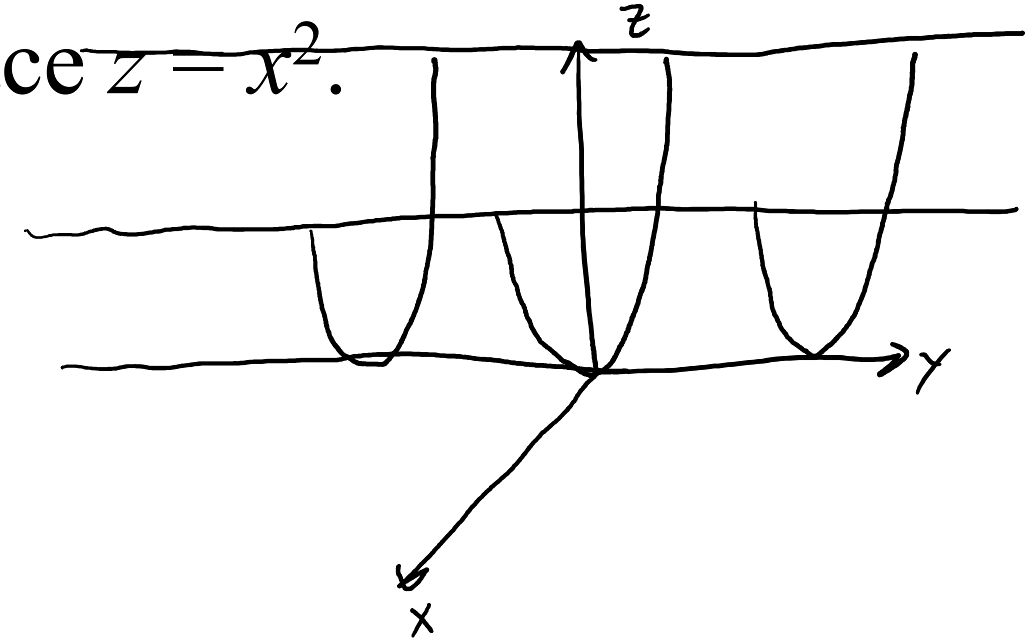
Cylinders and Quadratic Surfaces

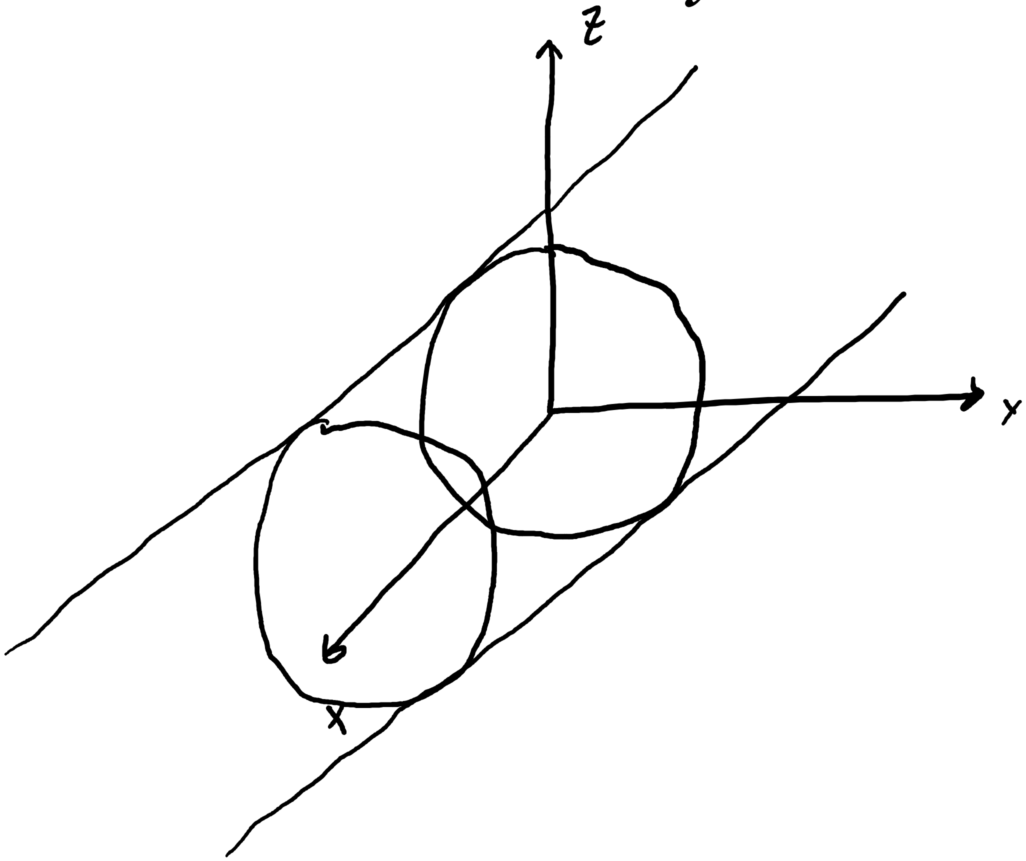
A <u>cylinder</u> is the continuation of a 2-D curve into 3-D.

→No longer just a soda can.

<u>Ex.</u> Sketch the surface $z = x^2$.



Ex. Sketch the surface $y^2 + z^2 = 1$.



A <u>quadratic surface</u> has a second-degree equation. The general equation is:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

A <u>trace</u> is the intersection of a surface with a plane.

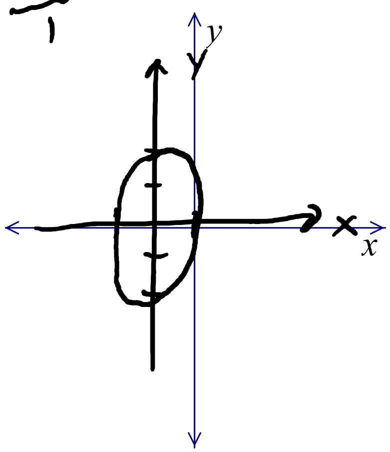
→We will use the traces of the quadratic surfaces with the coordinate planes to identify the surface.

Ex Sketch the surface $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$

xy-trace

xy trace

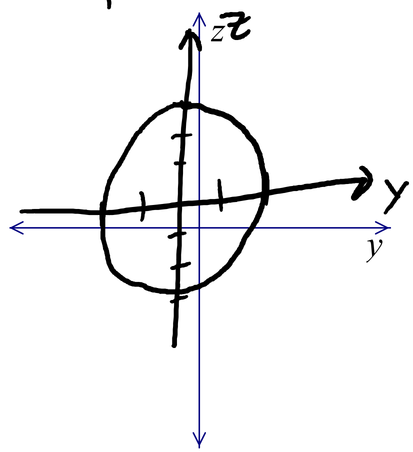
$\frac{x^2}{1} + \frac{y^2}{4} = 1$



yz-trace

yz trace

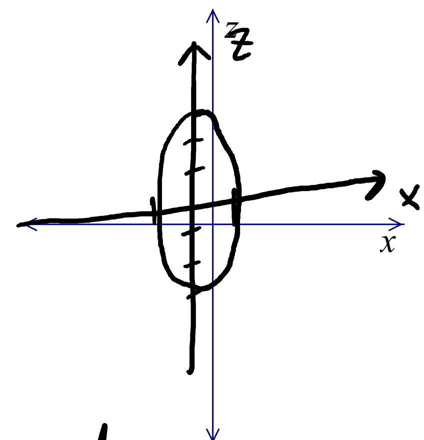
$\frac{y^2}{4} + \frac{z^2}{9} = 1$

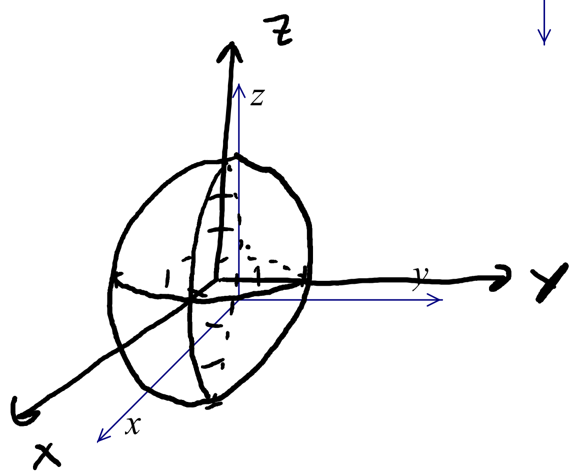


xz-trace

xz trace

$x^2 + \frac{z^2}{9} = 1$





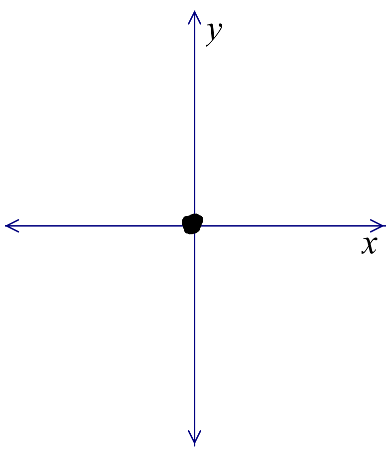
ellipsoid

[The Graph]

Ex Sketch the surface $z = 4x^2 + y^2$

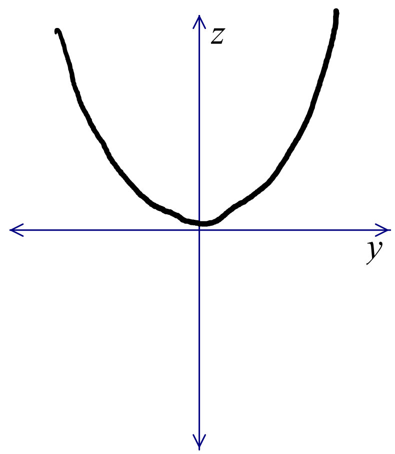
xy-trace $\rightarrow z=0$

$0 = 4x^2 + y^2$



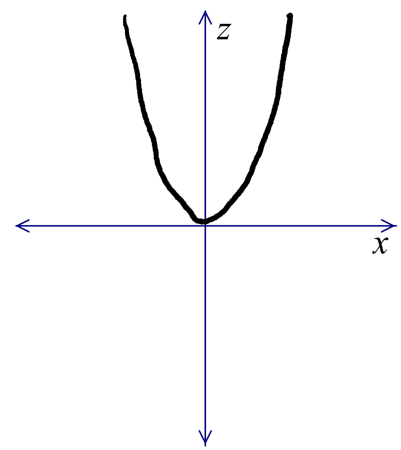
yz-trace

$z = y^2$



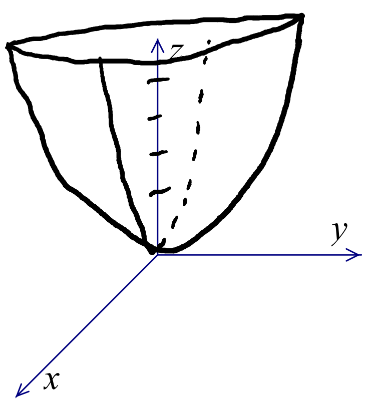
xz-trace

$z = 4x^2$



$z = 4$

$4 = 4x^2 + y^2$



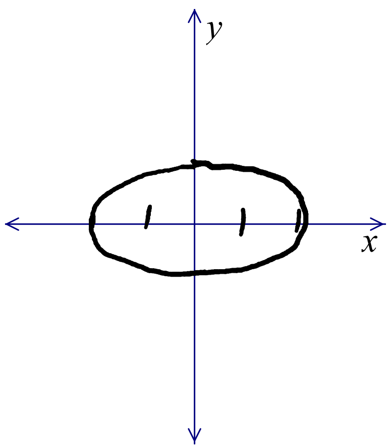
paraboloid

The Graph

Ex Sketch the surface $\dfrac{x^2}{4} + y^2 - \dfrac{z^2}{4} = 1$

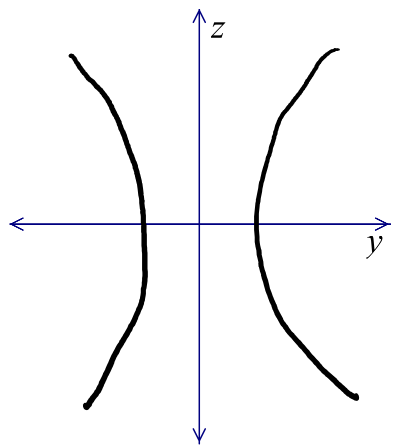
xy-trace

$\dfrac{x^2}{4} + y^2 = 1$



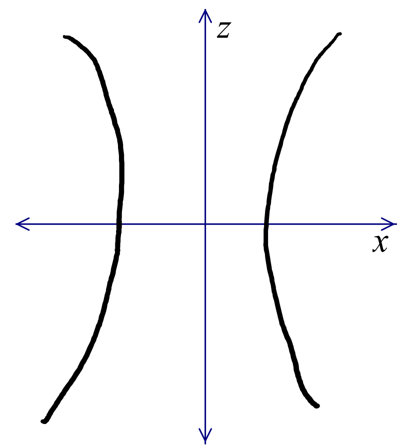
yz-trace

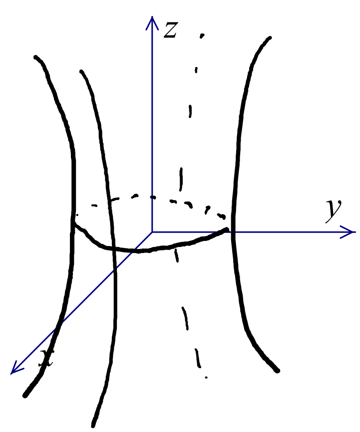
$y^2 - \dfrac{z^2}{4} = 1$



xz-trace

$\dfrac{x^2}{4} - \dfrac{z^2}{4} = 1$





hyperboloid in one sheet

The Graph

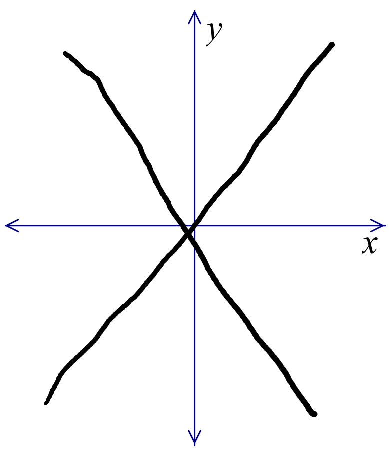
Ex Sketch the surface $z = x^2 - y^2$

xy-trace

$0 = x^2 - y^2$

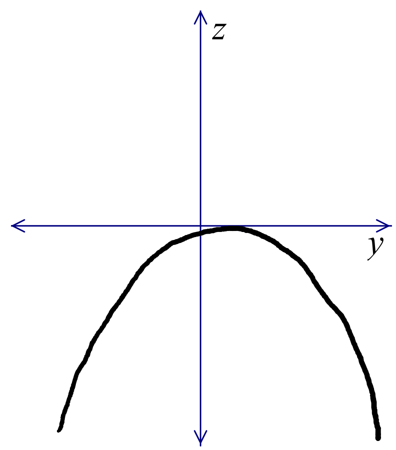
$x^2 = y^2$

$y = \pm x$



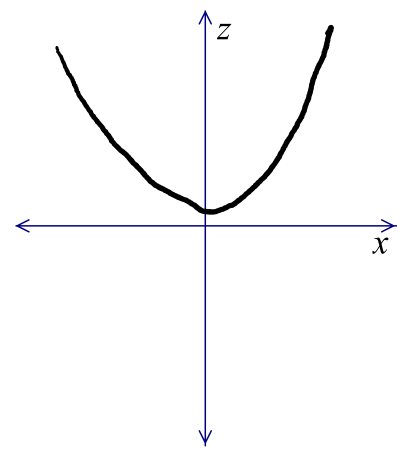
yz-trace

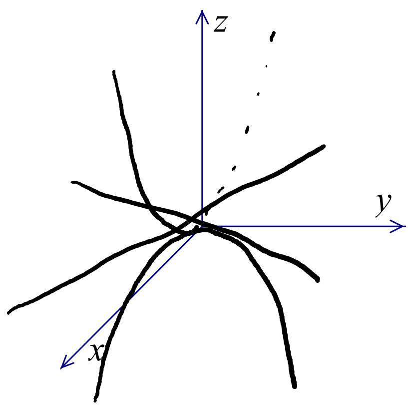
$z = -y^2$



xz-trace

$z = x^2$





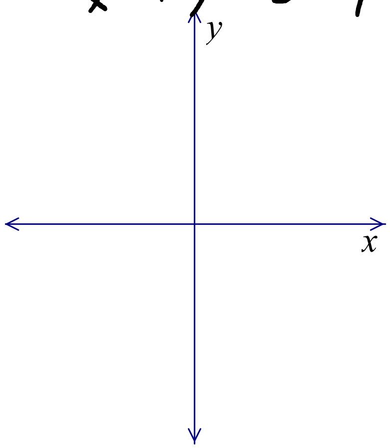
The Graph

<u>Ex</u> Sketch the surface $x^2 + y^2 - 2z^2 + 4 = 0$

<u>*xy*-trace</u>

$x^2 + y^2 + 4 = 0$

$x^2 + y^2 = -4$

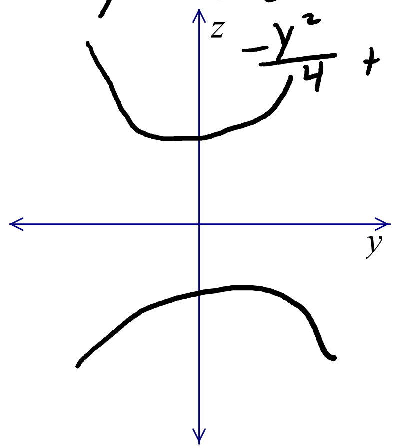


<u>*yz*-trace</u>

$y^2 - 2z^2 + 4 = 0$

$y^2 - 2z^2 = -4$

$-\frac{y^2}{4} + \frac{z^2}{2} = 1$

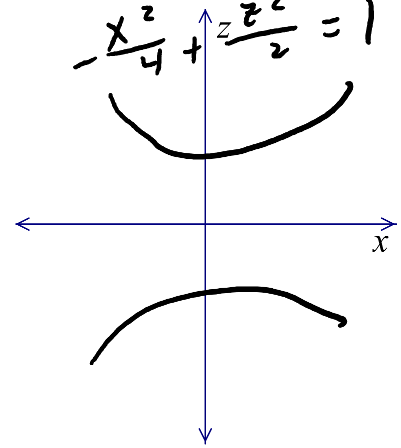


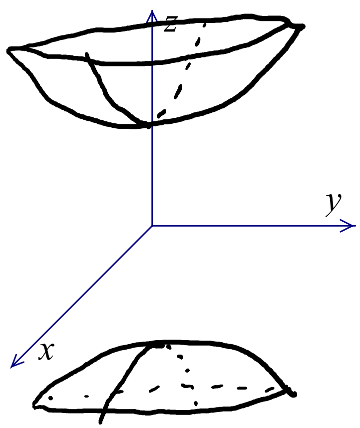
<u>*xz*-trace</u>

$x^2 - 2z^2 + 4 = 0$

$x^2 - 2z^2 = -4$

$-\frac{x^2}{4} + \frac{z^2}{2} = 1$





hyperboloid in two sheets

[The Graph]

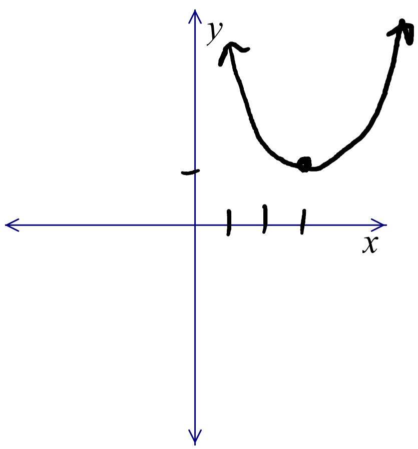
Ex Sketch the surface $x^2 + 2z^2 - 6x - y + 10 = 0$

$$y - 10 + \underline{9} = x^2 - 6x + \underline{9} + 2z^2$$

$$y - 1 = (x-3)^2 + 2z^2$$

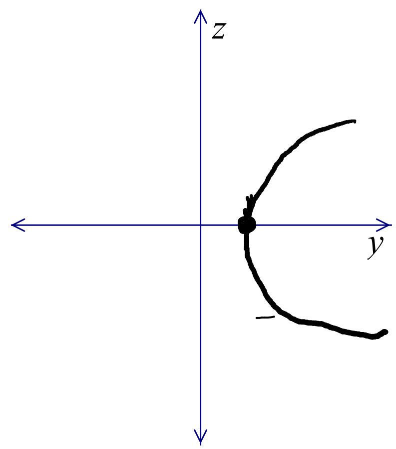
$z=0$ -trace

$$(y-1) = (x-3)^2$$



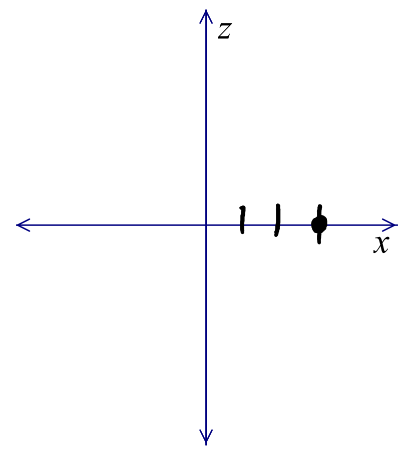
$x=3$ -trace

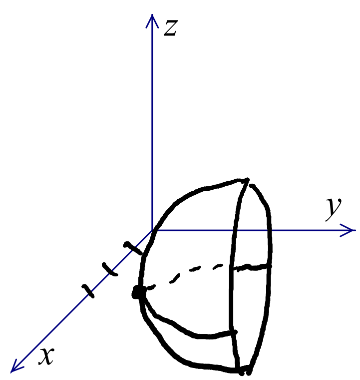
$$y-1 = 2z^2$$



$y=1$ -trace

$$0 = (x-3)^2 + 2z^2$$





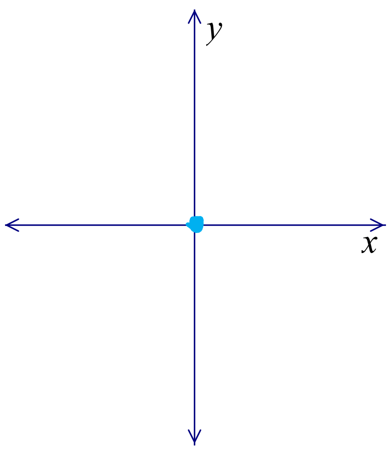
$(3, 1, 0)$

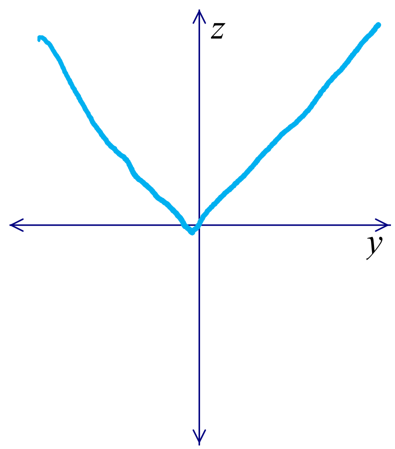
The Graph

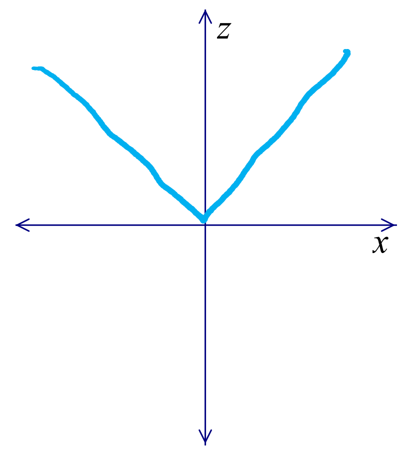
Ex Determine the region bounded by

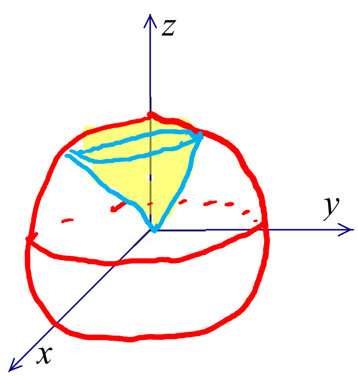
$$z = \sqrt{x^2 + y^2} \text{ and } x^2 + y^2 + z^2 = 1$$

xy-trace	*yz*-trace	*xz*-trace
$0 = \sqrt{x^2 + y^2}$	$z = \sqrt{y^2} \rightarrow z^2 = y^2$	$z = \sqrt{x^2} \rightarrow z^2 = x^2$









The Graph

Surfaces of rotation

x-axis: $y^2 + z^2 = [r(x)]^2$

y-axis: $x^2 + z^2 = [r(y)]^2$

z-axis: $x^2 + y^2 = [r(z)]^2$

Ex. Write the equation of the surface generated by revolving $y = \dfrac{1}{z}$ about the y-axis.

$$x^2 + z^2 = \left[\frac{1}{y}\right]^2$$

$$z = \frac{1}{y}$$