Inverse Functions

Def. A function is invertible if no two points have the same y-coordinate.

- The function is called one-to-one
- Each y corresponds to at most one x
- The graph passes the horizontal line test
- To find the inverse, switch x and y, and then solve for y.

→You may not find the equation for the inverse, even if the function is invertible

Ex. Let $\cancel{f(x)} = \dfrac{1}{2x-5}$, find $f^{-1}(x)$

y

$$x = \frac{1}{2y-5}$$

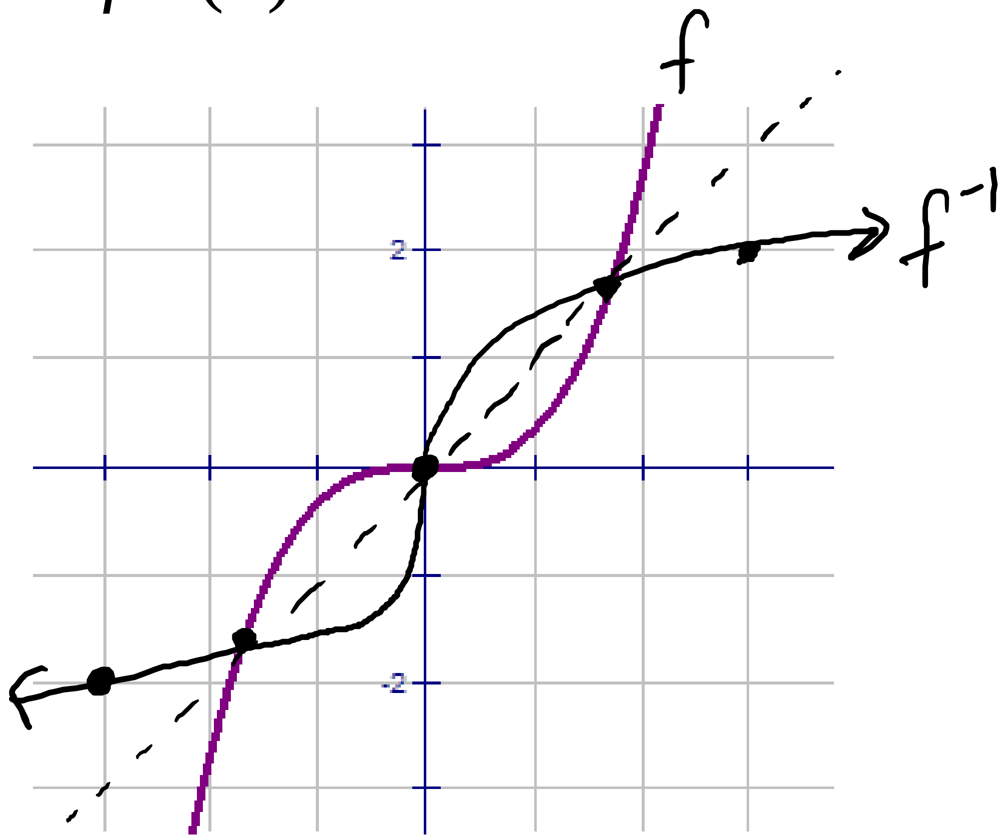
$$\frac{1}{x} = 2y-5$$

$$\frac{1}{x} + 5 = 2y$$

$$\frac{\frac{1}{x}+5}{2} = y$$

$$f^{-1}(x) = \frac{\frac{1}{x}+5}{2}$$

Ex. Sketch $f^{-1}(x)$



Ex. Let $\cancel{f(x)} = x^3 + x$, find $f^{-1}(10)$

y

On f^{-1} graph

$x = y^3 + y$

$10 = y^3 + y$

$y = 2$

$f^{-1}(10) = 2$

$x \qquad y$

Domain of $f \leftarrow\rightarrow$ Range of f^{-1}

Range of $f \leftarrow\rightarrow$ Domain of f^{-1}

Logarithms

$$\cancel{\log}_a x = y \Leftrightarrow a^y = x$$

$$\ln x = \log_e x$$

$\ln(x+5) = ??$

<u>Laws of Logarithms</u>

$$\ln(AB) = \ln A + \ln B$$

$$\log_a a^x = x$$

$$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$$

$$a^{\log_a x} = x$$

$$\ln A^n = n \ln A$$

$$\ln 1 = 0$$

Ex. Evaluate by hand.

a) $\log_2 32 = x \longrightarrow 32 = 2^x \longrightarrow x = 5$

b) $\log_3 1 = 0$

c) $\log_9 3 = x \longrightarrow 3 = 9^x \longrightarrow x = \frac{1}{2}$

d) $\log_{10} \frac{1}{100} = x \longrightarrow \frac{1}{100} = 10^x \longrightarrow 10^{-2} = 10^x$

$x = -2$

Ex. Express as a single logarithm

$$2\log x - 3\log y - \log z$$

$$\log(x^2) - \log(y^3) - \log z$$

$$\log\left(\frac{x^2}{y^3 z}\right)$$

If we want to evaluate a logarithm on the calculator, we may need to change the base

$$\log_a x = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

Ex. Evaluate $f(x) = \log_4 x$ at $x = 25$.

$$f(25) = \log_4 25 = \frac{\ln 25}{\ln 4} = 2.322$$

Ex. Solve $10e^x = 7^x$

$$\ln(10 \cdot e^x) = \ln(7^x)$$

$$\ln 10 + \ln(e^x) = x \ln 7$$

$$\ln 10 + x = x \ln 7$$

$$\ln 10 = x \ln 7 - x$$

$$\ln 10 = x(\ln 7 - 1)$$

$$x = \frac{\ln 10}{\ln 7 - 1}$$

$$12 + x = 5x$$

$$12 = 4x$$

Ex. The half-life of a substance is 12 days. If there are 10.32g initially, write an equation that represents the amount, *A*, of the substance after *t* days. When will there be 1g left?

$$\boxed{A = A_0\left(\tfrac{1}{2}\right)^{t/\lambda}}$$

$$A = A_0 e^{kt}$$

$$5.16 = 10.32\, e^{k(12)}$$

$$\ln .5 = \ln e^{12k}$$

$$\ln(.5) = 12k$$

$$k = \frac{\ln(.5)}{12} = -.058$$

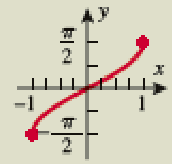
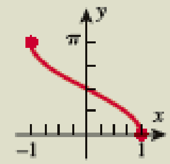
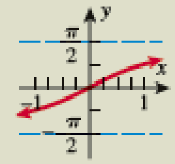
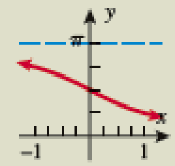
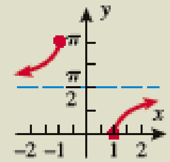
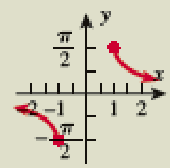
$$A = 10.32\, e^{-.058t}$$

$$1 = 10.32\, e^{-.058t}$$

$$\ln \frac{1}{10.32} = \ln e^{-.058t}$$

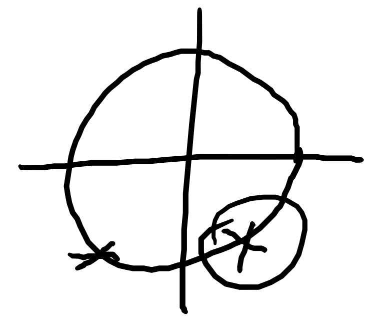
$$\ln\left(\frac{1}{10.32}\right) = -.058t$$

$$t = 40.408$$

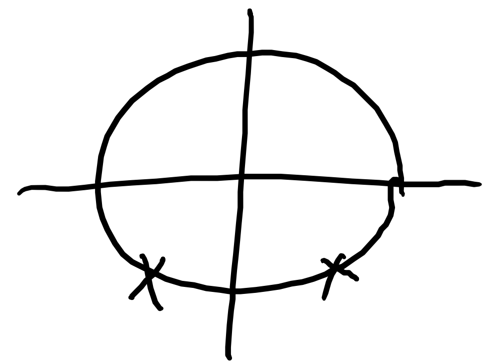
Inverse Function	Domain	Range	Graph
$y = \sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ or $[-90°, 90°]$	
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$ or $[0°, 180°]$	
$y = \tan^{-1}x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ or $(-90°, 90°)$	
$y = \cot^{-1}x$	$(-\infty, \infty)$	$(0, \pi)$ or $(0°, 180°)$	
$y = \sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ or $[0°, 90°) \cup (90°, 180°]$	
$y = \csc^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ or $[-90°, 0°) \cup (0°, 90°]$	

Ex. Evaluate by hand.

a) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$



b) $\sin^{-1}\left(\underbrace{\sin\frac{5\pi}{4}}_{-\frac{\sqrt{2}}{2}}\right) = -\frac{\pi}{4}$



Ex. Use a calculator to find the zeroes of

$f(x) = x^3 - 2x^2 - 19x + 10$

$x = -3.760,\ .506,\ 5.254$

Ex. Use a calculator to find all solutions to $x^3 - 3x - 6 = 3\cos x$ on the interval (-3,3).

$$x^3 - 3x - 6 - 3\cos x = 0$$

$$x = 2.213$$