

Geometric Mean & Similarity in Right Triangles

Def. The geometric mean between a and b is the number x where

$$\frac{x}{a} = \frac{b}{x}$$

Ex. Find the geometric mean between 14 and 10.

$$\frac{14}{x} = \frac{x}{10} \quad \sqrt{x^2} = \sqrt{140}$$
$$x = \sqrt{140}$$

Ex. If 15 is the geometric mean between 9 and x , find x .

$$\frac{9}{15} = \frac{15}{x} \quad 9x = 225$$
$$x = 25$$

Find the geometric mean.

6. 6 and 24

$$\frac{x}{6} \begin{array}{l} \nearrow 24 \\ \searrow x \end{array}$$

$$\sqrt{x^2} = \sqrt{144}$$

$$x = 12$$

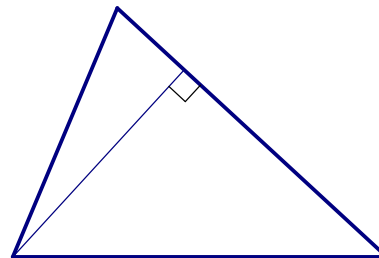
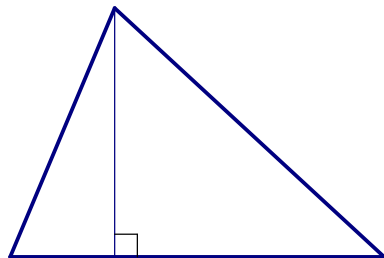
7. 5 and 12

$$\frac{x}{5} \begin{array}{l} \nearrow 12 \\ \searrow x \end{array}$$

$$\sqrt{x^2} = \sqrt{60}$$

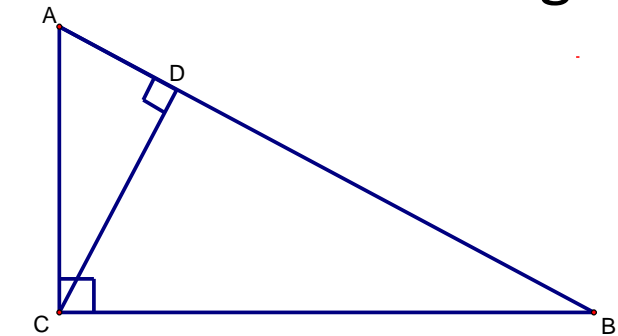
$$x = \sqrt{60}$$

Def. An altitude of a triangle is a line segment from a vertex that is perpendicular to the opposite side.

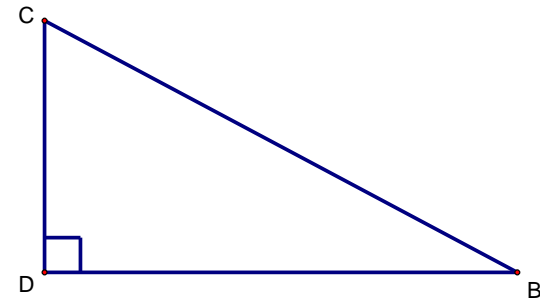
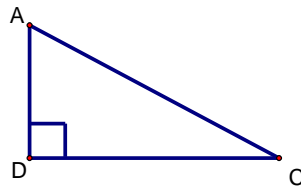
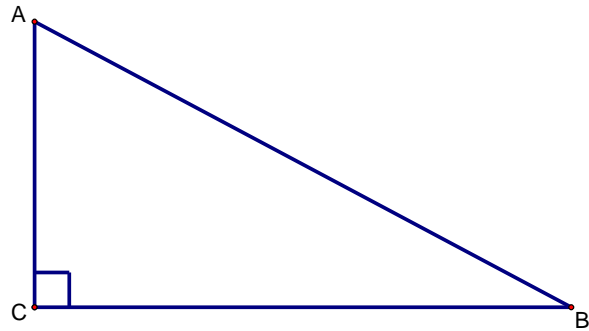


Similar Right Triangles.

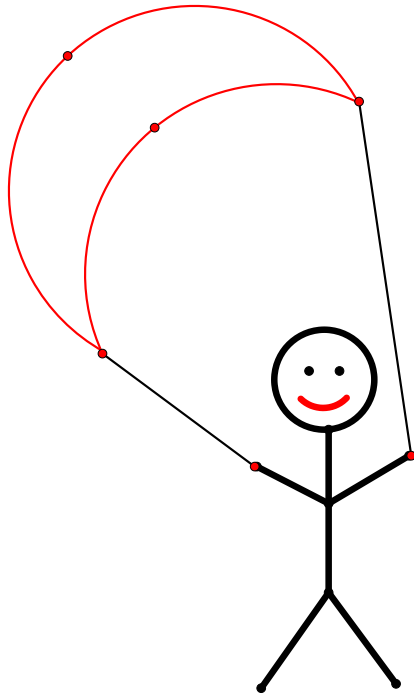
If the **altitude** is drawn from the vertex of a right triangle to its hypotenuse, then the two triangles are **similar** to the given triangle and to each other.



$$\triangle ACB \sim \triangle ADC \sim \triangle CDB$$

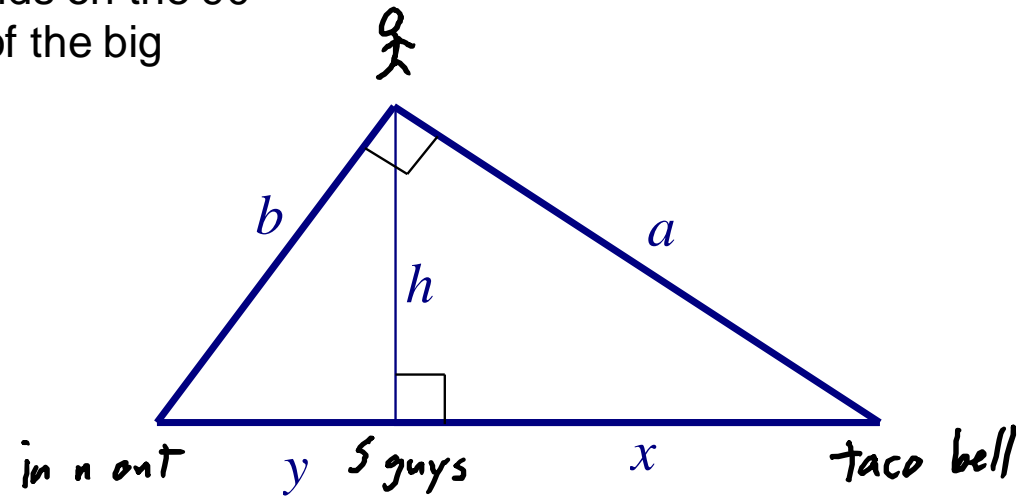


Meet Bob...



Bob is a hungry skydiver who has just landed on top of a mountain and is looking for a “means” down.

Bob always lands on the 90 degree angle of the big triangle.



$$\frac{x}{h} = \frac{h}{y}$$

$$\frac{x}{a} = \frac{a}{x+y}$$

$$\frac{y}{b} = \frac{b}{x+y}$$

Ex. Find x .



6

x

3

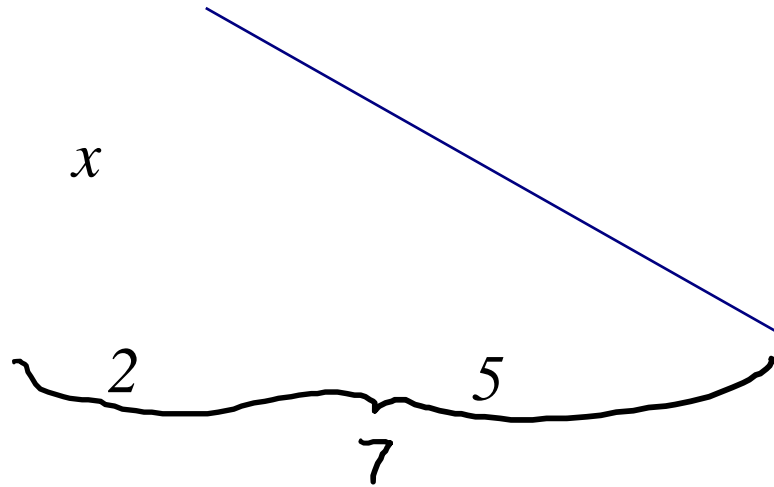
$$\frac{x}{3} \rightarrow \frac{6}{x}$$

The diagram shows a fraction $\frac{x}{3}$ on the left and $\frac{6}{x}$ on the right. A horizontal line connects the two fractions. Two arrows originate from the horizontal line: one points from the x in the denominator of the first fraction to the 6 in the numerator of the second fraction, and the other points from the 3 in the denominator of the first fraction to the x in the denominator of the second fraction. The entire diagram is crossed out with a large 'X'.

$$\sqrt{x^2} = \sqrt{18}$$

$$x = \sqrt{18}$$

Ex. Find x .



$$\frac{x}{2} \quad \begin{array}{l} \nearrow 7 \\ \searrow x \end{array}$$

$$\sqrt{x^2} = \sqrt{14}$$

$$x = \sqrt{14}$$

Ex. Find a and b .

$$\frac{10}{4} \xrightarrow{\text{cancel}} \frac{a}{10}$$

$$4a = 100$$

$$a = 25$$

$$\frac{b}{4} \xrightarrow{\text{cancel}} \frac{29}{b}$$

$$\sqrt{b^2} = \sqrt{116}$$

$$b = \sqrt{116}$$

