Functions

<u>Def.</u> A <u>function</u> is a relationship where no two points have the same *x*-coordinate.

- Each *x*-coordinate is associated with at most one *y*-coordinate
- The graph passes the vertical line test

Domain → all possible values of *x* where the function is defined.

Range → all possible values that the function attains

Ex. Find the domain and range.

a) $y = \sqrt{x-4} + 2$

Domain: $x \geq 4$

$[4, \infty)$

Range $y \geq 2$

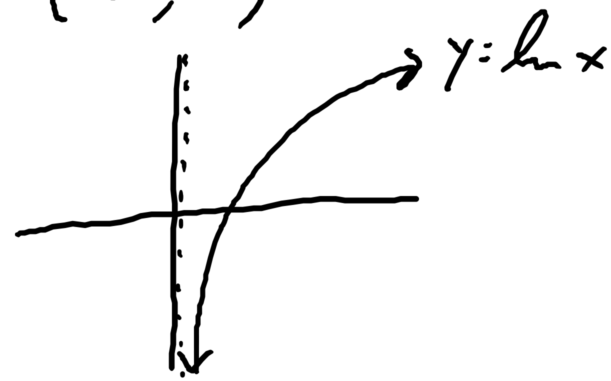
$[2, \infty)$

b) $y = \ln(x-1)$

Domain: $x > 1$

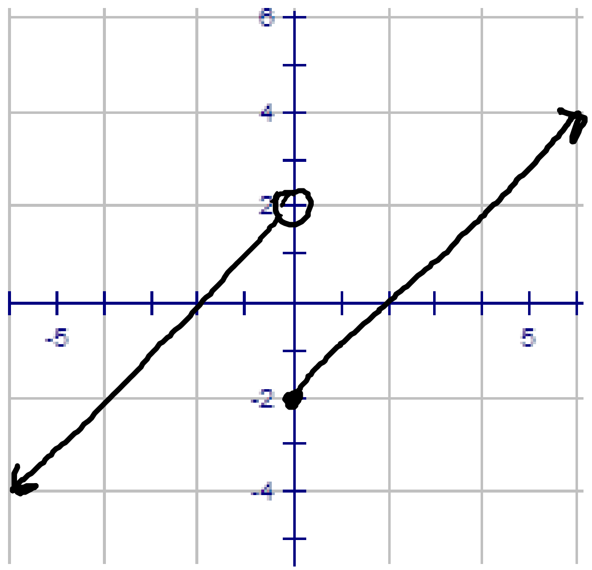
Range: all reals $\mathbb{R}$

$(-\infty, \infty)$



Def. A piecewise function is a function whose equation depends of the value of *x* where it is being evaluated.

Ex. Graph $f(x) = \begin{cases} x+2 & x<0 \\ x-2 & x \geq 0 \end{cases}$



The absolute value function is an example of a piecewise function:

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Linear and Polynomial Functions

$$y = mx + b \quad \text{or} \quad y - y_1 = m(x - x_1)$$

Either form is fine, you don't need to simplify your equation.

Ex. Find the equations of the lines parallel to and perpendicular to $y - 5x = 3$ that contain the point (2,1).

$y = 5x + 3$

parallel

$m = 5$

$(2, 1)$

$y - y_1 = m(x - x_1)$

$y - 1 = 5(x - 2)$

perp.

$m = -\frac{1}{5}$

$(2, 1)$

$y - 1 = -\frac{1}{5}(x - 2)$

The <u>standard form</u> of a quadratic function is

$$f(x) = a(x-h)^2 + k$$

The vertex of the parabola is the point (h,k).

If $a > 0$, the parabola opens upward.

If $a < 0$, the parabola opens downward.

Ex. Write the equation of the parabola whose vertex is (1,2) and that contains the point (3,5).

$$y = a(x-h)^2 + k$$

$$y = a(x-1)^2 + 2$$

$$5 = a(3-1)^2 + 2$$

$$5 = 4a + 2$$

$$3 = 4a$$

$$a = \frac{3}{4}$$

$$\boxed{y = \frac{3}{4}(x-1)^2 + 2}$$

Exponential Functions

$$P = P_0 a^t \quad \text{or} \quad P = P_0 e^{kt} \quad \text{or} \quad P = P_0 (1 + r)^t$$

P_0 = initial value
a = base
k = continuous growth rate
r = annual growth rate

Laws of Exponents can be found on p. 52

Ex. The population of Quahog is 10,000,000 and it has an annual growth rate of 2%. Find the doubling time.

$$P = P_0(1+r)^t$$

$$20{,}000{,}000 = 10{,}000{,}000(1+.02)^t$$

$$2 = 1.02^t$$

$$\ln 2 = \ln(1.02^t)$$

$$\frac{\ln 2}{\ln 1.02} = \frac{t \ln 1.02}{\ln 1.02}$$

$$t = \frac{\ln 2}{\ln 1.02}$$

$$= 35.003 \text{ yrs.}$$

Ex. Assume that housing prices grow exponentially and that Mr. Burns' mansion cost $50,000 in 1970 and $200,000 in 1990. If *t* is years since 1970, write an equation that represents the cost of the mansion as a function of *t*. How much would the house cost in 2012?

$t = 0 \quad P = 50{,}000$

$t = 20 \quad P = 200{,}000$

$\to t = 42$

$$P = P_0 e^{kt}$$

$$200{,}000 = 50{,}000 e^{k(20)}$$

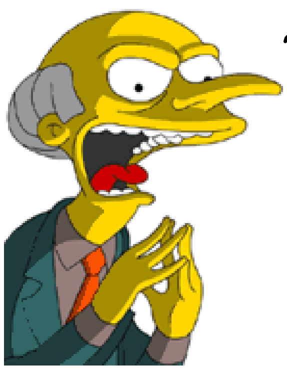
$$4 = e^{20k}$$

$$\ln 4 = \ln(e^{20k})$$

$$\ln 4 = 20k$$

$$k = \frac{\ln 4}{20}$$

$$k = .069$$



$$P = 50{,}000 e^{.069t}$$

$$P = 50{,}000 e^{.069(42)}$$

$$= \$918{,}958.68$$

- Exponential functions dominate power functions as $x \to \infty$.

<u>Ex.</u> $\lim_{x \to \infty} \frac{2^x}{x^2}$ As $x \to \infty$, $y = \frac{2^x}{x^2} \to \infty$.

$\frac{\text{BIGGER}}{\text{BIG}}$

Ex. Solve for *x*.

a) $9^x = 3^{x+1}$

$(3^2)^x = 3^{x+1}$

$3^{2x} = 3^{x+1}$ → $2x = x+1$

$x = 1$

b) $(\frac{1}{2})^x = 8$

$(2^{-1})^x = 2^3$

$2^{-x} = 2^3$ → $-x = 3$

$x = -3$