Warm-up Problems

1. Determine if the DE $\left(x^2-1\right)\frac{dy}{dx}-2x=0$ has a unique solution on the interval [0,5].
2. San Diego has a population of 3 million people, and the rate of change of this population is proportional to the square root of the population. Express this as an IVP.

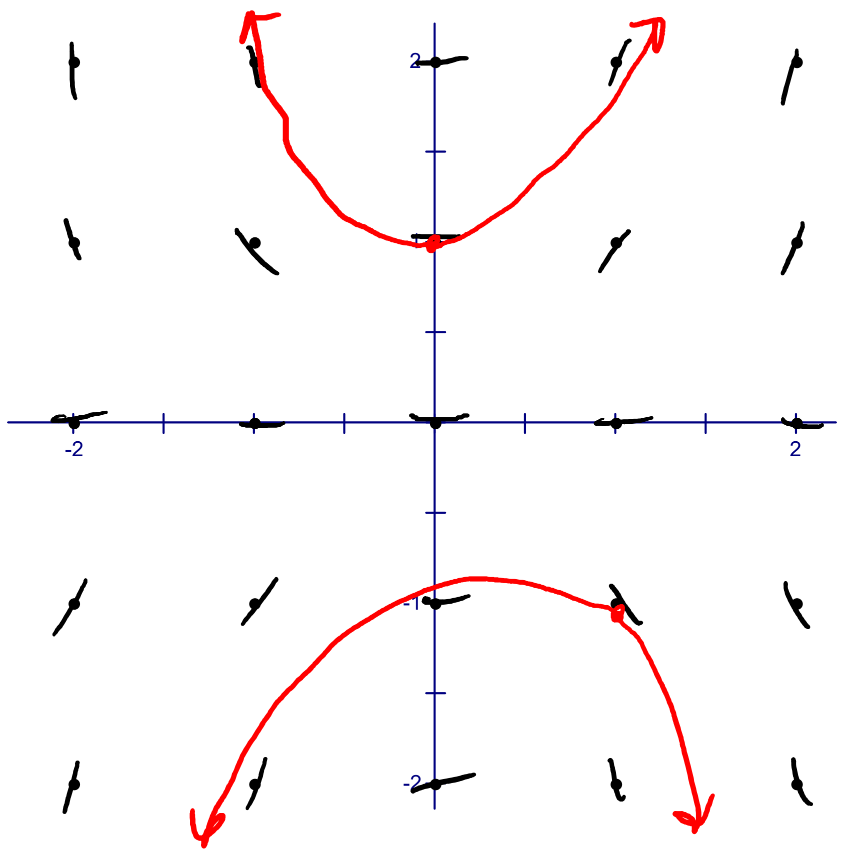
Sketching Solution Curves

Ex. $\frac{dy}{dx} = xy$

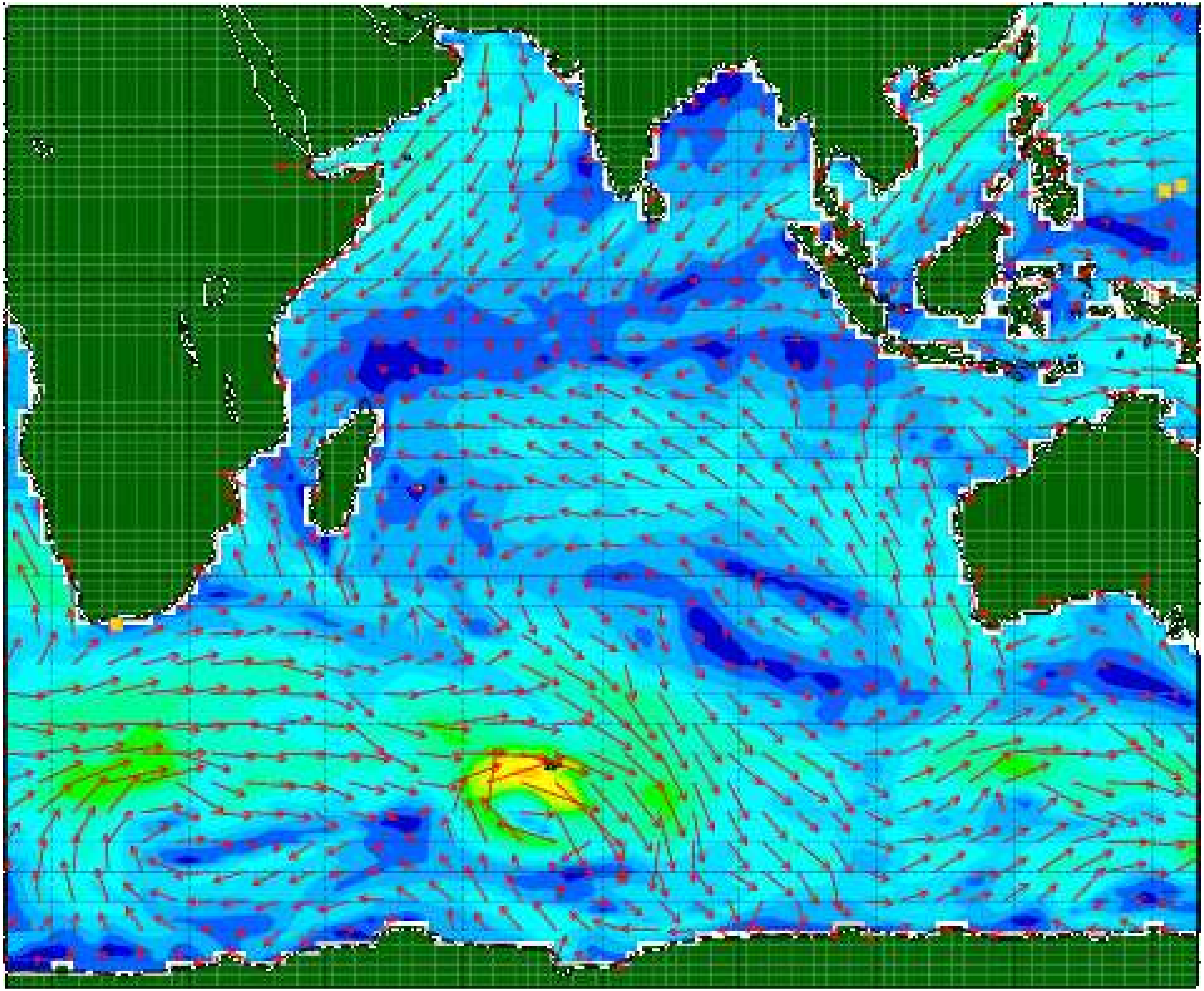
While we can't find a solution yet, we can find the slope of the solution at (0,2), assuming it passes through this point.

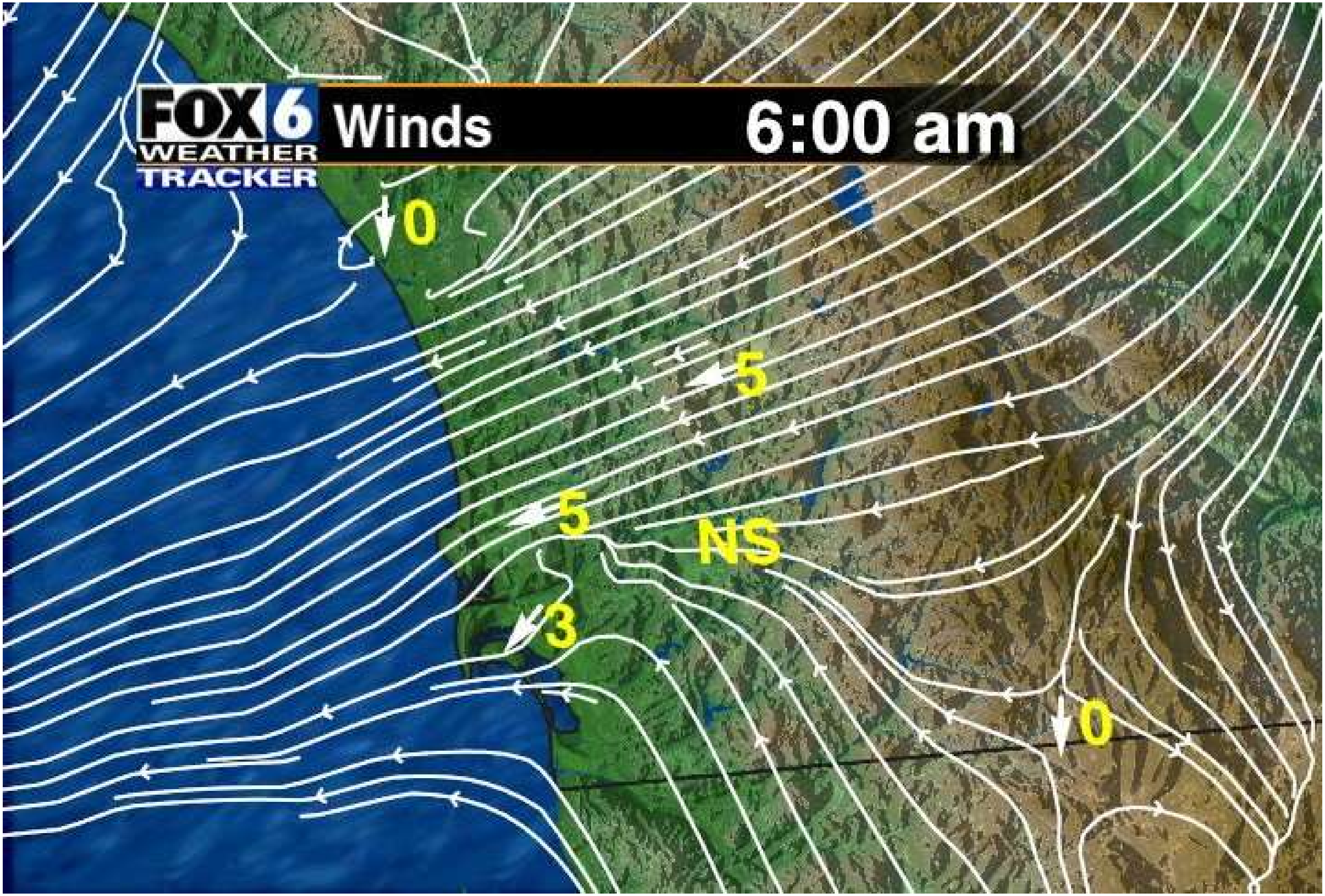
- We can draw a segment through the point that has the appropriate slope: called a lineal element.
- If we draw several of these lines, we get a good idea of what a solution would look like. This is called a slope field or direction field.

Ex. Draw a slope field for $\frac{dy}{dx} = xy$.



Sketch some solution curves.



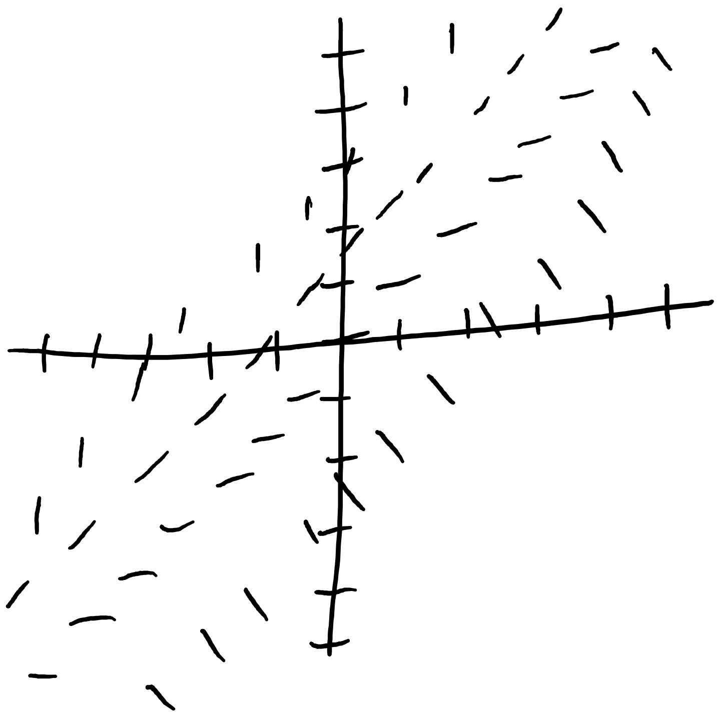

FOX 6
WEATHER
TRACKER
Winds
6:00 am
0
5
5
NS
3
0

A shortcut would be to find all points that have the same slope

i.e. For the DE $\frac{dy}{dx} = f(x,y)$, find all points that cause $f(x,y) = c$ for some constant.

→This is called the <u>method of isoclines</u>, and $f(x,y) = c$ is called an <u>isocline</u>.

Ex. Sketch the isoclines for the DE $\frac{dy}{dx} = y - x$ for integer values of c, $-5 \le c \le 5$.



$c = 3: \quad 3 = y - x$

$y = x + 3$

$c = 0: \quad 0 = y - x$

$y = x$

$c = -2: \quad -2 = y - x$

$y = x - 2$

$c = 1: \quad 1 = y - x$

$y = x + 1$

Def. A DE is autonomous if the independent variable does not appear.

$$\frac{dy}{dx} = 1 + y^2 \qquad \frac{dy}{dx} = 5\sin y$$

$$\frac{dy}{dx} = f(y)$$

We say that $y = c$ is a <u>critical point</u> of the DE $\frac{dy}{dx} = f\left(y\right)$ if $f(c) = 0$.

This is also called an <u>equilibrium point</u> or <u>stationary point</u>.

→The constant function $y = c$ is a solution to the DE, called the <u>equilibrium solution</u>.

Notice that the critical point divides the plane into regions where a solution will be monotonic (either increases or decreases).

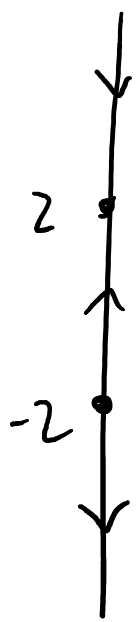
Solutions can’t cross these horizontal lines, which serve as asymptotes of the solutions.

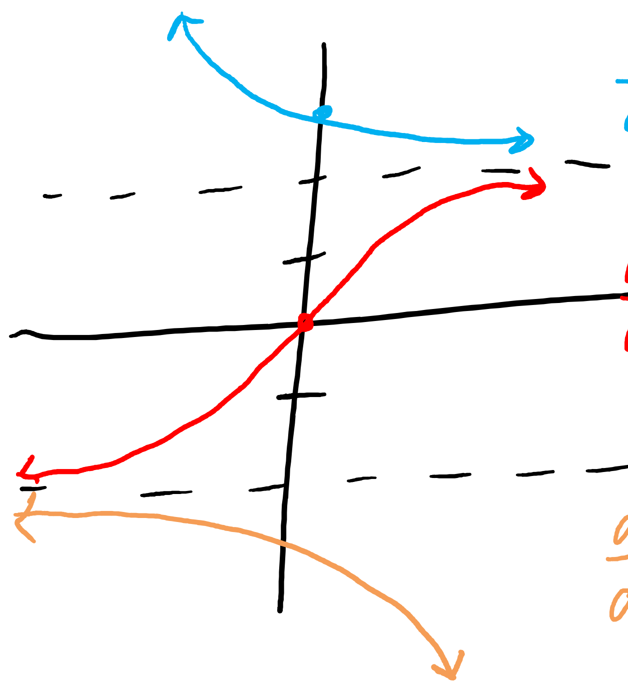
A <u>phase portrait</u> of an autonomous DE is a vertical number line that shows the increasing/decreasing nature of solution curves.

Ex. Sketch a phase portrait of $\frac{dy}{dx} = 4 - y^2$, and then sketch typical solution curves in the regions in the xy-plane determined by the graphs of the equalibrium solutions.

$$4 - y^2 = 0$$

$$y = \pm 2$$

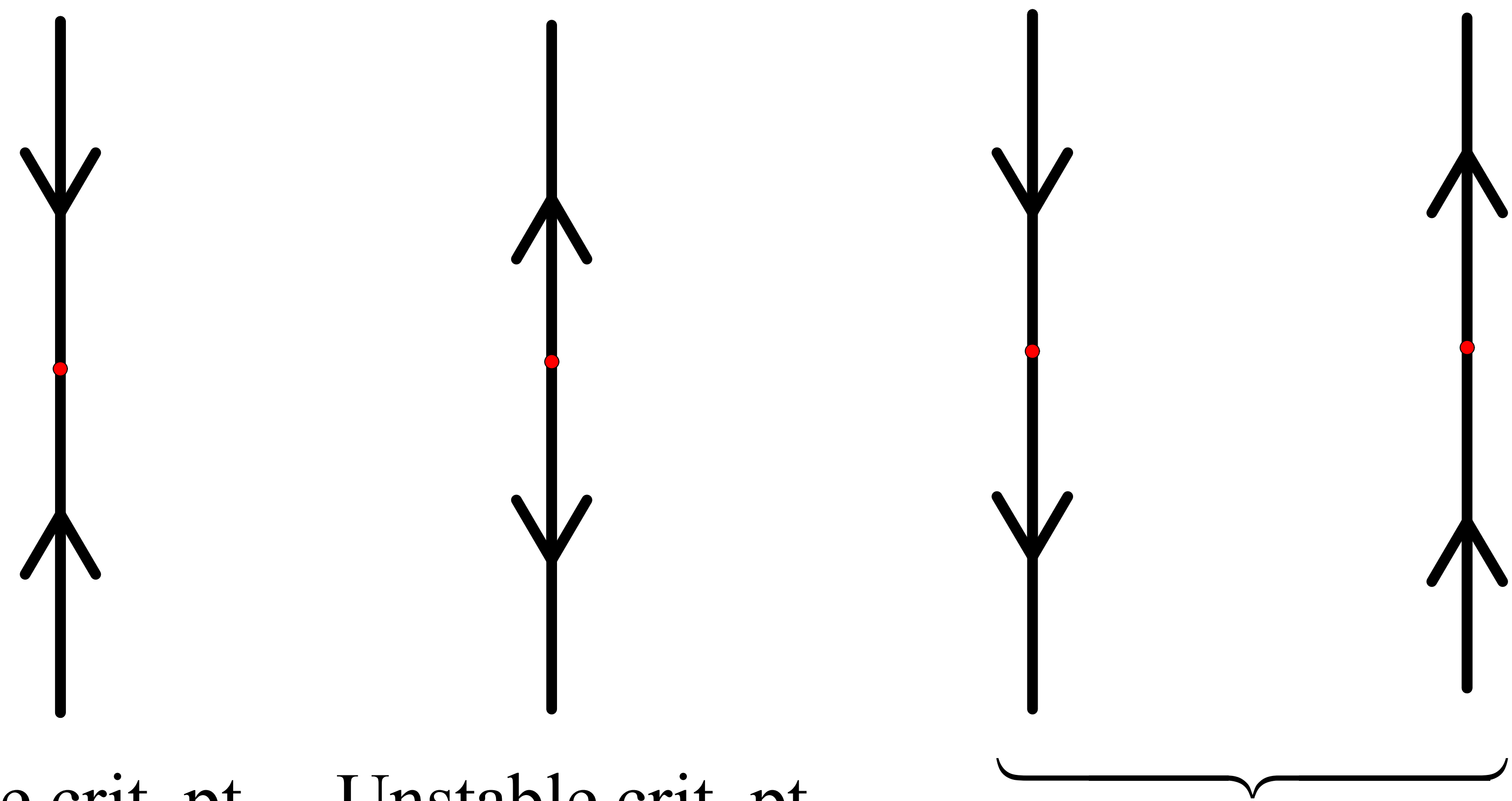




$\frac{dy}{dx} = 4 - y^2, \; y(0) = 3$

$\frac{dy}{dx} = 4 - y^2; \; y(0) = 0$

$\frac{dy}{dx} = 4 - y^2, \; y(0) = -3$



Stable crit. pt.

Attractor

Unstable crit. pt.

Repeller

Semistable

Note: An autonomous DE such as $\frac{dy}{dx} = 2y - 2$ depends only on the y-coordinate of the point. When sketching slope fields, all lineal elements along a horizontal line will have the same slope, since the slope is independent of x.

If the DE had been $\frac{dy}{dx} = 3x + 2$, all lineal elements along a vertical line would be parallel.