

LESSON
9-1

Properties of Parallelograms

Practice and Problem Solving: A/B

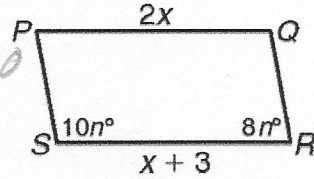
$2x = x + 3$
 $x = 3$

PQRS is a parallelogram. Find each measure.

1. $RS = 3 + 3 = 6$

2. $m\angle S = 10(10) = 100$

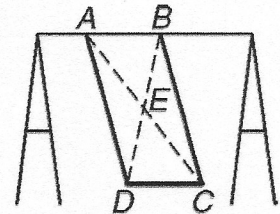
3. $m\angle R = 8(10) = 80$



$10n + 8n = 180$
 $18n = 180$
 $n = 10$

The figure shows a swing blown to one side by a breeze. As long as the seat of the swing is parallel to the top bar, the swing makes a parallelogram. In

$\square ABCD$, $DC = 2$ ft, $BE = 4\frac{1}{2}$ ft, and $m\angle BAD = 75^\circ$.



Find each measure.

4. $AB = 2$

5. $ED = 4\frac{1}{2}$

6. $BD = 9$

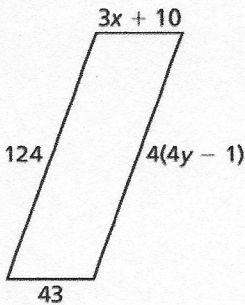
7. $m\angle ABC = 180 - 75 = 105$

8. $m\angle BCD = 75$

9. $m\angle ADC = 105$

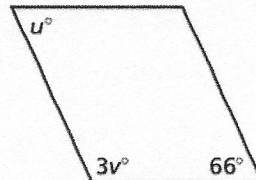
In Exercises 10–13, find the value of each variable in the parallelogram.

10.



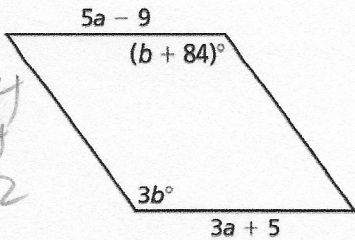
$3x + 10 = 43$
 $3x = 33$
 $x = 11$
 $124 = 4(4y - 1)$
 $124 = 16y - 4$
 $128 = 16y$
 $y = 8$

11.



$u = 66$
 $3v + 66 = 180$
 $3v = 114$
 $v = 38$

12.



$3b = b + 84$
 $2b = 84$
 $b = 42$
 $5a - 9 = 3a + 5$
 $2a = 14$
 $a = 7$

Three vertices of $\square GHIJ$ are $G(0, 0)$, $H(2, 3)$, and $J(6, 1)$. Use the grid to the right to complete Problems 14–20.

14. Plot vertices G , H , and J on the coordinate plane.

15. Find the rise (difference in the y -coordinates) from

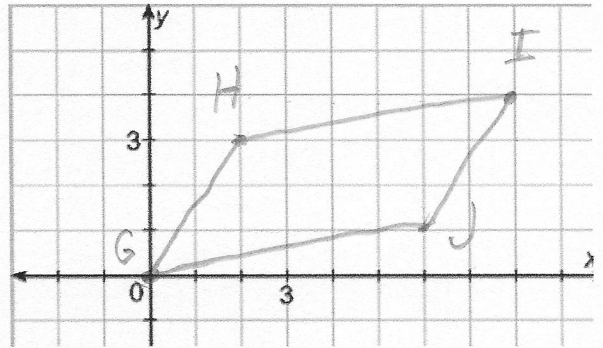
G to H .

3

16. Find the run (difference in the x -coordinates) from

G to H .

2



17. Using your answers from Problems 15 and 16, find the coordinates of vertex I .

(8 , 4)

18. Plot vertex I . Connect the points to draw $\square GHIJ$.

19. Check your answer by finding the slopes of \overline{IH} and \overline{JG} . slope of $\overline{IH} =$ $\frac{1}{6}$

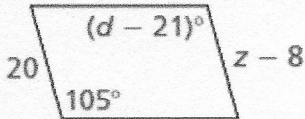
slope of $\overline{JG} =$ $\frac{1}{6}$

20. What do the slopes tell you about \overline{IH} and \overline{JG} ?

parallel

In Exercises 21–26, find the value of each variable in the parallelogram.

21.



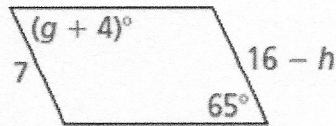
$$z - 8 = 20$$

$$z = 28$$

$$d - 21 = 105$$

$$d = 126$$

22.



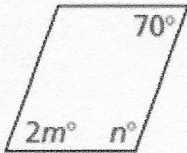
$$16 - h = 7$$

$$h = 9$$

$$g + 4 = 65$$

$$g = 61$$

23.



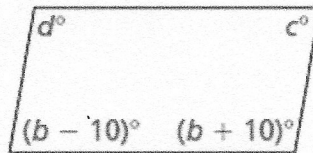
$$2m = 70$$

$$m = 35$$

$$n + 70 = 180$$

$$n = 110$$

24.



$$d = 90 + 10$$

$$d = 100$$

$$b - 10 + b + 10 = 180$$

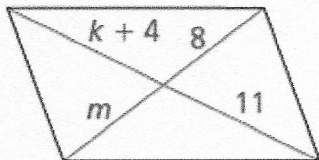
$$2b = 180$$

$$b = 90$$

$$c = 90 - 10$$

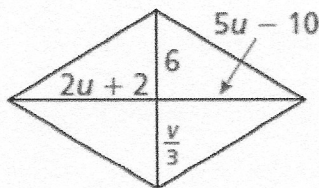
$$c = 80$$

25.



$m = 8$
 $k + 4 = 11$
 $k = 7$

26.



$\frac{v}{3} = 6$
 $v = 18$

$5u - 10 = 2u + 2$
 $3u = 12$
 $u = 4$

In Exercises 27 – 34, find the indicated measure in $\square LMNQ$. Explain your reasoning.

27. $LM = 13$

28. $LP = 7$

29. $LQ = 8$

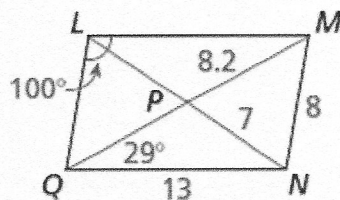
30. $MQ = (8, 2) \cdot 2 = 16.4$

31. $m\angle LMN = 180 - 100 = 80$

32. $m\angle NQL = 80$

33. $m\angle MNQ = 100$

34. $m\angle LMQ = 29$



36. $\angle J$ and $\angle K$ are consecutive angles in a parallelogram, $m\angle J = (3x + 7)^\circ$, and $m\angle K = (5x - 11)^\circ$. Find the measure of each angle.

$3x + 7 + 5x - 11 = 180$
 $8x - 4 = 180$
 $8x = 184$
 $x = 23$

$m\angle J = 3(23) + 7 = 76$
 $m\angle K = 5(23) - 11 = 104$

37. The sides of $\square MNPQ$ are represented by the expressions below. Sketch $\square MNPQ$ and find its perimeter.

$-2x + 37 = x - 5$
 $-3x = -42$
 $x = 14$

$MQ = -2x + 37$

$QP = y + 14$

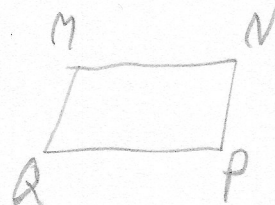
$NP = x - 5$

$MN = 4y + 5$

$MQ = -2(14) + 37 = 9$
 $NP = 9$

$QP = 3 + 14 = 17$
 $MN = 17$

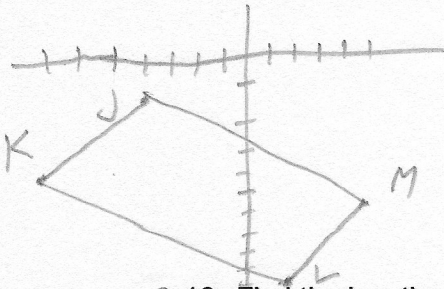
$4y + 5 = y + 14$
 $3y = 9$
 $y = 3$



$perim = 9 + 9 + 17 + 17 = 52$

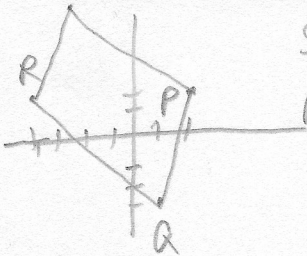
Use the given method to determine whether the quadrilateral with the given vertices is a parallelogram.

15. Find the slopes of all four sides: $J(-4, -1)$, $K(-7, -4)$, $L(2, -10)$, $M(5, -7)$.



$$\begin{aligned} \text{slope } JK &= \frac{-1 - (-4)}{-4 - (-7)} = \frac{3}{3} = 1 & \text{slope } LM &= \frac{-7 - (-10)}{5 - 2} = \frac{3}{3} = 1 \\ \text{slope } JM &= \frac{-1 - (-7)}{-4 - 5} = \frac{6}{-9} = -\frac{2}{3} & \text{slope } LK &= \frac{-4 - (-10)}{-7 - 2} = \frac{6}{-9} = -\frac{2}{3} \end{aligned}$$

16. Find the lengths of all four sides: $P(2, 2)$, $Q(1, -3)$, $R(-4, 2)$, $S(-3, 7)$.



$$\begin{aligned} SR &= \sqrt{1^2 + 5^2} = \sqrt{26} \\ PQ &= \sqrt{1^2 + 5^2} = \sqrt{26} \end{aligned}$$

$$\begin{aligned} PS &= \sqrt{5^2 + 5^2} = \sqrt{50} \\ RQ &= \sqrt{5^2 + 5^2} = \sqrt{50} \end{aligned}$$

In Exercises 9–12, find the values of x and y that make the quadrilateral a parallelogram.

18.

$$\begin{aligned} 4x + 6 &= 7x - 3 \\ 9 &= 3x \\ x &= 3 \\ 4y - 3 &= 3y + 1 \\ y &= 4 \end{aligned}$$

19.

$$\begin{aligned} 4x + 13 &= 5x - 12 \\ 25 &= x \\ 4y + 7 &= 3(25) - 8 \\ 4y + 7 &= 67 \\ 4y &= 60 \\ y &= 15 \end{aligned}$$

20.

$$\begin{aligned} 5x - 6 &= 4x + 2 \\ x &= 8 \end{aligned}$$

21.

$$\begin{aligned} 6x &= 3x + 2 \\ 3x &= 2 \\ x &= \frac{2}{3} \end{aligned}$$