Warm up Problems

A particle moves along the x-axis with velocity $v(t) = e^{t/2}\cos\left(\frac{t^2}{8}\right)$ for $0 \le t \le 7$.

1. Find times when particle is at rest. $v(t)=0$ $t=3.545,\ 6.140$

2. Is particle moving left or right at time $t = 4$? $v(4)=-3.075$ left

3. Is velocity increasing or decreasing at time $t = 4$? $v'(4)=-8.256$ dec.

4. Is speed increasing or decreasing at time $t = 4$?

inc. because $v(4)$ and $a(4)$ are same sign

Linear Approximation

We can use the tangent line to approximate the value of a function at a difficult point.

$$f(x) \approx f(a) + f'(a)(x - a) \text{ near } x = a$$

→ This is just the equation of the tangent line.

Called linear approximation, tangent line approximation, or local linearization.

Ex. Find the linearization of $f(x) = \frac{1}{x^2}$ at $x = 1$.

$= x^{-2} \rightarrow f(1) = \frac{1}{1^2} = 1$

$f'(x) = -2x^{-3} \rightarrow f'(1) = \frac{-2}{1^3} = -2$

$y = f(1) + f'(1)(x-1)$

$L(x) = 1 + (-2)(x-1)$

Ex. Find the linear approx. of $f(x) = \sqrt{x+1}$ near $x = 3$, then use it to approx. $f(3.04)$.

$f(3) = \sqrt{4} = 2$

$f'(x) = \frac{1}{2}(x+1)^{-1/2}$

$f'(3) = \frac{1}{2}(4)^{-1/2} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$y = f(3) + f'(3)(x-3)$

$y = 2 + \frac{1}{4}(x-3)$

$f(3.04) \approx 2 + \frac{1}{4}(3.04-3)$

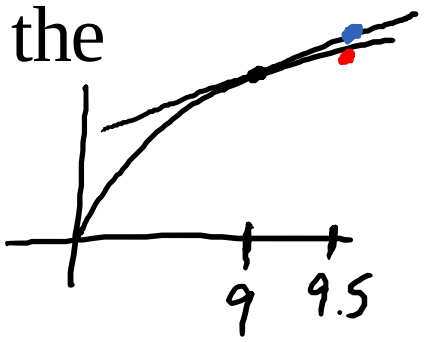
$\approx 2 + \frac{1}{4}(.04)$

$\approx 2 + .01$

≈ 2.01

Ex. Use linear approximation to estimate $\sqrt{9.5}$.

Is this estimate higher or lower than the actual value?



$f(x) = \sqrt{x}$, $a = 9$

$\rightarrow f(9) = \sqrt{9} = 3$

$f'(x) = \frac{1}{2}x^{-1/2} \rightarrow f'(9) = \frac{1}{2}(9)^{-1/2} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$y = f(9) + f'(9)(x-9)$

$y = 3 + \frac{1}{6}(x-9)$

$f(9.5) \approx 3 + \frac{1}{6}(9.5-9) = 3 + \frac{1}{6} \cdot \frac{1}{2} = 3 + \frac{1}{12} = \boxed{\frac{37}{12}}$

higher because $f(x)$ is conc. down

Ex. It is known that $f(5) = 27$ and $f'(5) = 6$. Use the tangent line to find an approximation of $f(5.2)$.

$$y = f(5) + f'(5)(x-5)$$

$$y = 27 + 6(x-5)$$

$$f(5.2) \approx 27 + 6(5.2-5)$$

$$\approx 27 + 6(.2)$$

$$\approx 27 + 1.2$$

$$\approx 28.2$$

Unit 2 Progress Check: MCQ Part B

- Do #4-5, 7-15

Unit 3 Progress Check: MCQ

- Do them all

Unit 4 Progress Check: MCQ

- Do #2, 5-7, 14-15