Similarity

Similar figures have the same shape but not the same size

- $\sim$ is the symbol for similar
- In $\sim$ figures corresponding angles are congruent and corresponding sides are proportional.

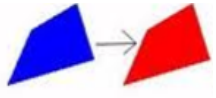
Angles are equal

Side fractions are equal

→ scale factor = $\frac{\text{image}}{\text{preimage}}$

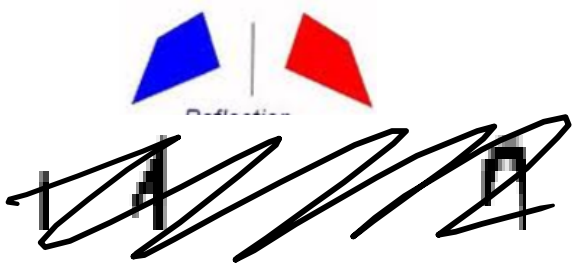
A similarity transformation is one in which the image has the **same shape** as it's pre-image

Translation: A shift

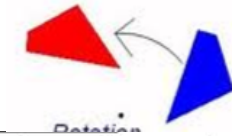


$(x, y) \rightarrow (x + 1, y - 2)$

Reflection: A mirror image

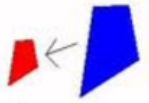


Rotation: Turn around a point



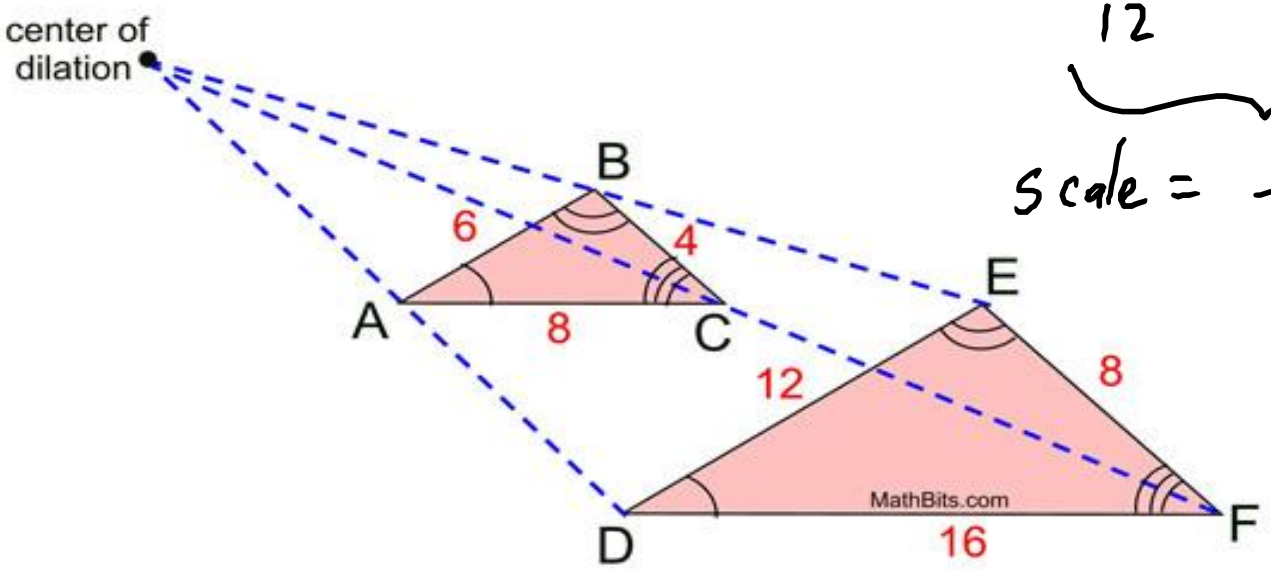
$90°: (x, y) \rightarrow (-y, x)$
$180°: (x, y) \rightarrow (-x, -y)$
$270°: (x, y) \rightarrow (y, -x)$

Dilation: Make bigger or smaller



$\frac{6}{12}$ $\frac{4}{8}$ $\frac{8}{16}$

scale = $\frac{1}{2}$



The triangles are similar. $\Delta ABC \sim \Delta DEF$

ΔDEF is a dilation of ΔABC by a scale factor of 2.

Coordinate Notation for a Dilation

(with the origin as the center)

$$(x, y) \rightarrow (kx, ky)$$

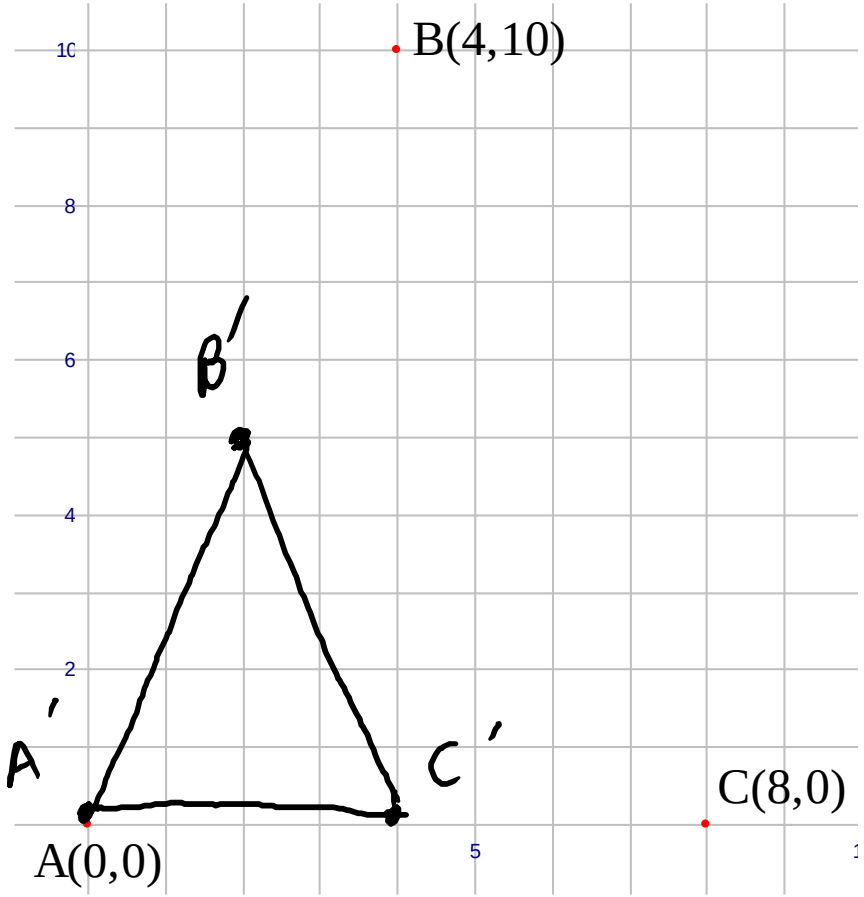
k is the scale factor

Example:

A dilation with scale factor 4 that is centered at the origin can be expressed as: $(x, y) \rightarrow (4x, 4y)$

Example:

Dilate the triangle with a scale factor of $\frac{1}{2}$.



$(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

$A(0,0) \rightarrow A'(0,0)$

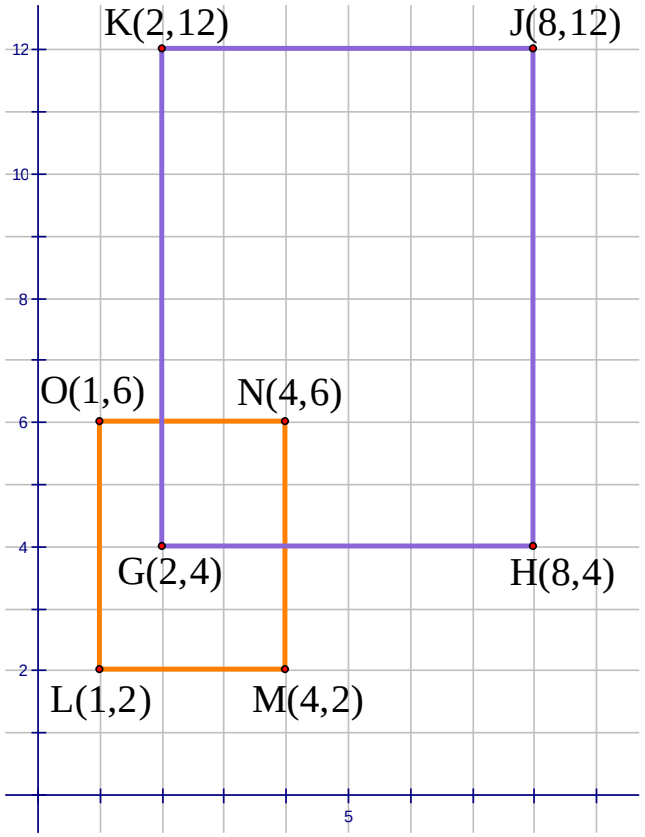
$B(4,10) \rightarrow B'(2,5)$

$C(8,0) \rightarrow C'(4,0)$

Figures are similar if you can map one onto the other using one or more similarity or congruence transformation.

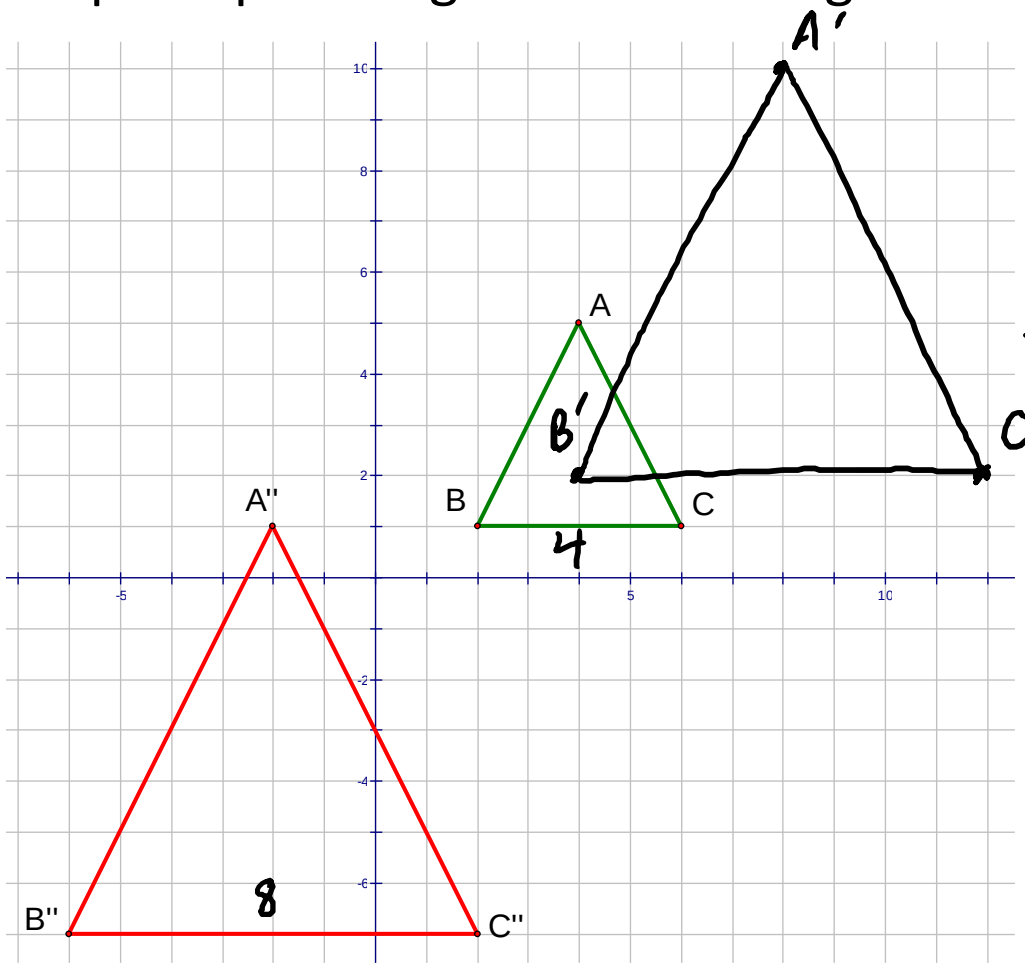
Ex. Determine whether the two figures are similar using similarity transformations.

$(x, y) \rightarrow (2x, 2y)$



Give the coordinate notation for the sequence of similarity transformations that will map the pre-image onto the image.

a)



Dilate: scale factor $= \frac{8}{4} = 2$

$(x, y) \rightarrow (2x, 2y)$

Translate: left 10, down 9

$(x, y) \rightarrow (x-10, y-9)$

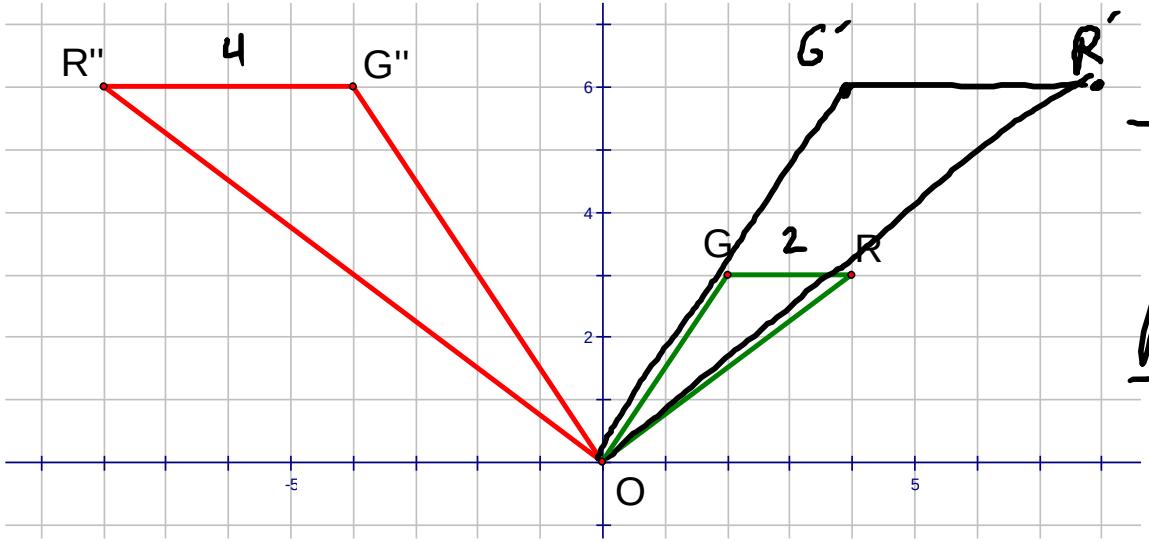
$(x, y) \rightarrow (2x, 2y)$

$A(4, 5) \rightarrow A'(8, 10)$

$B(2, 1) \rightarrow B'(4, 2)$

$C(6, 1) \rightarrow C'(12, 2)$

b)



Dilate: scale factor $= \frac{4}{2} = 2$

$(x, y) \rightarrow (2x, 2y)$

Reflect: over y-axis

$(x, y) \rightarrow (-x, y)$

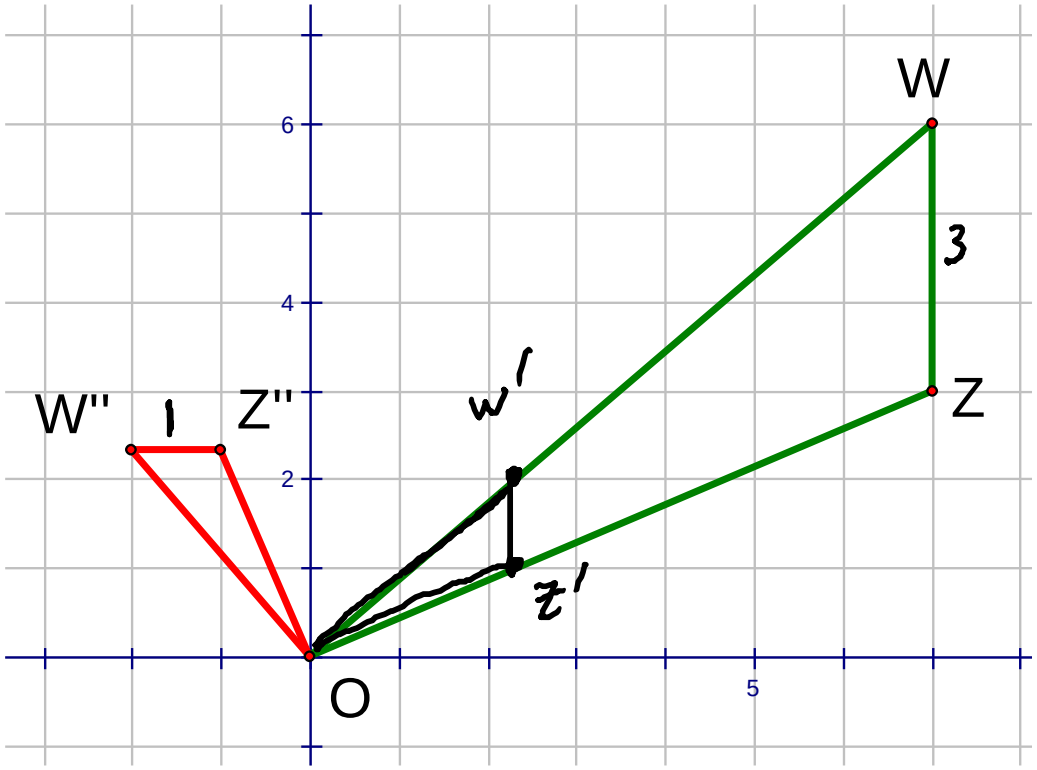
$(x, y) \rightarrow (2x, 2y)$

$G(2, 3) \rightarrow G'(4, 6)$

$R(4, 3) \rightarrow R'(8, 6)$

$O(0, 0) \rightarrow O'(0, 0)$

c)



Dilate: scale factor = $\frac{1}{3}$

$(x, y) \rightarrow (\frac{1}{3}x, \frac{1}{3}y)$

Rotate: 90° CCW

$(x, y) \rightarrow (-y, x)$

$(x, y) \rightarrow (\frac{1}{3}x, \frac{1}{3}y)$

$W(7,6) \rightarrow W'(\frac{7}{3}, 2)$

$Z(7,3) \rightarrow Z'(\frac{7}{3}, 1)$

$O(0,0) \rightarrow O'(0,0)$