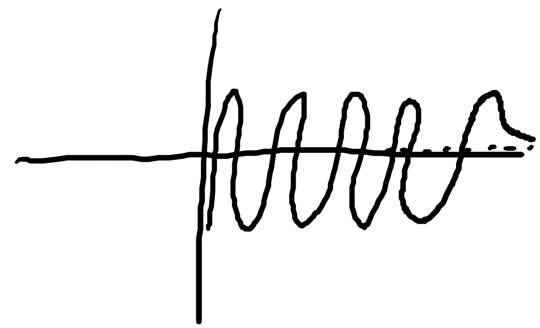
# Asymptotes and Continuity

Def. A horizontal asymptote of $f(x)$ occurs at $y = L$ if $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$

- A graph can cross a horizontal asymptote

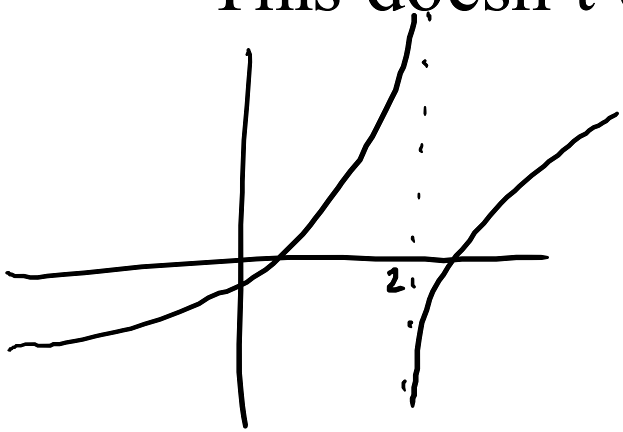
Ex. Consider $f(x) = \sin\left(\frac{1}{x}\right)$.

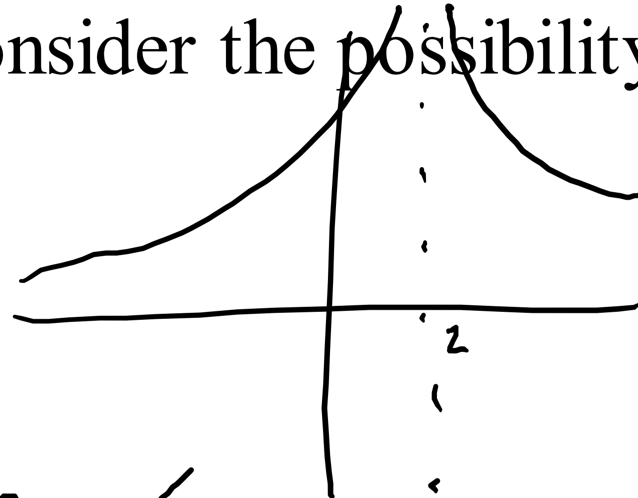
$\lim_{x\to\infty} \sin\left(\frac{1}{x}\right) = 0 \Rightarrow y=0$ is horiz. asymp.

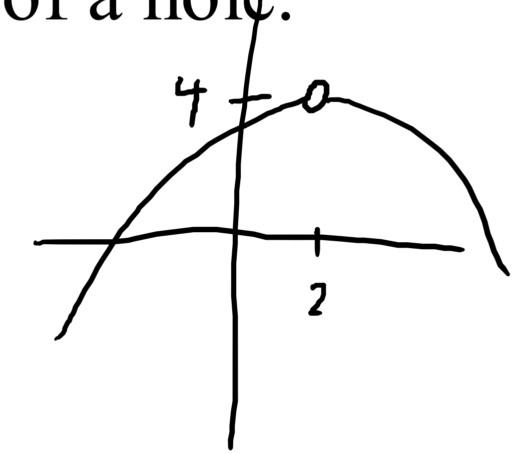


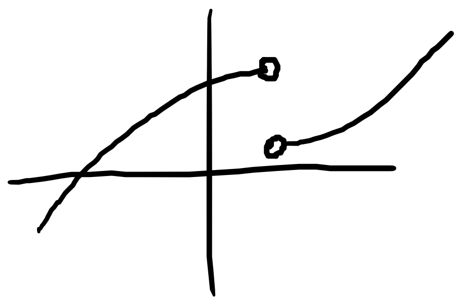
Def. (informal) A vertical asymptote of $f(x)$ occurs at values of $x$ where $f(x)$ is undefined (sort of).

- This doesn't consider the possibility of a hole.



Def. (formal) Consider the point $x = a$, such that $f(a)$ is undefined.

- The graph has a vertical asymptote if $\lim_{x \to a} f(x) = \infty$ or $-\infty$
- The graph has a hole if $\lim_{x \to a} f(x) =$ a finite value.

Ex. Find all asymptotes of $f(x) = \frac{x^2-6x+5}{x^2-7x+10}$.

$x^2-7x+10 = (x-2)(x-5)$

$x=2 \quad x=5$

horiz.

$$\lim_{x\to\infty} \frac{x^2-6x+5}{x^2-7x+10} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$$

$\boxed{y=1}$

vert.

$$\lim_{x\to5} \frac{x^2-6x+5}{x^2-7x+10} = \cancel{\frac{0}{0}} \quad \lim_{x\to5} \frac{(x-1)\cancel{(x-5)}}{(x-2)\cancel{(x-5)}} = \lim_{x\to5} \frac{x-1}{x-2} = \frac{4}{3}$$

$\boxed{\text{hole } (5, \tfrac{4}{3})}$

$$\lim_{x\to2} \frac{x^2-6x+5}{x^2-7x+10} = \frac{-3}{0} = \pm\infty$$

$\boxed{\text{vert. asymp. } x=2}$

Ex. Find all asymptotes of $f(x) = \frac{1}{e^x+1}$. $\neq 0$

no vert. asymp.

horiz.

$$\lim_{x\to\infty} \frac{1}{e^x+1} = \frac{1}{\infty} = 0$$

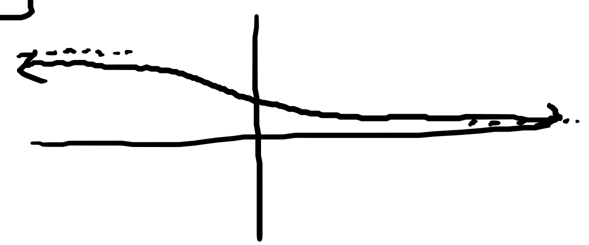
$$\boxed{y=0}$$

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$\lim_{x\to-\infty} \frac{1}{e^x+1} = 1$$

$e^x \to 0$

$$\boxed{y=1}$$



Summary

For a horizontal asymptote,

$$x \to \infty \text{ and } f(x) \to \text{finite}$$

For a vertical asymptote,

$$x \to \text{finite and } f(x) \to \infty$$

Def. (informal) A function is continuous on an interval if the graph has no gaps, jumps, or breaks on the interval.

Ex. Is $f(x) = \frac{1}{x+2}$ continuous on $[0,5]$?

yes

Def. (formal) A function $f(x)$ is continuous on an interval if, for all points $c$ on the interval:

i. $\lim_{x \to c} f(x)$ exists

ii. $f(c)$ exists

iii. $\lim_{x \to c} f(x) = f(c)$

Ex. Let $f(x) = \begin{cases} \frac{e^x - 1}{2x}, & x \neq 0 \\ B, & x = 0 \end{cases}$

Find a value of $B$ so that $f(x)$ is continuous at $x = 0$.

i) $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{e^x - 1}{2x} = \cancel{\frac{0}{0}} = \frac{1}{2}$

ii) $f(0) = B$

iii) $\lim_{x \to 0} f(x) = f(0)$

$\boxed{\frac{1}{2} = B}$

Unit 1 Progress Check: MCQ Part A

- Skip #2-4, 16, 18

Unit 1 Progress Check: MCQ Part B

- Skip #1, 3-6

Unit 1 Progress Check: MCQ Part C

- Skip #3, 13-15