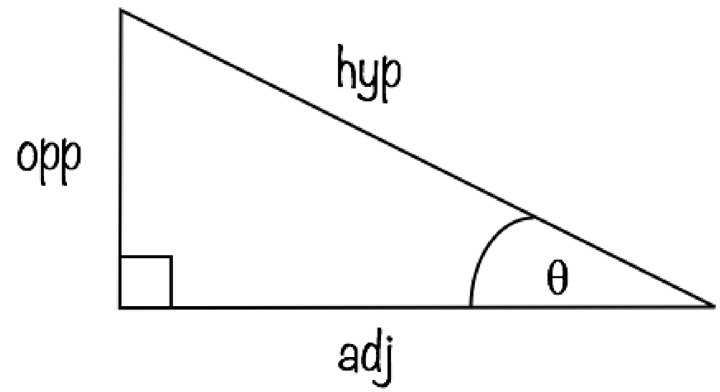
Trigonometry

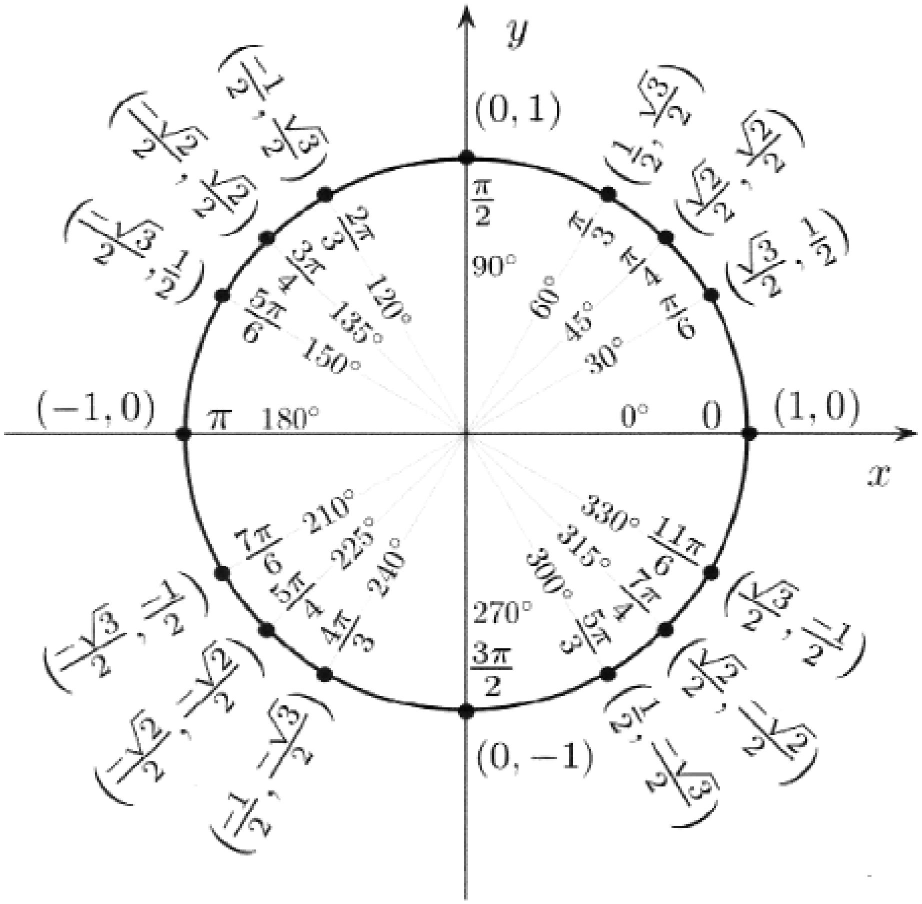
$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$



$(x, y) = (\cos\theta, \sin\theta)$



$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sin 2x = 2\sin x \cos x$$

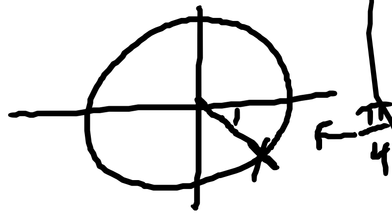
$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

Ex. Find the trig ratios for the given angle.

a) $\frac{7\pi}{4}$ rad

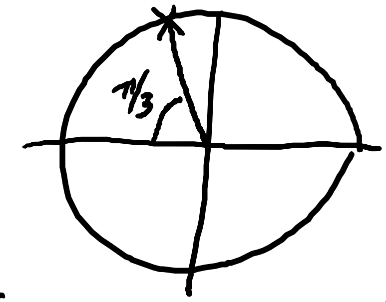


$\sin\frac{7\pi}{4} = \frac{-\sqrt{2}}{2}$ $\quad$ $\csc\frac{7\pi}{4} = \frac{-2}{\sqrt{2}}$

$\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad$ $\sec\frac{7\pi}{4} = \frac{2}{\sqrt{2}}$

$\tan\frac{7\pi}{4} = -1$ $\quad$ $\cot\frac{7\pi}{4} = -1$

b) $-\frac{4\pi}{3}$ rad



$\sin\frac{-4\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad$ $\csc\frac{-4\pi}{3} = \frac{2}{\sqrt{3}}$

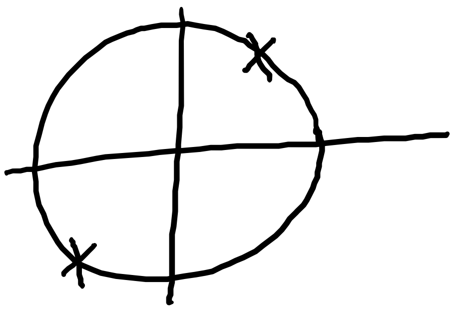
$\cos\frac{-4\pi}{3} = -\frac{1}{2}$ $\quad$ $\sec\frac{-4\pi}{3} = -2$

$\tan\frac{-4\pi}{3} = -\sqrt{3}$ $\quad$ $\cot\frac{-4\pi}{3} = -\frac{1}{\sqrt{3}}$

Ex. Find all solutions on $[0, 2\pi]$.

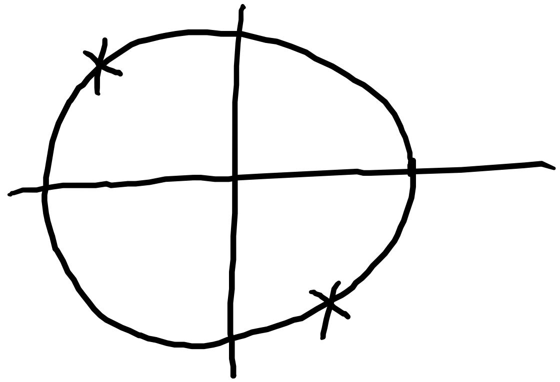
a) $|\tan x| = 1$

$\tan x = 1$



$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\tan x = -1$



$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Ex. Find all solutions on [0,2π].

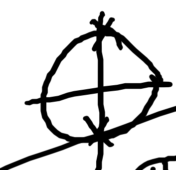
b) $2\cos x + \sin 2x = 0$

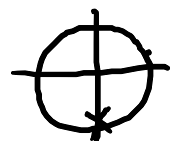
$$2\cos x + 2\sin x\cos x = 0$$

$$2\cos x\,(1 + \sin x) = 0$$

$$2\cos x = 0 \qquad\qquad 1 + \sin x = 0$$

$$\cos x = 0 \qquad\qquad \sin x = -1$$





$$x = \frac{\pi}{2},\ \frac{3\pi}{2} \qquad\qquad x = \frac{3\pi}{2}$$

Ex. Find all solutions on [0,2π].

c) $2 + \cos 2x = 3\cos x$

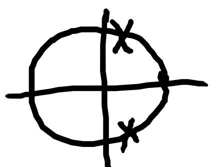
$$2y^2 - 3y + 1$$
$$(2y-1)(y-1)$$

$$2 + 2\cos^2 x - 1 = 3\cos x$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

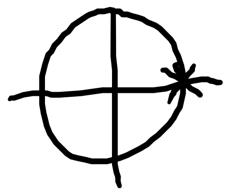
$$(2\cos x - 1)(\cos x - 1) = 0$$

$\cos x = \frac{1}{2}$



$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\cos x = 1$



$x = 0, 2\pi$

Moving and Combining Functions

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift c units *upward:* $h(x) = f(x) + c$
2. Vertical shift c units *downward:* $h(x) = f(x) - c$

outside → vert.

3. Horizontal shift c units to the *right:* $h(x) = f(x - c)$
4. Horizontal shift c units to the *left:* $h(x) = f(x + c)$

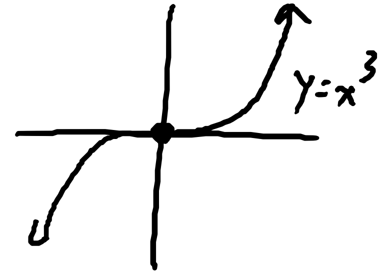
inside → horiz. (opp.)

Reflections in the Coordinate Axes

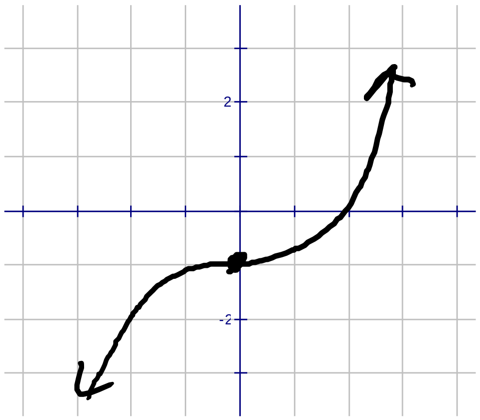
Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the x-axis: $h(x) = -f(x)$ → outside = vert. flip
2. Reflection in the y-axis: $h(x) = f(-x)$ → inside = horiz. flip

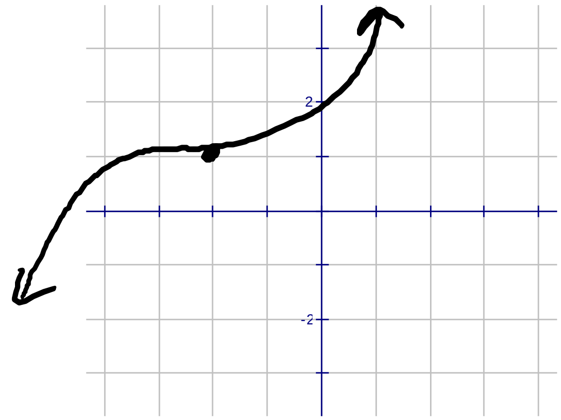
Ex. Use the graph of $f(x) = x^3$ to sketch:



a. $g(x) = x^3 - 1$

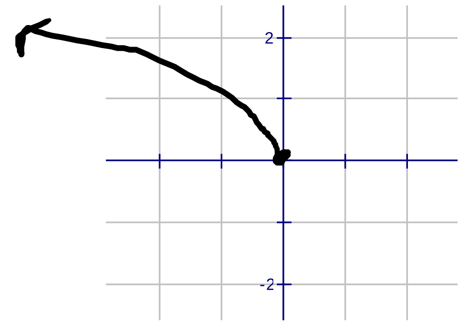


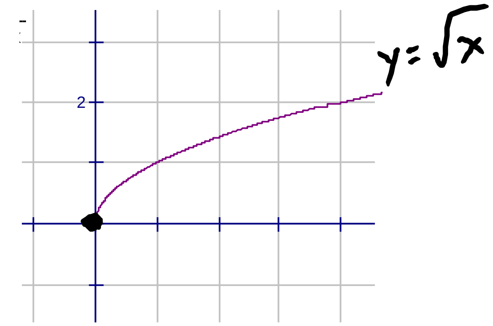
b. $g(x) = \overbrace{(x + 2)}^{\text{left 2}}{}^3 \overbrace{+ 1}^{\text{up 1}}$



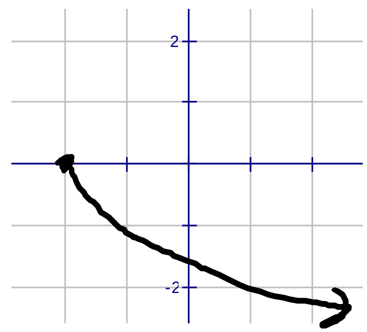
Ex. Use the graph of $f(x)=\sqrt{x}$ below to describe and draw the graph of:

a) $f(x)=\sqrt{-x}$

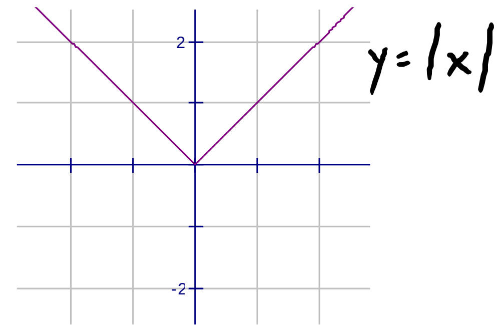




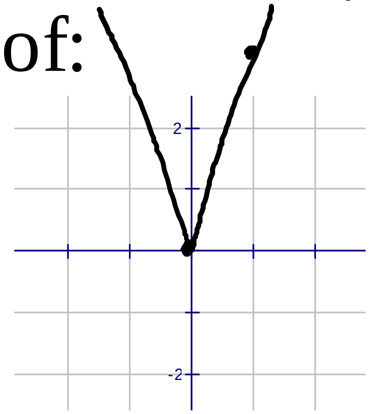
b) $f(x)=-\sqrt{\underbrace{x+2}_{\text{left}}}$



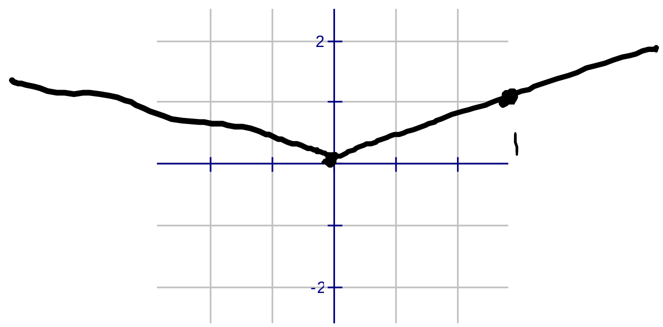
Ex. Use the graph of $f(x)=|x|$ below to draw the graph of:



a) $f(x)=3|x|$



b) $f(x)=\frac{1}{3}|x|$



$$(f \circ g)(x) \text{ means } f(g(x))$$

Ex. Given $f(x) = x + 2$ and $g(x) = 4 - x^2$

a) $(f \circ g)(x) = f(g(x)) = f(4-x^2) = 4-x^2+2$

b) $(g \circ f)(x) = g(f(x)) = g(x+2) = 4-(x+2)^2$

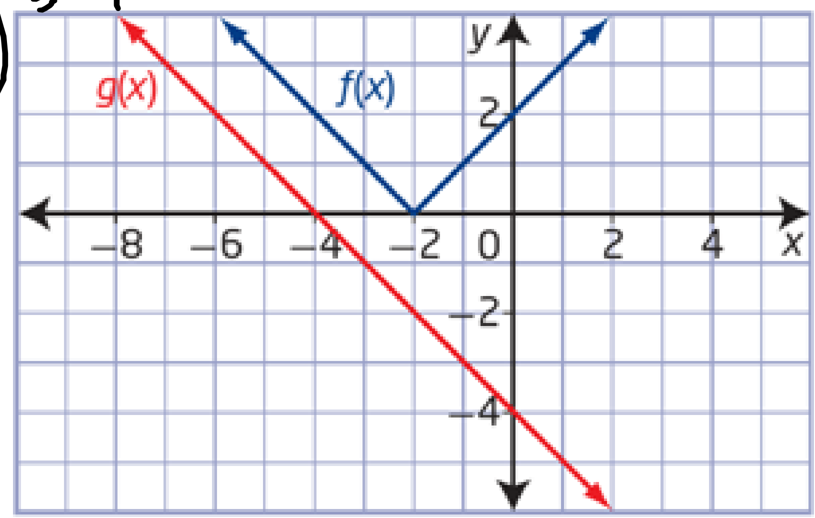
c) $(g \circ g)(x) = g(g(x)) = g(4-x^2) = 4-(4-x^2)^2$

Ex. Use the graphs below to evaluate.

a) $(f \circ g)(-4) = f(g(-4)) = f(0) = 2$

b) $(f \circ g)(0) = f(g(0)) = f(-4) = 2$

c) $(g \circ f)(-2) = g(f(-2)) = g(0) = -4$



Ex. Write $h(x) = \dfrac{1}{(x-2)^2}$ as the composition of two functions.

$$h(x) = f(g(x)) = \frac{1}{(x-2)^2}$$

$$f(x) = \frac{1}{x^2}$$

$$g(x) = x - 2$$