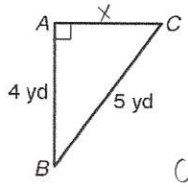
LESSON 13-4

Problem Solving with Trigonometry

Practice and Problem Solving: A/B

Find the following. Round lengths to the nearest hundredth and angle measures to the nearest degree.

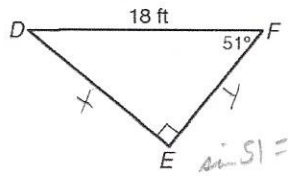
1. 

$x^2+4^2=5^2$
$x=3$
$\cos B = \frac{4}{5}$
$B=37$
$37+C=90$
$C=53$

$AC = 3$

$m\angle B = 37°$

$m\angle C = 53°$

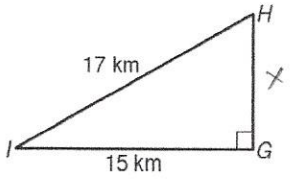
2. 

$\sin 51 = \frac{x}{18}$
$x = 13,99$
$\cos 51 = \frac{y}{18}$
$y = 11,33$
$51+D=90$
$D=39$

$DE = 13,99$

$EF = 11,33$

$m\angle D = 39$

3. 

$15^2+x^2=17^2$
$x=8$
$\sin H = \frac{15}{17}$
$H=62$
$I+62=90$
$I=28$

$GH = 8$

$m\angle H = 62$

$m\angle I = 28$

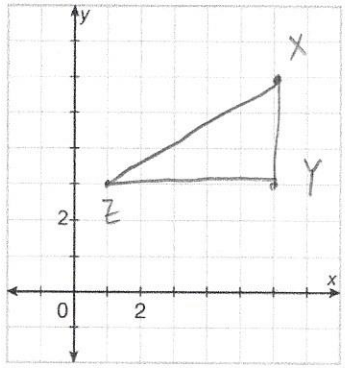
$\triangle XYZ$ has vertices $X(6, 6)$, $Y(6, 3)$, and $Z(1, 3)$. Complete Problems 4–6 to find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

4. Plot the points and draw $\triangle XYZ$

5. Find XY and YZ from the graph. Use the Pythagorean Theorem to find XZ.

$3^2+5^2=x^2$
$x=5,83$

$XY = 3$ $YZ = 5$ $XZ = 5,83$



6. Find the angle measures.

$m\angle X$ $\tan X = \frac{5}{3}$ $X = 59$

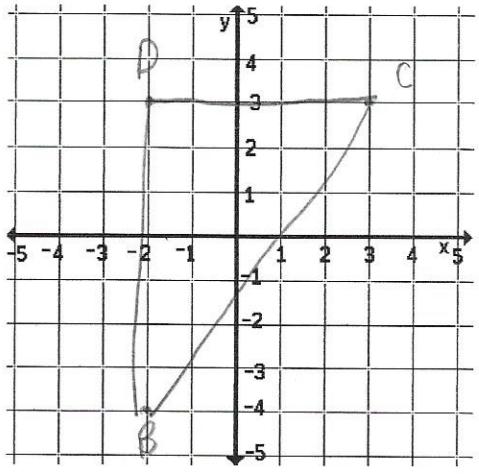
$m\angle Z = 90-59 = 31$

For each triangle, find all three side lengths to the nearest hundredth and all three angle measures to the nearest degree.

7. $B(-2, -4)$, $C(3, 3)$, $D(-2, 3)$

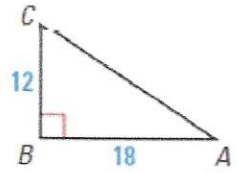
$CD = 5$
$DB = 7$
$CB = 8.60$

$5^2+7^2=x^2$
$x=8.60$
$\tan B = \frac{5}{7}$
$B=36$
$C=90-36=54$



INVERSE TANGENTS Use a calculator to approximate the measure of A to the nearest tenth of a degree.

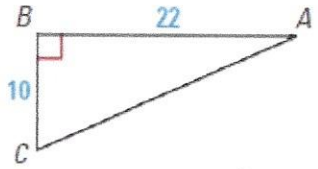
8.



$\tan A = \frac{12}{18}$

$A = 33.7$

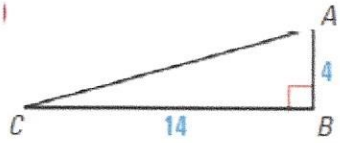
9.



$\tan A = \frac{10}{22}$

$A = 24.4$

10.

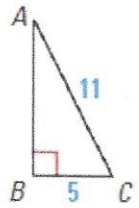


$\tan A = \frac{14}{4}$

$A = 74.1$

INVERSE SINES AND COSINES Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

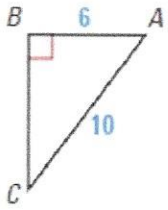
11.



$\sin A = \frac{5}{11}$

$A = 27.0$

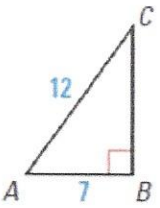
12.



$\cos A = \frac{6}{10}$

$A = 53.1$

13.

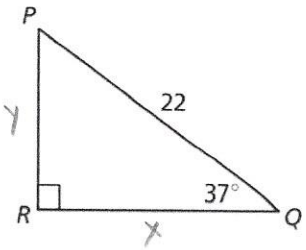


$\cos A = \frac{7}{12}$

$A = 54.3$

In Exercises 14 – 24, find all unknown sides and angles. Round decimal answers to the nearest tenth.

14.

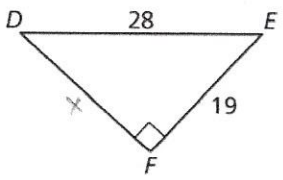


$\cos 37 = \frac{x}{22}$ $\quad \sin 37 = \frac{y}{22}$

$x = 17.6$ $\quad y = 13.2$

$P = 90 - 37 = 53$

15.



$x^2 + 19^2 = 28^2$

$x = 20.6$

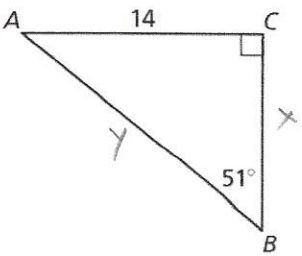
$\sin D = \frac{19}{28}$

$D = 42.7$

$E = 90 - 42.7$

$E = 47.3$

16.



$\sin 51 = \frac{14}{y}$

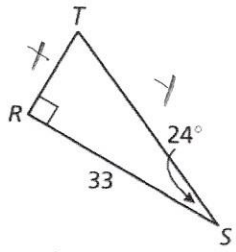
$y = 18.0$

$\tan 51 = \frac{14}{x}$

$x = 11.3$

$A = 90 - 51 = 39$

17.

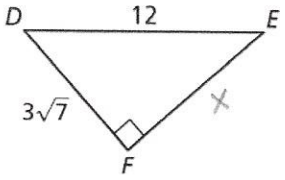


$\tan 24 = \frac{x}{33}$ $\cos 24 = \frac{33}{y}$

$x = 14.7$ $y = 36.1$

$T = 90 - 24 = 66$

18.



$x^2 + (3\sqrt{7})^2 = 12^2$

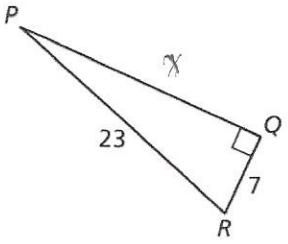
$x = 9$

$\sin E = \frac{3\sqrt{7}}{12}$

$E = 41.4$

$D = 90 - 41.4 = 48.6$

19.



$x^2 + 7^2 = 23^2$

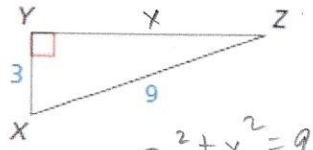
$x = 21.9$

$\cos R = \frac{7}{23}$

$R = 72.3$

$P = 90 - 72.3 = 17.7$

20.



$3^2 + x^2 = 9^2$

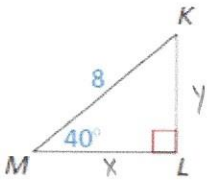
$x = 8.5$

$\cos X = \frac{3}{9}$

$X = 70.5$

$Y = 90 - 70.5 = 19.5$

21.



$K = 90 - 40 = 50$

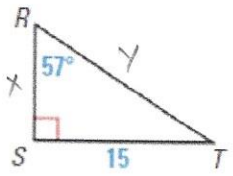
$\sin 40 = \frac{y}{8}$

$y = 5.1$

$\cos 40 = \frac{x}{8}$

$x = 6.1$

22.



$T = 90 - 57 = 33$

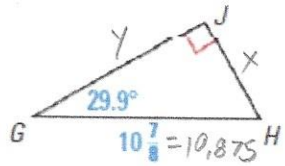
$\sin 57 = \frac{15}{y}$

$y = 17.9$

$\tan 57 = \frac{15}{x}$

$x = 9.7$

23.



$H = 90 - 29.9 = 60.1$

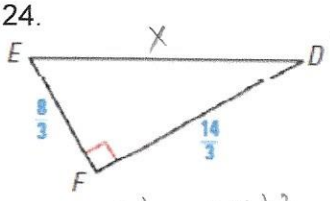
$\sin 29.9 = \frac{x}{10.875}$

$x = 5.4$

$\cos 29.9 = \frac{y}{10.875}$

$y = 9.4$

24.



$\left(\frac{8}{3}\right)^2 + \left(\frac{14}{3}\right)^2 = x^2$

$x = 5.4$

$\tan D = \frac{8/3}{14/3}$

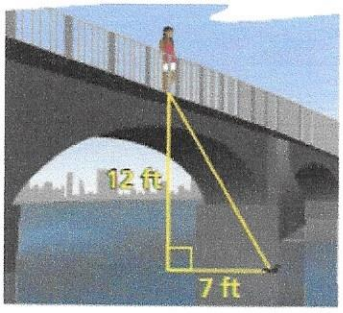
$D = 29.7$

$E = 90 - 29.7 = 60.3$

26. You are standing on a footbridge that is 12 feet above a lake. You look down and see a duck in the water. The duck is 7 feet away from the footbridge. What is the angle of elevation from the duck to you?

$\tan x = \frac{12}{7}$

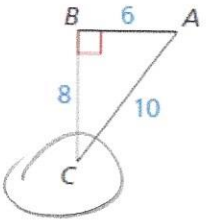
$x = 59.7$



In Exercises 27 - 28, determine which of the two acute angles has the given trigonometric ratio.

27.

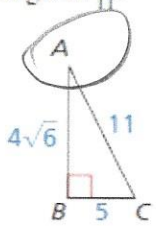
The cosine of the angle is $\frac{4}{5}$.



$\cos C = \frac{8}{10} = \frac{4}{5}$

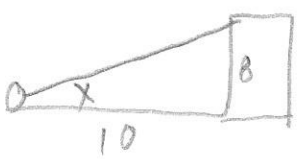
28.

The sine of the angle is $\frac{5}{11}$.



$\sin A = \frac{5}{11}$

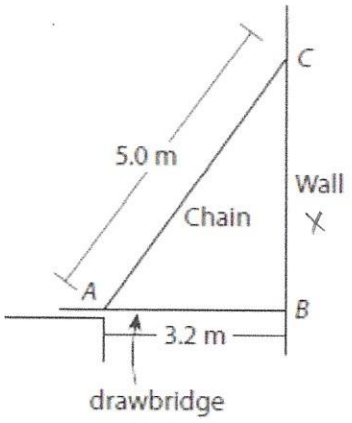
29. A soccer ball is placed 10 feet away from the goal, which is 8 feet high. You kick the ball and it hits the crossbar along the top of the goal. What is the angle of elevation of your kick?



$\tan x = \frac{8}{10}$

$A = 38.7$

10. **History** A drawbridge at the entrance to an ancient castle is raised and lowered by a pair of chains. The figure represents the drawbridge when flat. Find the height of the suspension point of the chain, to the nearest tenth of a meter, and the measures of the acute angles the chain makes with the wall and the drawbridge, to the nearest degree.



$x^2 + 3.2^2 = 5.0^2$

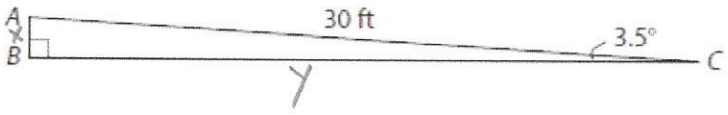
$x = 3.8$

$\cos A = \frac{3.2}{5.0}$

$A = 50$

$C = 90 - 50 = 40$

11. **Building** For safety, the angle a wheelchair ramp makes with the horizontal should be no more than 3.5°. What is the maximum height of a ramp of length 30 ft? What distance along the ground would this ramp cover? Round to the nearest tenth of a foot.



$\sin 3.5 = \frac{x}{30}$

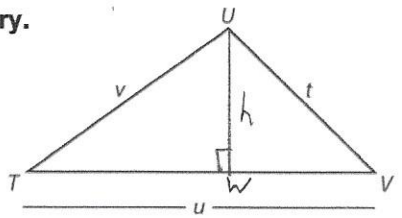
$x = 1.8$

$\cos 3.5 = \frac{y}{30}$

$y = 29.9$

Follow the steps to find the area of the triangle using trigonometry.

33. Draw a line from vertex U perpendicular to the base $\overline{TV}$ at a point W. Label its length h. Write the sine of $\angle T$ as a ratio using variables in the figure. Solve for h. Then write the area of the triangle using your value for h.



$\sin T = \frac{h}{v}$ $h = v \sin T$ $Area = \frac{1}{2}(u)(v \sin T)$

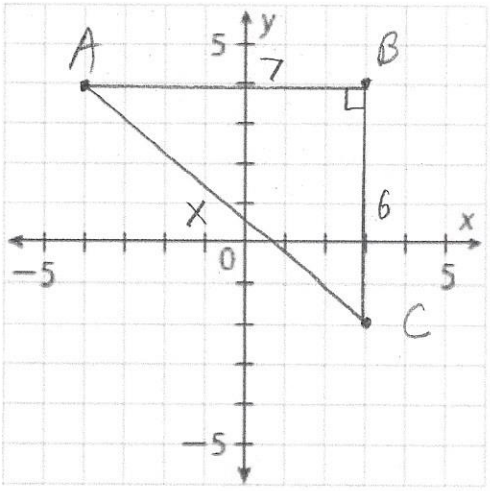
Area of a triangle = $\frac{1}{2}$ base × height

34. What is the area of the triangle if $\angle T = 37°$, $u = 14$, and $v = 10$?

$A = \frac{1}{2}(14)(10)\sin 37 = 42.1$

Solve each triangle. Find the side lengths to the nearest hundredth and the angle measures to the nearest degree.

6. Triangle ABC with vertices $A(-4, 4)$, $B(3, 4)$, and $C(3, -2)$



$AB = 7$
$BC = 6$
$AC = 9.22$

$m\angle A = 41$
$m\angle C = 49$
$m\angle B = 90$

$7^2 + 6^2 = x^2$
$85 = x^2$
$x = \sqrt{85} = 9.22$

$\tan A = \frac{6}{7}$
$A = 41$

$41 + 90 + B = 180$
$B = 49$