700 • (desired %) = total points needed

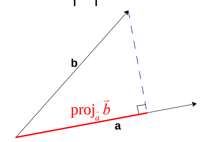
Total points needed
– 4 undropped exams
– quiz percent
Points needed on final (out of 200)

Semester 1 Review

Unit vector = $\dfrac{\vec{v}}{|\vec{v}|}$ $\text{proj}_{\vec{a}}\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$ or $\dfrac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a}$

$\vec{u}\parallel\vec{v}$ iff $\vec{u}=c\vec{v}$

$\vec{u}\cdot\vec{v}=|\vec{u}||\vec{v}|\cos\theta$



$\vec{u}\times\vec{v}$ is orthogonal to $\vec{u}$ and $\vec{v}$

A line that contains the point $P(x_0,y_0,z_0)$ and direction vector $\langle a,b,c\rangle$:

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

parametric

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

symmetric

The equation of the plane containing point $P(x_0,y_0,z_0)$ with normal vector $\vec{n}=\langle a,b,c\rangle$,

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

Ex. Let θ be the angle between the planes $5y + 6z = 2$ and $2x + 8y - 3z = 1$. Find $\cos\theta$.

Sketching 3-D

Ex. Sketch $-9x^2 + y^2 - 9z^2 = 9$

Cylind → Rect

$x = r\cos\theta$
$y = r\sin\theta$
$z = z$

Rect → Cylind

$x^2 + y^2 = r^2$
$\tan\theta = \frac{y}{x}$
$z = z$

Spher → Rect

$x = \rho\sin\varphi\cos\theta$
$y = \rho\sin\varphi\sin\theta$
$z = \rho\cos\varphi$

Rect → Spher

$x^2 + y^2 + z^2 = \rho^2$
$\tan\theta = \frac{y}{x}$
$\cos\varphi = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

Parameterizing curves: $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

Derivative and integral of vector functions

Ex. Find the parametric equations of the line tangent to $\vec{r}(t) = \langle \cos t, 4e^{2t}, 4e^{-2t} \rangle$ at (1,4,4).

Ex. Find $\vec{r}(t)$ if $\vec{r}\,'(t) = \sin t\,\vec{\mathrm{i}} - \cos t\,\vec{\mathrm{j}} + 16t\,\vec{\mathrm{k}}$ and $\vec{r}(0) = \vec{\mathrm{i}} + \vec{\mathrm{j}} + 5\vec{\mathrm{k}}$

If **r**(t) = displacement vector, then

r′(t) = velocity **v**(t)

r″(t) = acceleration **a**(t)

|**r**′(t)| = speed $v(t)$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\left|\vec{r}\,'(t)\right|} = \text{unit tangent vector}$$

$$s(t) = \int_a^t \sqrt{\left[f'(u)\right]^2 + \left[g'(u)\right]^2 + \left[h'(u)\right]^2}\,du$$

$= \text{arc length of } \vec{r} \text{ on } [a,b]$

$$\kappa = \frac{\left|\vec{r}\,'(t) \times \vec{r}\,''(t)\right|}{\left|\vec{r}\,'(t)\right|^3} = \text{curvature of } \vec{r}$$

First and second partial derivatives

Implicit differentiation:

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} \qquad \frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$$

Total Differential: $dz = f_x dx + f_y dy$

Let $w = f(x,y)$, with $x = g(t)$ and $y = h(t)$:

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$$

$$\nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle$$

Gradient is orthogonal to level curve of f

Directional derivative of f in the direction of unit vector **u** is $D_u f(x,y) = \nabla f(x,y) \cdot \vec{u}$

Ex. Find the parametric equations of the line tangent to the curve of intersection of $4x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2$ at (-1,1,2).

Critical point = where f_x and f_y are both zero, or where one is undefined

$$d = f_{xx}(a,b)\,f_{yy}(a,b) - [\,f_{xy}(a,b)]^2$$

If $d > 0$ and $f_{xx}(a,b) < 0$, then (a,b) is a rel. max.

If $d > 0$ and $f_{xx}(a,b) > 0$, then (a,b) is a rel. min.

If $d < 0$, then (a,b) is a saddle point.

If $d = 0$, then the test fails.

Ex. Find the critical points of $f(x,y) = 6 - 20x + 96y - 5x^2 - 8y^2$ and classify them as relative maximum, relative minimum, or saddle point.

Ex. Use Lagrange multipliers to find the maximum value of $f(x,y) = x + 2xy + 2y$ subject to the constraint $x + 2y = 80$

Multiple Integrals – changing order

Ex. $\int_0^1 \int_{y^2}^1 y\cos\left(x^2\right)dxdy$

$$\iint_R \quad dA = \iint \quad dydx = \iint \quad rdrd\theta$$

$$V = \iint_R f\left(x,y\right)dA = \iiint_Q dV$$

<u>Ex.</u> Find the volume bounded by $x^2 + y^2 = 64$, under the plane $z = y$, and in the first octant.

$$m = \iint_R \rho(x, y)\, dA \quad \text{where } \rho \text{ is density}$$

Surface area: $S = \iint_R \sqrt{1 + f_x^{\,2} + f_y^{\,2}}\, dA$

Jacobian: $\dfrac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$

<u>Ex.</u> Evaluate $\iint_R (3x + 2y)\, dA$, where R is the region bounded by $x - y = 0$, $x - y = 2$, $2x + y = 0$, and $2x + y = 3$

If you would like your final exam mailed to you, please bring a self addressed, stamped envelope with you to the final.

→ You can get them from me next semester if you want