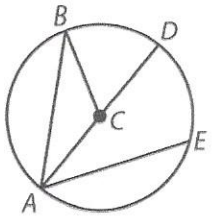
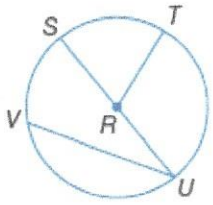
15-1 Central Angles and Inscribed Angles

Refer to the figure for Problems 1–3. *C* is the center of the circle.



1. Name the chord(s). $\overline{AB}$, $\overline{AD}$, $\overline{AE}$
2. Name the central angle(s). $\angle BCD$, $\angle ACB$
3. Name the inscribed angle(s). $\angle BAD$, $\angle EAD$, $\angle BAE$

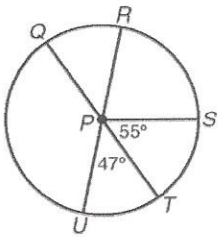
For Exercises 4 – 6, refer to $\odot R$.



4. Name the center of the circle. R
5. Identify a chord that is also a diameter. $\overline{SU}$
6. Is $\overline{VU}$ a radius? Explain? no, an endpoint is not the center

For each figure, determine the indicated measures.

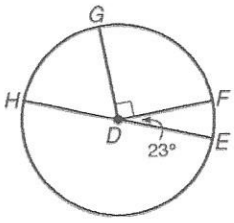
7.



$m\overset{\frown}{QS} = 180 - 55 = 125$

$m\overset{\frown}{RQT} = 180 + 47 = 227$

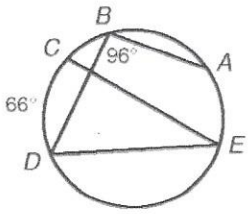
8.



$m\overset{\frown}{HG} = 180 - 90 - 23 = 67$

$m\overset{\frown}{FEH} = 180 + 23 = 203$

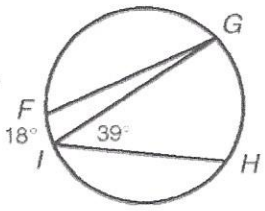
9.



$m\angle CED = \frac{1}{2}(66) = 33$

$m\overset{\frown}{DEA} = 2(96) = 192$

10.

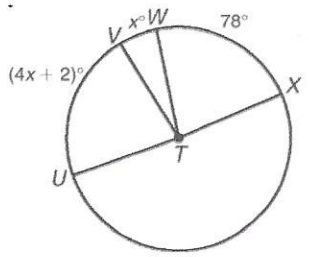


$m\angle FGI = \frac{1}{2}(18) = 9$

$m\overset{\frown}{GH} = 2(39) = 78$

Find the unknown value.

11. $x =$



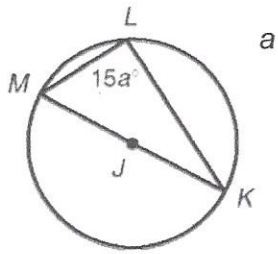
$4x+2+x+78=180$

$5x+80=180$

$5x=100$

$x=20$

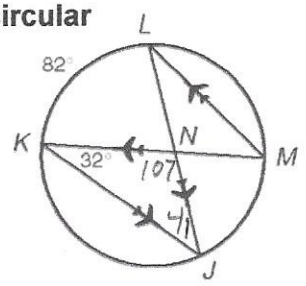
12. $a =$



$15a=90$

$a=6$

The figure shows a passenger airplane's flight path on a circular radar screen in an air traffic control tower.



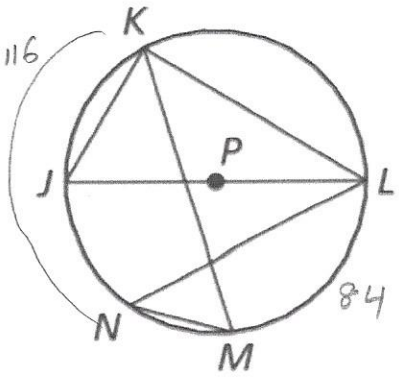
$180-32-41=107$

13. What is $m\overset{\frown}{MJ}$? $2(32)=64$

14. What is $m\angle LJK$? $\frac{1}{2}(82)=41$

15. What is $m\angle LNK$? $180-107=73$

In Exercises 16–23, find the measure of the indicated arc or angle in $\odot P$ given $m\overset{\frown}{LM}$ = 84° and $m\overset{\frown}{KN}$ = 116°.



16. $m\angle JKL = \frac{1}{2}(180)=90$

17. $m\angle MKL$ $\frac{1}{2}(84)=42$

18. $m\angle KMN = \frac{1}{2}(116)=58$

19. $m\angle JKM$ $\frac{1}{2}(96)=48$

20. $m\angle KLN = \frac{1}{2}(116)=58$

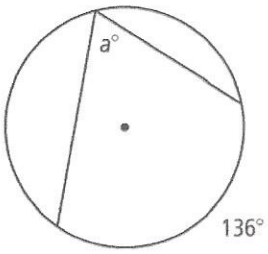
21. $m\angle LNM$ $\frac{1}{2}(84)=42$

22. $m\overset{\frown}{MJ} = 180-84=96$

23. $m\overset{\frown}{LKJ} = 180$

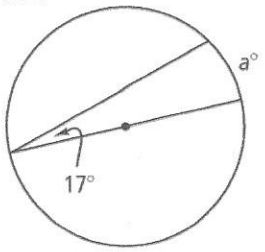
Find the value of each variable. For each circle, the dot represents the center.

24.



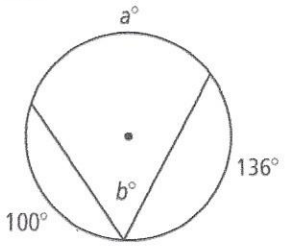
$a = \frac{1}{2}(136) = 68$

25.



$a = 2(17) = 34$

26.

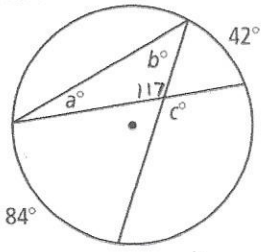


$a = 360 - 100 - 136 = 124$

$b = \frac{1}{2}(124) = 62$

180
−21
−42
117

27.

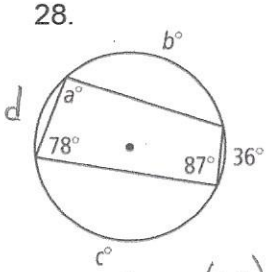


$a = \frac{1}{2}(42) = 21$

$b = \frac{1}{2}(84) = 42$

$c = 117$

28.



$b + 36 = 2(78)$

$b = 120$

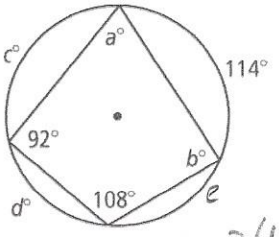
$d + 120 = 2(87)$

$d = 54$

$c = 360 - 120 - 36 - 54 = 150$

$a = \frac{1}{2}(150) = 75$

29.



$114 + c = 2(108) \rightarrow c = 102$

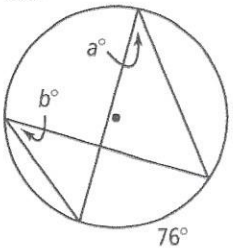
$114 + e = 2(92) \rightarrow e = 70$

$d = 360 - 102 - 114 - 70 = 74$

$b = \frac{1}{2}(102 + 74) = 88$

$a = \frac{1}{2}(74 + 70) = 72$

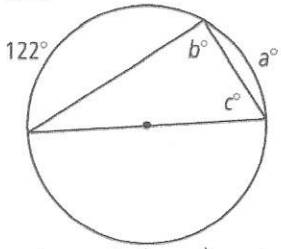
30.



$a = \frac{1}{2}(76) = 38$

$b = \frac{1}{2}(76) = 38$

31.

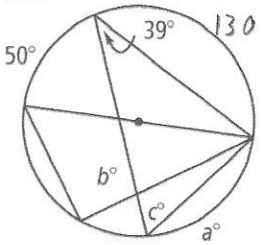


$b = \frac{1}{2}(180) = 90$

$c = \frac{1}{2}(122) = 61$

$a = 180 - 122 = 58$

32.

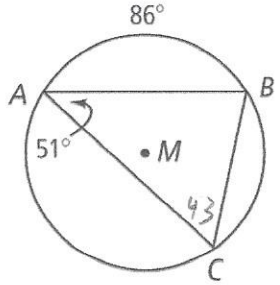


$a = 2(39) = 78$

$b = \frac{1}{2}(180) = 90$

$c = \frac{1}{2}(130) = 65$

Find each indicated measure for ⊙*M*



33. a. $m\angle B$ = 180 − 51 − 43 = 86

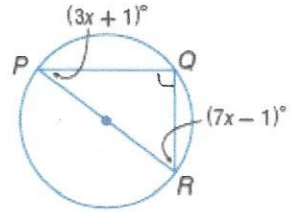
b. $m\angle C = \frac{1}{2}(86) = 43$

c. $m\overset{\frown}{BC}$ = 2(51) = 102

d. $m\overset{\frown}{AC} = 2(86) = 172$

Find each value.

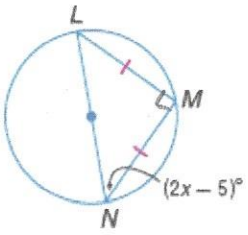
34. $m\angle R$



$3x+1+7x-1+90=180$
$10x+90=180$
$10x=90$
$x=9$

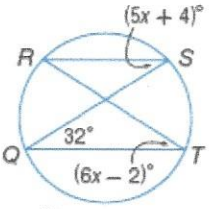
$m\angle R = 7(9)-1 = 62$

35. x



$2x-5+2x-5+90=180$
$4x+80=180$
$4x=100$
$x=25$

36. $m\angle R$, $m\angle S$

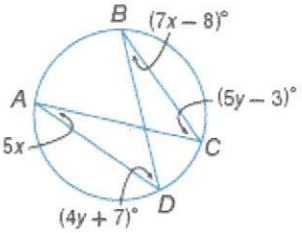


$5x+4=6x-2$
$6=x$

$m\angle R = 32$

$m\angle S = 5(6)+4$
$= 34$

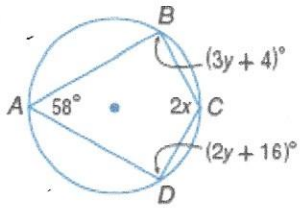
37. $m\angle A$, $m\angle C$



$7x-8=5x$
$-8=-2x$
$x=4$
$m\angle A = 5(4)$
$= 20$

$4y+7=5y-3$
$10=y$
$m\angle C = 5(10)-3$
$= 47$

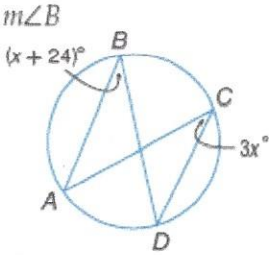
38. $m\angle C$ and $m\angle D$



$2y+16+3y+4=180$
$5y+20=180$
$5y=160$
$y=32$
$m\angle D = 2(32)+16 = 80$

$2x+58=180$
$2x=122$
$x=61$

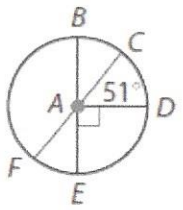
39. $m\angle B$



$x+24=3x$
$24=2x$
$x=12$

$m\angle B = 12+24 = 36$

The center of the circle is A. Find each measure using the appropriate theorems and postulates.

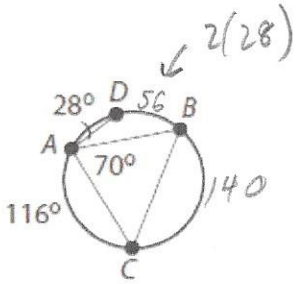


7. $m\widehat{CE} = 51 + 90 = 141$

8. $m\widehat{DF} = 180 - 51 = 129$

9. $m\widehat{BEC} = 180 + 90 + 51 = 321$

Find each measure using the appropriate theorems and postulates. $m\widehat{AC} = 116°$

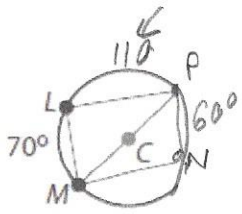


10. $m\widehat{BC} = 2(70) = 140$

11. $m\widehat{AD} = 360 - 56 - 140 - 116 = 48$

The center of the circle is C. Find each measure using the appropriate theorems and postulates. $m\widehat{LM} = 70°$ and $m\widehat{NP} = 60°$.

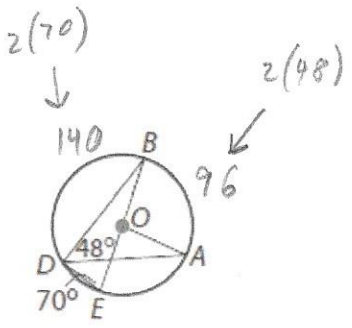
180-70



12. $m\angle MNP = \frac{1}{2}(180) = 90$

13. $m\angle LMN = 110 + 60 = 170$

The center of the circle is O. Find each arc or angle measure using the appropriate theorems and postulates.



14. $m\angle BDE = \frac{1}{2}(180) = 90$

15. $m\overset{\frown}{ABD}$
$= 140 + 96$
$= 236$

16. $m\overset{\frown}{ED} = 180 - 140 = 40$

17. $m\angle DBE = \frac{1}{2}(40) = 20$

Represent Real-World Problems The circle graph shows how a typical household spends money on energy. Use the graph to find the measure of each arc.

18. $m\overset{\frown}{PQ} = (.45)(360) = 162$

19. $m\overset{\frown}{UPT} = (.90)(360) = 324$