

## 15.2 and 15.3

Each quadrilateral described is inscribed in a circle. Determine the angle measures.

1. Quadrilateral  $ABCD$  has  $m\angle A = 53^\circ$  and  $m\angle B = 82^\circ$ .

$$m\angle C = \frac{180 - 53}{127}$$

$$m\angle D = \frac{180 - 82}{98}$$

2. Quadrilateral  $RSTU$  has  $m\angle S = 104^\circ$  and  $m\angle T = 55^\circ$ .

$$m\angle R = \frac{180 - 55}{125} \quad m\angle U = \frac{180 - 104}{76}$$

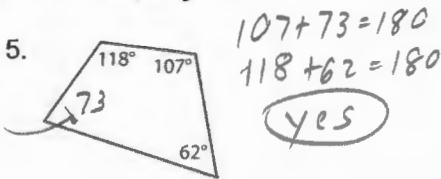
3. Quadrilateral  $JKLM$  has  $m\angle J = 90^\circ$  and  $\angle K \cong \angle M$ .

$$m\angle K = \frac{1}{2}(180) = 90 \quad m\angle L = 180 - 90 = 90 \quad m\angle M = \frac{1}{2}(180) = 90$$

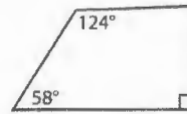
4. Quadrilateral  $RSTU$  has  $m\angle S = 35^\circ$  and  $m\angle T = 120^\circ$ .

$$m\angle R = \frac{180 - 120}{60} \quad m\angle U = \frac{180 - 35}{145}$$

Determine whether each quadrilateral can be inscribed in a circle. If it cannot be determined, say so.

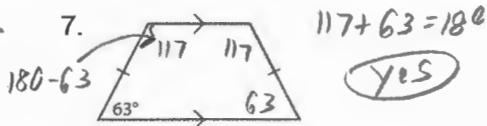


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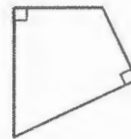


$$129 + 90 \neq 180$$

no



8.

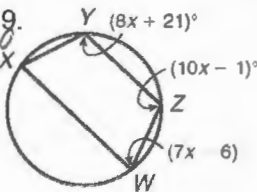


$$90 + 90 = 180$$

yes

For each inscribed quadrilateral, determine the angle measures.

$$\begin{aligned} 7x - 6 + 8x + 2 &= 180 \\ 15x + 15 &= 180 \\ 15x &= 165 \\ x &= 11 \end{aligned}$$

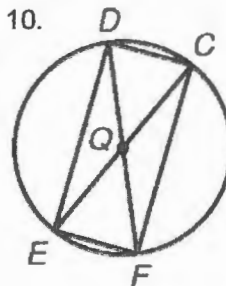


$$m\angle X = 180 - 109 = 71$$

$$m\angle Y = 8(11) + 21 = 109$$

$$m\angle Z = 10(11) - 1 = 109$$

$$m\angle W = 7(11) - 6 = 71$$

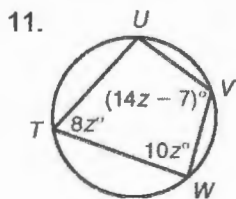


$$m\angle C = 90$$

$$m\angle D = 90$$

$$m\angle E = 90$$

$$m\angle F = 90$$

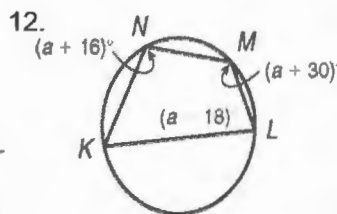


$$m\angle T = 8(8.5) = 68$$

$$m\angle U = 180 - 85 = 95$$

$$m\angle V = 14(8.5) - 7 = 117$$

$$m\angle W = 10(8.5) = 85$$



$$m\angle K = 180 - 121 = 59$$

$$m\angle L = 91 - 18 = 73$$

$$m\angle M = 91 + 30 = 121$$

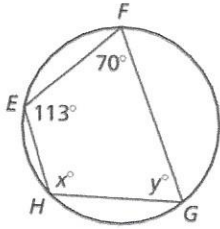
$$m\angle N = 91 + 16 = 107$$

$$\begin{aligned} 14z - 7 + 8z &= 180 \\ 22z &= 187 \\ z &= 8.5 \end{aligned}$$

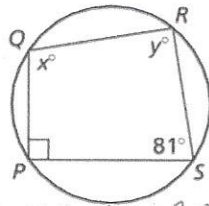
$$\begin{aligned} a + 16 + a - 18 &= 180 \\ 2a - 2 &= 180 \\ 2a &= 182 \\ a &= 91 \end{aligned}$$

In Exercises 13-15, find the value of each variable.

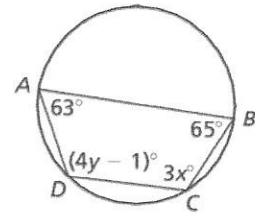
13.  
 $x = 180 - 70 = 110$   
 $y = 180 - 113 = 67$



14.  
 $y = 180 - 90 = 90$   
 $x = 180 - 81 = 99$

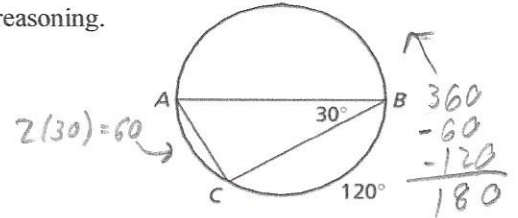


15.  
 $3x + 63 = 180$   
 $3x = 117$   
 $x = 39$   
 $4y - 1 + 65 = 180$   
 $4y + 64 = 180$   
 $4y = 116$   
 $y = 29$



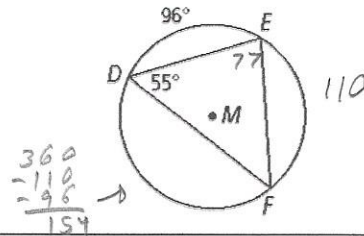
16. Determine whether  $\overline{AB}$  is a diameter of the circle. Explain your reasoning.

$180^\circ$  is a semicircle, so  $\overline{AB}$  is a diameter



Find each indicated measure for  $\odot M$ .

- 17. a.  $m\widehat{EF} = 2(55) = 110$
- b.  $m\angle E = \frac{1}{2}(154) = 77$
- c.  $m\angle F = \frac{1}{2}(96) = 48$
- d.  $m\widehat{DF} = 154$



Find the values of the variables.

18.  
 $x = 180 - 80 = 100$   
 $y = 180 - 95 = 85$

19.  
 $m = 180 - 60 = 120$   
 $2k + 60 = 180$   
 $2k = 120$   
 $k = 60$

20.  
 $3a = \frac{1}{2}(54 + 66)$   
 $3a = 60$   
 $a = 20$   
 $4b = \frac{1}{2}(66 + 110)$   
 $4b = 88$   
 $b = 22$

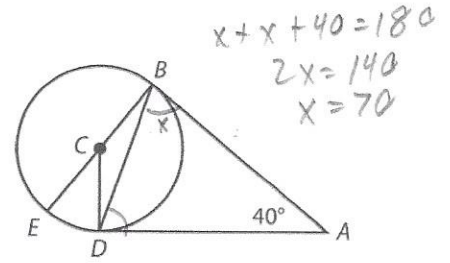
21.  
 $m\angle T = 180 - 45 = 135$   
 $m\angle Z = 2(25) + 30 = 80$   
 $2x + 30 + 4x = 180$   
 $6x = 150$   
 $x = 25$

22.  
 $m\angle H = 2(53) = 106$   
 $m\angle G = 4(26) - 11 = 93$   
 $x + 21 + 2x = 180$   
 $3x = 159$   
 $x = 53$   
 $3y + 9 + 4y - 11 = 180$   
 $7y - 2 = 180$   
 $7y = 182$   
 $y = 26$

23.  $m\angle D$  and  $m\angle B$   
 $21x - 8 + 23x + 12 = 180$   
 $44x + 4 = 180$   
 $44x = 176$   
 $x = 4$   
 $m\angle B = 21(4) - 8 = 76$   
 $m\angle D = 23(4) + 12 = 104$

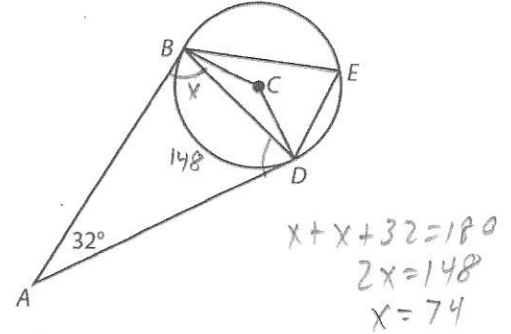
Refer to the figure for Problems 1-4.  $\overline{AB}$  is tangent to  $\odot C$  at point  $B$  and  $\overline{AD}$  is tangent to  $\odot C$  at point  $D$ . Determine the angle measures.

- $m\angle ABC = 90$
- $m\angle DCB = 180 - 40 = 140$
- $m\angle BDA = 70$
- $m\angle CDB = 90 - 70 = 20$



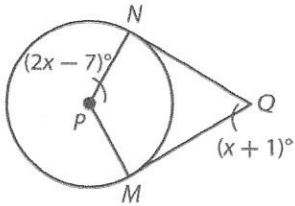
Refer to the figure for Problems 5-8.  $\overline{AB}$  is tangent to  $\odot C$  at point  $B$  and  $\overline{AD}$  is tangent to  $\odot C$  at point  $D$ . Determine the angle measures.

- $m\angle BCD = 180 - 32 = 148$
- $m\angle CDA = 90$
- $m\angle BED = \frac{1}{2}(148) = 74$
- $m\angle DBA = 74$



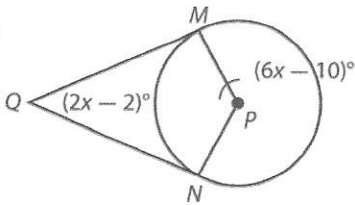
In Problems 9 and 10,  $\overline{QM}$  is tangent to  $\odot P$  at point  $M$  and  $\overline{QN}$  is tangent to  $\odot P$  at point  $N$ . Solve for the variable and determine the angle measures.

$$\begin{aligned} 2x - 7 + x + 1 &= 180 \\ 3x - 6 &= 180 \\ 3x &= 186 \\ x &= 62 \end{aligned}$$



- $$x = 62$$
- $$m\angle NQM = 62 + 1 = 63$$
- $$m\angle PNQ = 90$$
- $$m\angle NPM = 2(62) - 7 = 117$$

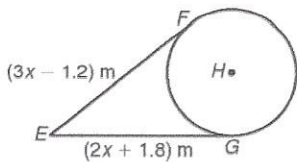
$$\begin{aligned} 2x - 2 + 6x - 10 &= 180 \\ 8x - 12 &= 180 \\ 8x &= 192 \\ x &= 24 \end{aligned}$$



- $$x = 24$$
- $$m\angle MQN = 2(24) - 2 = 46$$
- $$m\angle QMP = 90$$
- $$m\angle NPM = 6(24) - 10 = 134$$

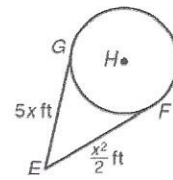
In Problems 11 and 12,  $\overline{EF}$  is tangent to  $\odot H$  at point  $F$  and  $\overline{EG}$  is tangent to  $\odot H$  at point  $G$ . Determine the length of  $\overline{EF}$ .

11.  $EF = 2(3) + 1.8 = 7.8$



$$\begin{aligned} 3x - 1.2 &= 2x + 1.8 \\ x &= 3 \end{aligned}$$

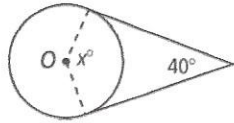
12.  $EF =$



$$\begin{aligned} \frac{x^2}{2} &= 5x \\ x^2 &= 10x \\ x^2 - 10x &= 0 \\ x(x - 10) &= 0 \\ x - 10 &= 0 \\ \boxed{x = 10} \end{aligned}$$

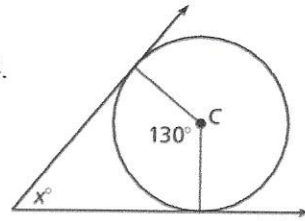
In Exercises 13 - 14, find the value of  $x$

13.



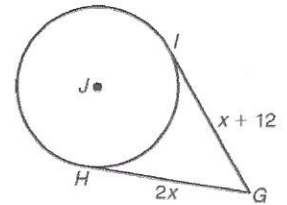
$$x = 180 - 40 = 140$$

14.



$$x = 180 - 130 = 150$$

Refer to the figure for Problems 15 - 18.  $\overline{GH}$  is tangent to  $\odot J$  at point  $H$  and  $\overline{GI}$  is tangent to  $\odot J$  at point  $I$ . Answer the questions to determine the length of  $\overline{GH}$ . The first one is done for you.



15. How are  $\overline{GH}$  and  $\overline{GI}$  related? *They are congruent*

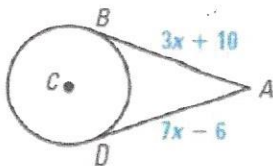
16. Write an equation to solve for  $x$ .  $2x = x + 12$

17. Solve the equation. What is the value of  $x$ ?  $x = 12$

18. What is  $\overline{GH}$ ?  $2(12) = 24$

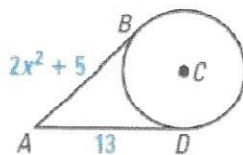
Find the value(s) of the variable

19.



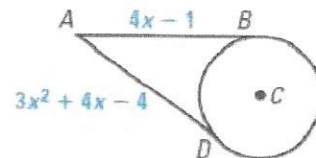
$$\begin{aligned} 7x - 6 &= 3x + 10 \\ 4x &= 16 \\ x &= 4 \end{aligned}$$

20.



$$\begin{aligned} 2x^2 + 5 &= 13 \\ 2x^2 &= 8 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

21.



$$\begin{aligned} 3x^2 + 4x - 4 &= 4x - 1 \\ 3x^2 - 3 &= 0 \\ 3x^2 &= 3 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

15-3 A (Geometry) Factoring and Solving Quadratic Equations Review.

Factor each trinomial.

1.  $x^2 + 13x + 36$   
 $(x+4)(x+9)$

2.  $x^2 + 11x + 24$   
 $(x+3)(x+8)$

3.  $x^2 + 14x + 40$   
 $(x+4)(x+10)$

7.  $x^2 - 7x + 6$   
 $(x-6)(x-1)$

8.  $x^2 - 9x + 14$   
 $(x-2)(x-7)$

9.  $x^2 - 11x + 24$   
 $(x-3)(x-8)$

13.  $x^2 - x - 2$   
 $(x-2)(x+1)$

14.  $x^2 - 3x - 18$   
 $(x-6)(x+3)$

15.  $x^2 - 4x - 45$   
 $(x-9)(x+5)$

19.  $x^2 - 6x + 5$   
 $(x-1)(x-5)$

20.  $x^2 - 9x + 18$   
 $(x-3)(x-6)$

21.  $x^2 - 12x + 32$   
 $(x-4)(x-8)$

Solve each quadratic equation by factoring.

26.  $x^2 - 3x + 2 = 0$   
 $(x-1)(x-2) = 0$   
 $x-1=0 \quad x-2=0$   
 $x=1 \quad x=2$

27.  $x^2 - 4x + 3 = 0$   
 $(x-1)(x-3) = 0$   
 $x-1=0 \quad x-3=0$   
 $x=1 \quad x=3$

30.  $x^2 - 6x + 5 = 0$   
 $(x-1)(x-5) = 0$   
 $x-1=0 \quad x-5=0$   
 $x=1 \quad x=5$

31.  $x^2 + 16x + 28 = 0$   
 $(x+14)(x+2) = 0$   
 $x+14=0 \quad x+2=0$   
 $x=-14 \quad x=-2$

32.  $x^2 + 10x + 9 = 0$   
 $(x+1)(x+9) = 0$   
 $x+1=0 \quad x+9=0$   
 $x=-1 \quad x=-9$

33.  $x^2 - 12x + 32 = 0$   
 $(x-4)(x-8) = 0$   
 $x-4=0 \quad x-8=0$   
 $x=4 \quad x=8$

34.  $x^2 + 13x + 42 = 0$   
 $(x+6)(x+7) = 0$   
 $x+6=0 \quad x+7=0$   
 $x=-6 \quad x=-7$

35.  $x^2 - 7x + 12 = 0$   
 $(x-3)(x-4) = 0$   
 $x-3=0 \quad x-4=0$   
 $x=3 \quad x=4$