Laplace Transform

- A transform is an operation that changes a function into a new function.
- →Examples of this are derivatives and antiderivatives.
- → These are linear transforms:

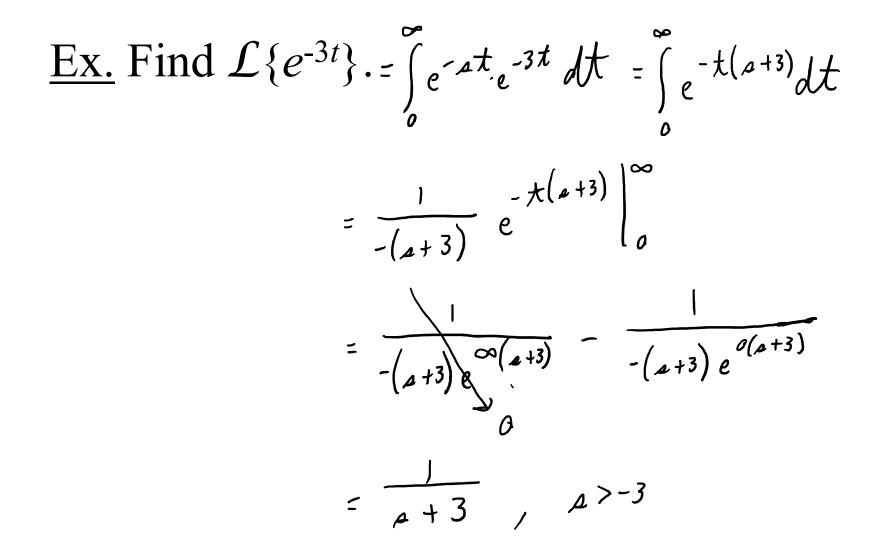
$$\int (\alpha f(x) + \beta g(x)) dx = \alpha \int f(x) dx + \beta \int g(x) dx$$

<u>Def.</u> Let f(t) be defined for $t \ge 0$, then $\mathcal{L}\left\{f(t)\right\} = \int_{0}^{\infty} e^{-st} f(t) dt$ is called the <u>Laplace Transform</u> of f. *** $\mathcal{L}\left\{f(t)\right\} = F(s)$

<u>Ex.</u> Find $\mathcal{L}\{1\}$. = $\int e^{-\alpha t} \cdot | dt$ $=\lim_{R\to\infty}\int_{0}^{R}e^{-st}dt=\lim_{R\to\infty}\frac{1}{-s}e^{-st}\Big|_{0}^{R}$ $= \lim_{\substack{n \to \infty \\ n \to \infty}} \frac{1}{-p} e^{-pR} + \frac{1}{p} e^{-pR} = \lim_{\substack{n \to \infty \\ R \to \infty}} \frac{1}{p} e^{-pR} + \frac{1}{p} e^{-pR} = \lim_{\substack{n \to \infty \\ R \to \infty}} \frac{1}{p} e^{-pR} + \frac{1}{p} e^{-pR} = \frac{1}{p}$

Find the domain.

Ex. Find
$$\mathcal{L}\lbrace t\rbrace$$
. = $\int_{0}^{\infty} e^{-at} \cdot t \, dt$
 $\downarrow_{n=t}$ $dv = e^{-at} dt$
 $du = dt$ $v = \frac{1}{-a} e^{-at}$
 $= \frac{t}{-a} e^{-at} + \frac{1}{-a} \cdot \frac{1}{-a} e^{-at} \int_{0}^{\infty} e^{-at} \int_{0}^{$



Some basic transforms are on p. 277

$\rightarrow \mathcal{L}$ is linear

 $\underline{\text{Ex. Find } \mathcal{L}\{1+5t\}} = \mathcal{L}\{i\} + \mathcal{L}\{5t\}$ $= \mathcal{L}\{i\} + 5\mathcal{L}\{t\}$ $= \frac{1}{\rho} + \frac{5}{\rho^2}$

- A function *f* is of <u>exponential order</u> *c* if there exist constants *c*, M > 0, and T > 0 such that $|f(t)| < Me^{ct}$ for all t > T.
- <u>Thm.</u> If *f* is a piecewise continuous function on $[0,\infty)$ and of exponential order *c*, then $\mathcal{L}{f(t)}$ exists for s > c.
- → This is not a requirement for the existence of \mathcal{L} , others also have an \mathcal{L} .

<u>Thm.</u> If *f* is piecewise continuous on $(0,\infty)$ and of exponential order, and if $F(s) = \mathcal{L}\{f(t)\}, \text{ then } \lim_{s \to \infty} F(s) = 0$

$$F(a) = a + 2$$

$$f$$

<u>Ex.</u> Find $\mathcal{L}{f(t)}$, where $f(t) = \begin{cases} 0 & 0 \le t \le 3\\ 2 & t > 3 \end{cases}$

