Warm up Problem

If $f(x) = \frac{1}{x^2}$, find $f'(2)$.

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\frac{1}{(2+h)^2} - \frac{1}{2^2}}{h}$$

$$= \lim_{h \to 0} \left(\frac{1}{(2+h)^2} - \frac{1}{2^2} \right) \cdot \frac{1}{h} = \lim_{h \to 0} \frac{2^2 - (2+h)^2}{(2+h)^2 \cdot 2^2} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{4 - (4 + 4h + h^2)}{(2+h)^2 \cdot 2^2} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-4h - h^2}{(2+h)^2 \cdot 2^2} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{h(-4-h)}{(2+h)^2 \cdot 2^2} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-4-h}{(2+h)^2 \cdot 2^2} = \frac{-4}{2^2 \cdot 2^2} = \boxed{-\frac{1}{4}}$$

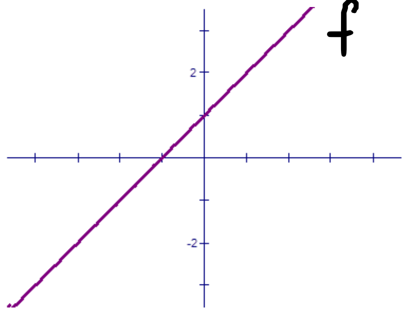
The Derivative Function

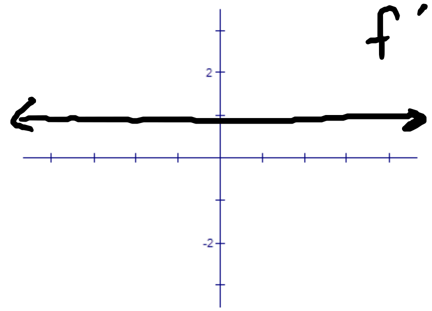
Def. For any function $f(x)$, we can find the derivative function $f'(x)$ by:

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

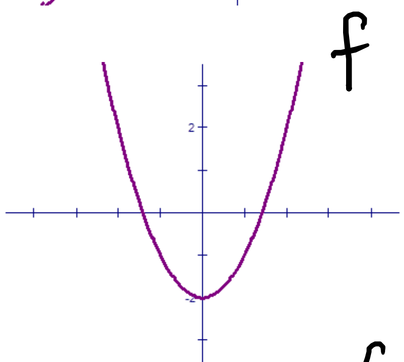
Ex. Given the graph of $f(x)$, sketch $f'(x)$.

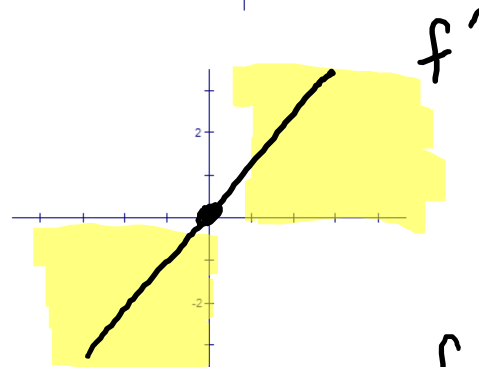
a.



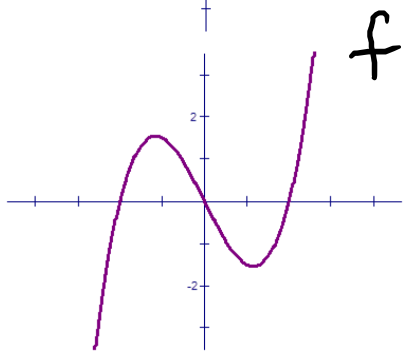


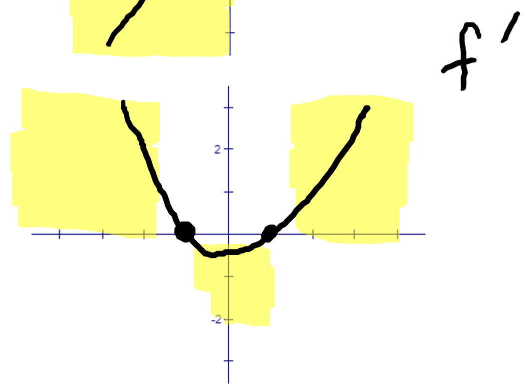
b.



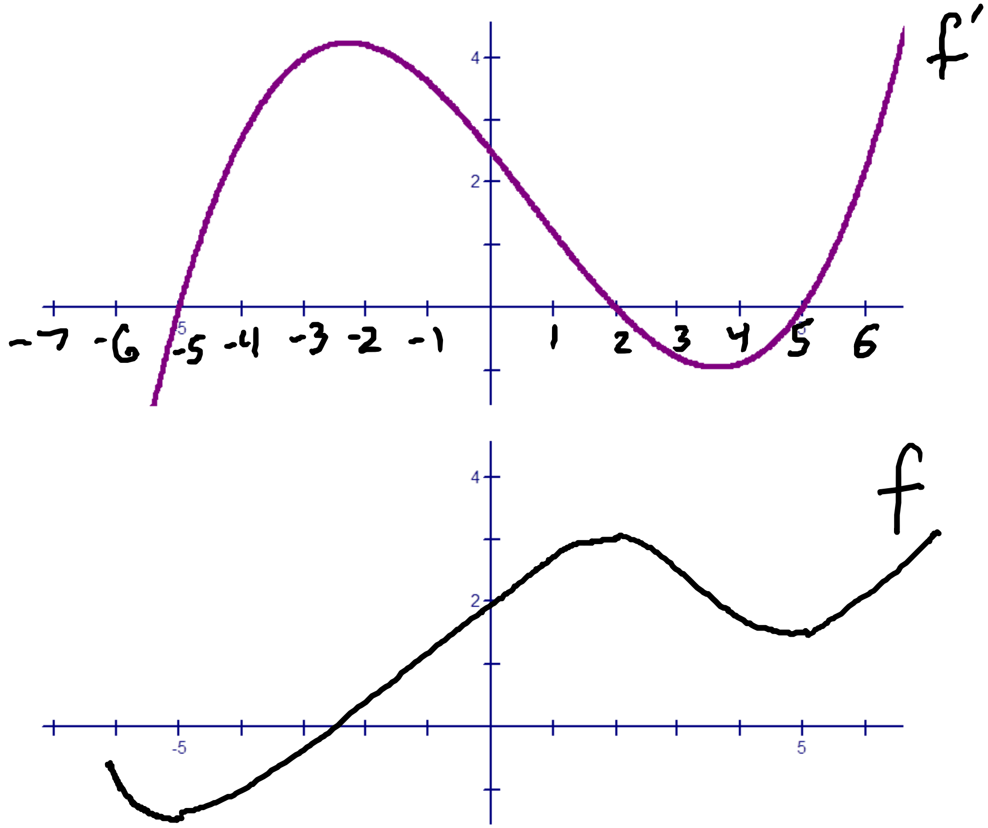


c.





<u>Ex.</u> Given the graph of $f'(x)$, sketch $f(x)$.



<u>Note:</u> $f'(a)$ is a number

$f'(x)$ is a function

<u>Algebraically</u>

<u>Ex.</u> If $f(x) = 5$, find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{5 - 5}{h} = 0$$

$$f'(x) = 0$$

Ex. If $f(x) = 3x - 2$, find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[3(x+h) - 2] - [3x - 2]}{h}$$

$$= \lim_{h \to 0} \frac{3x + 3h - 2 - 3x + 2}{h} = \lim_{h \to 0} \frac{3h}{h} = 3$$

$$f'(x) = 3$$

Ex. If $f(x) = x^2$, find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h \to 0} (2x+h) = 2x$$

$$f'(x) = 2x$$

Pract. If $f(x) = x^3$, find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h}$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2) = 3x^2$$

$$\boxed{f'(x) = 3x^2}$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

Rules

If $f(x) = \text{constant}$, then $f'(x) = 0$.

If $f(x) = mx + b$, then $f'(x) = m$.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.

→Only use these rules, don't make up your own.

→If the problem says "Use definition of derivative", you must use the limit.

Ex. If $f(x) = \frac{1}{x^2}$, find $f'(2)$.

$f(x) = x^{-2}$

$f'(x) = -2x^{-3}$

$f'(2) = -2(2)^{-3} = \frac{-2}{2^3} = -\frac{1}{4}$

Note: A function is differentiable at a point if the function and its derivative are continuous at the point.

<u>Ex.</u> Is the function $f(x) = \begin{cases} x^3, & x < 2 \\ 12x - 16, & x \geq 2 \end{cases}$ differentiable at $x = 2$?

$f'(x) = \begin{cases} 3x^2 & x < 2 \\ 12 & x \geq 2 \end{cases}$

Is f cont.?	Is f′ cont.?
$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (12x - 16) = 8$	$\lim\limits_{x \to 2^+} f'(x) = \lim\limits_{x \to 2^+} 12 = 12$
$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} x^3 = 8$	$\lim\limits_{x \to 2^-} f'(x) = \lim\limits_{x \to 2^-} 3x^2 = 12$
✓	✓

yes