LESSON 10-4

Coordinate Proof Using Distance with Quadrilaterals

Practice and Problem Solving: A/B

1. The vertices of a quadrilateral are given by the coordinates $W(3, 5)$, $X(5, 0)$, $Y(-3, -4)$, and $Z(-5, 1)$. Is the quadrilateral a parallelogram? a trapezoid? Explain your reasoning.

slope $\overline{WX} = \frac{5-0}{3-5} = -\frac{5}{2}$
slope $\overline{YZ} = \frac{-4-1}{-3-(-5)} = -\frac{5}{2}$ } parallel

slope $\overline{XY} = \frac{-4-0}{-3-5} = \frac{-4}{-8} = \frac{1}{2}$
slope $\overline{WZ} = \frac{5-1}{3-(-5)} = \frac{4}{8} = \frac{1}{2}$ } parallel

parallelogram

2. Two friends see a drawing of quadrilateral *PQRS* with vertices *P*(0, 2), *Q*(3, −4), *R*(1, −5), and *S*(−2, 1). Joe says the quadrilateral is a parallelogram but not a rectangle. Carl says the quadrilateral is a rectangle. Who is correct? Show the work that supports your answer.

slope $\overline{PQ} = \frac{2-(-4)}{0-3} = \frac{6}{-3} = -2$
slope $\overline{RS} = \frac{-5-1}{1-(-2)} = \frac{-6}{3} = -2$

slope $\overline{QR} = \frac{-4-(-5)}{3-1} = \frac{1}{2}$
slope $\overline{PS} = \frac{2-1}{0-(-2)} = \frac{1}{2}$

-2 & $\frac{1}{2}$ are perp., so rectangle → Carl is right

For Problems 3-6, consider quadrilateral ABCD with vertices A(2,4), B(4,-1), C(-1,-3), and D(-3,2).

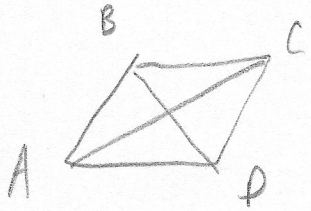
3. Show that ABCD is a parallelogram. Include the work that leads to your conclusion.

slope $\overline{AB} = \frac{4-(-1)}{2-4} = -\frac{5}{2}$
slope $\overline{CD} = \frac{2-(-3)}{-3-(-1)} = -\frac{5}{2}$ } parallel

slope $\overline{BC} = \frac{-1-(-3)}{4-(-1)} = \frac{2}{5}$
slope $\overline{AD} = \frac{4-2}{2-(-3)} = \frac{2}{5}$ } parallel

so parallelogram

4. Show that ABCD is a rectangle. Include the work that leads to your conclusion.



$-\frac{5}{2}$ & $\frac{2}{5}$ are perp., so rectangle

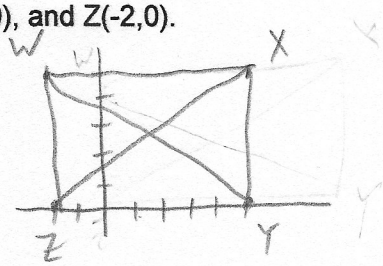
5. Show that ABCD is a rhombus. Include the work that leads to your conclusion.

slope $\overline{AC} = \frac{4-(-3)}{2-(-1)} = \frac{7}{3}$
slope $\overline{BD} = \frac{2-(-1)}{-3-4} = \frac{3}{-7}$ } perp. diagonals, so rhomb.

6. Explain why ABCD must be a square.

if rhomb. & rect, then square

For Problems 7-10, consider quadrilateral WXYZ with vertices W(-2,5), X(5,5), Y(5,0), and Z(-2,0).



7. Are the diagonals congruent? Include the work that leads to your conclusion.

WY: $5^2+7^2=x^2$, $25+49=x^2$, $74=x^2$, $x=\sqrt{74}$

XZ: $5^2+7^2=x^2$, $25+49=x^2$, $74=x^2$, $x=\sqrt{74}$

yes, so rectangle

8. Are the diagonals perpendicular? Include the work that leads to your conclusion.

slope $\overline{WY} = \frac{-5}{7}$

slope $\overline{XZ} = \frac{5}{7}$

not perp., so not rhombus

9. Do the diagonals bisect each other? Include the work that leads to your conclusion.

midpt $\overline{WY} = \left(\frac{-2+5}{2}, \frac{5+0}{2}\right) = (1.5, 2.5)$

midpt $\overline{XZ} = \left(\frac{5+-2}{2}, \frac{5+0}{2}\right) = (1.5, 2.5)$

} same, so diag. bisect each other $\Rightarrow$ parallelogram

10. Is WXYZ a square? Explain your reasoning. no, not a rhombus

slope $\overline{AC} = -\frac{1}{7}$
slope $\overline{BD} = 7$
rhombus

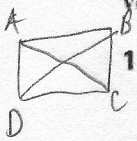
$AC = \sqrt{2^2+(-14)^2} = \sqrt{200}$
$BD = \sqrt{14^2+2^2} = \sqrt{200}$
rectangle

square

slope $\overline{JL} = \frac{1}{3}$
slope $\overline{KM} = -3$
rhomb

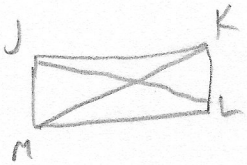
$JL = \sqrt{4^2+12^2} = \sqrt{160}$
$KM = \sqrt{6^2+2^2} = \sqrt{40}$
not rect.

Algebra Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.



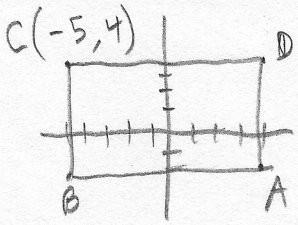
11. $A(-10, 4)$, $B(-2, 10)$, $C(4, 2)$, $D(-4, -4)$

12. $J(-9, -7)$, $K(-4, -2)$, $L(3, -3)$, $M(-2, -8)$

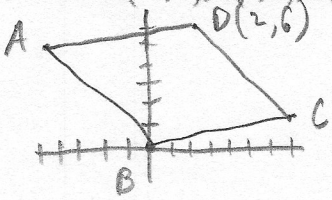


Analyze Relationships The coordinates of three vertices of parallelogram $ABCD$ are given. Find the coordinates of the fourth point so that the given type of figure is formed.

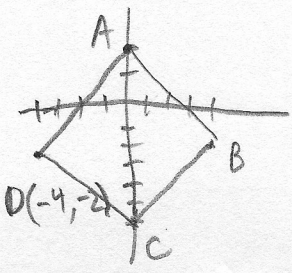
13. $A(4, -2)$, $B(-5, -2)$, $D(4, 4)$, rectangle



14. $A(-5, 5)$, $B(0, 0)$, $C(7, 1)$, rhombus



15. $A(0, 2)$, $B(4, -2)$, $C(0, -6)$, square



16. $A(2, 1)$, $B(-1, 5)$, $C(-5, 2)$, square