Warm-up Problems

Sketch the surface

$9x^2 + 4y^2 - 36z^2 - 18x - 144z = 171$

$9x^2 - 18x + 4y^2 - 36z^2 - 144z = 171$

$9(x^2 - 2x + \underline{1}) + 4y^2 - 36(z^2 + 4z + \underline{4}) = 171 + \underline{9} + \underline{-144}$

$9(x-1)^2 + 4y^2 - 36(z+2)^2 = 36$

$\frac{(x-1)^2}{4} + \frac{y^2}{9} - \frac{(z+2)^2}{1} = 1$

$(1, 0, -2)$

Cylindrical and Spherical

In polar coordinates, position is based on angle and distance from the origin…

(1,1) in rectangular is equivalent to $\left(\sqrt{2}, \frac{\pi}{4}\right)$ in polar

Cylindrical coordinates extends polar to 3-D

→Find the position of a point over the *xy*-plane using r and θ, then find the distance from the *xy*-plane (this is the z coordinate)

(r,θ,z) is the cylindrical coordinate of a point

$$r \in \mathbb{R}, \quad 0 \le \theta \le 2\pi, \quad z \in \mathbb{R}$$

Some relationships:

<u>Cylind → Rect</u>	<u>Rect → Cylind</u>
$x = r\cos\theta$	$x^2 + y^2 = r^2$
$y = r\sin\theta$	$\tan\theta = \frac{y}{x}$
$z = z$	$z = z$

Ex. Find the rectangular coordinates of the cylindrical point $\left(2, \frac{2\pi}{3}, 1\right)$

$r \quad \theta \quad z$

$x = 2\cos\frac{2\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$

$y = 2\sin\frac{2\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

$z = 1$

$(-1, \sqrt{3}, 1)$

Ex. Find the cylindrical coordinates of the rectangular point $(3,-3,-7)$

$x^2+y^2=r^2$

$3^2+(-3)^2=r^2$

$9+9=r^2$

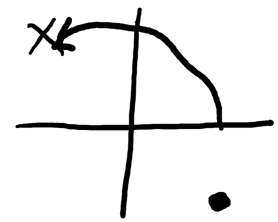
$18=r^2$

$r=\pm\sqrt{18}$

$=\pm 3\sqrt{2}$

$\tan\theta=\frac{-3}{3}$

$\tan\theta=-1$



$\left(3\sqrt{2},\ \frac{7\pi}{4},\ -7\right)$

$\left(-3\sqrt{2},\ \frac{3\pi}{4},\ -7\right)$

Some surfaces are easier to describe in cylindrical

Ex. Describe the surface $r = c$.

Ex. Describe the surface $\theta = c$.

Ex. Describe the surface $z = r$.

Ex. Find the cylindrical equation of the surface $x^2 + y^2 = 2z^2$.

$$\boxed{r^2 = 2z^2}$$

$$(r\cos\theta)^2 + (r\sin\theta)^2 = 2z^2$$

Ex. Find the cylindrical equation of the surface $y^2 = 2x$.

$$(r \sin \theta)^2 = 2 r \cos \theta$$

$$r^2 \sin^2 \theta = 2r \cos \theta$$

$$r \sin^2 \theta = 2 \cos \theta$$

$$r = \frac{2 \cos \theta}{\sin^2 \theta}$$

Ex. Find the rectangular equation of the surface $r^2 \cos 2\theta + z^2 + 1 = 0$.

$$r^2(\cos^2\theta - \sin^2\theta) + z^2 + 1 = 0$$

$$r^2\cos^2\theta - r^2\sin^2\theta + z^2 + 1 = 0$$

$$(r\cos\theta)^2 - (r\sin\theta)^2 + z^2 + 1 = 0$$

$$x^2 - y^2 + z^2 + 1 = 0$$

In <u>spherical coordinates</u>, points are represented by (ρ,θ,φ):

ρ = distance from origin ($\rho \geq 0$)

θ = angle in xy-plane ($0 \leq \theta \leq 2\pi$)

φ = angle with positive z-axis ($0 \leq \varphi \leq \pi$)

Ex. Describe the surface $\rho = c$.

Ex. Describe the surface $\theta = c$.

Ex. Describe the surface $\varphi = c$.

Some relationships:

<u>Spher → Rect</u>	<u>Rect → Spher</u>
$x = \rho \sin\varphi \cos\theta$	$x^2 + y^2 + z^2 = \rho^2$
$y = \rho \sin\varphi \sin\theta$	$\tan\theta = \frac{y}{x}$
$z = \rho \cos\varphi$	$\cos\varphi = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

Ex. Find the rectangular coordinates of the spherical point $\left(2, \frac{\pi}{4}, \frac{\pi}{3}\right)$

$$\rho \quad \theta \quad \varphi$$

$$x = \rho \sin\varphi \cos\theta = 2 \sin\frac{\pi}{3} \cos\frac{\pi}{4} = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{2}$$

$$y = \rho \sin\varphi \sin\theta = 2 \sin\frac{\pi}{3} \sin\frac{\pi}{4} = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{2}$$

$$z = \rho \cos\varphi = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$$

$$\boxed{\left(\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}, 1\right)}$$

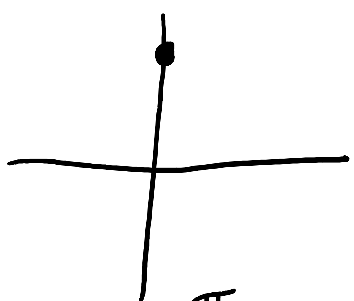
Ex. Find the spherical coordinates of the rectangular point $\left(\underset{x}{0}, \underset{y}{2\sqrt{3}}, \underset{z}{-2}\right)$

$x^2+y^2+z^2=\rho^2$

$0+12+4=\rho^2$

$\rho=4$

$\tan\theta=\frac{2\sqrt{3}}{0}$



$\theta=\frac{\pi}{2}$

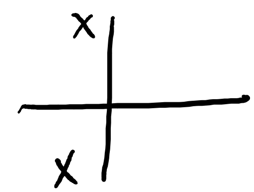
$\cos\varphi=\frac{z}{\rho}$

$\cos\varphi=\frac{-2}{4}$

$\cos\varphi=-\frac{1}{2}$

$\varphi=\frac{2\pi}{3}$

$$\boxed{\left(4, \frac{\pi}{2}, \frac{2\pi}{3}\right)}$$



Ex. Find the spherical equation of the surface

$$x^2 - y^2 - z^2 = 1.$$

$$(\rho \sin\varphi \cos\theta)^2 - (\rho \sin\varphi \sin\theta)^2 - (\rho \cos\varphi)^2 = 1$$

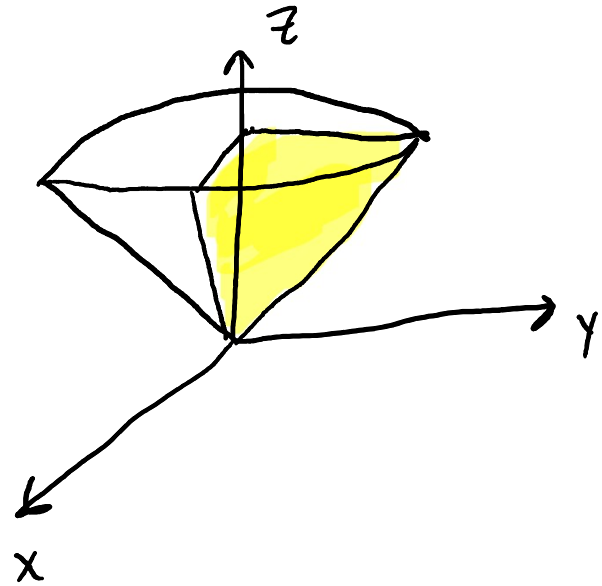
Ex. Find the rectangular equation of the surface $\rho = \sin \theta \sin \varphi$.

$$\rho^2 = \rho \sin \theta \sin \varphi$$

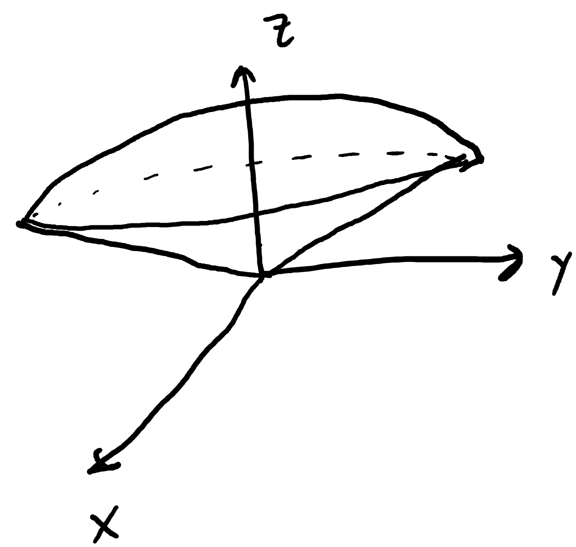
$$\boxed{x^2 + y^2 + z^2 = y}$$

Ex. Sketch the solid described by $0 \le \theta \le \frac{\pi}{2}$,

$$\underbrace{r \le z}_{z=r}\ \underbrace{\le 2}_{z=2}.$$



Ex. Sketch the solid described by $0 \leq \varphi \leq \frac{\pi}{3}$, $\rho \leq 2$.




I don't think this is what the professor meant by "polar coordinates"
igloos make a lot more sense in spherical coordinates, though.
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