrt{2}$ by the continuity of the cosine function at $\pi/4$. Thus $\lim\limits_{x\to(\pi/4)} f(x)$ exists and equals $1/\sqrt{2}$ which agrees with the value of $f(\pi/4)$. Therefore, f is continuous at $\pi/4$, so f is continuous on $(-\infty,\infty)$.

57. $f(x)=\begin{cases}\dfrac{x^3-a^3}{x-a} & \text{if } x\ne a\\ c & \text{if } x=a\end{cases}$

For $x\ne a$, $f(x)=x^2+ax+a^2$, so $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a}\left(x^2+ax+a^2\right)=a^2+a^2+a^2=3a^2$. In order for f to be continuous at a, we need $f(a)=c=\lim\limits_{x\to a} f(x)=3a^2$. Therefore, $c=3a^2$ and

$$f(x)=\begin{cases}\dfrac{x^3-a^3}{x-a} & \text{if } x\ne a\\ 3a^2 & \text{if } x=a\end{cases}.$$

58. $f(x)=\begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x<2 \\ ax^2-bx+3 & \text{if } 2\le x<3 \\ 2x-a+b & \text{if } x\ge 3 \end{cases}$

At $x=2$: $\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}\dfrac{x^2-4}{x-2}=\lim_{x\to 2^-}\dfrac{(x-2)(x+2)}{x-2}=\lim_{x\to 2^-}(x+2)=4$

$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}\left(ax^2-bx+3\right)=4a-2b+3.$

We must have $4a-2b+3=4$ or $4a-2b=1$ (1)

At $x=3$: $\lim_{x\to 2^-} f(x)=\lim_{x\to 3^-}\left(ax^2-bx+3\right)=9a-3b+3$

$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}\left(2x-a+b\right)=6-a+b.$

We must have $9a-3b+3=6-a+b$ or $10a-4b=3$ (2).

Now solve the system of equations by addition –2 times equation (1) to equation (2):

$$\begin{array}{r} -8a+4b=-2 \\ 10a-4b=3 \\ \hline 2a\quad\;\;=1 \end{array} \quad \text{so } a=\tfrac{1}{2}.$$

Substituting $a=\frac{1}{2}$ for a in (1) gives us $-2b=-1$, so $b=\frac{1}{2}$ as well. Thus, for f to be continuous on $(-\infty,\infty)$, $a=b=\frac{1}{2}$.

59. The function $f(x)=\begin{cases} \dfrac{3+4^{1/x}}{5+4^{1/x}} & \text{if } x\ne 0 \\ k & \text{if } x=0 \end{cases}$ cannot be made continuous at $x=0$ because

$$\lim_{x\to 0^-} f(x)=\lim_{t\to -\infty}\frac{3+4^t}{5+4^t}=\frac{3}{5}\ne 1=\lim_{t\to\infty}\frac{4^t}{4^t}=\lim_{t\to\infty}\frac{3+4^t}{5+4^t}=\lim_{x\to 0^+} f(x).$$

60. (a) Let $f(x)=\dfrac{2x^2-5x=3}{x^2-1}=\dfrac{(x-1)(2x-3)}{(x-1)(x+1)}=\dfrac{2x-3}{x+1}=g(x)$ for $x\ne 1$. But $g(1)=\dfrac{2(1)-3}{1+1}=\dfrac{-1}{2}=-\dfrac{1}{2}$. Observe that f is not defined at $x=1$, but if we define $f(1)=-\dfrac{1}{2}$, the function will be continuous at $x=1$ because $g(x)$ is continuous at $x=1$.

(b) Observe that $f(x)=\dfrac{2x^2-5x=3}{x^2-1}=\dfrac{(x-1)(2x-3)}{(x-1)(x+1)}$ has a vertical asymptote at $x=-1$ so it cannot be made continuous there.