Derivatives of Polynomials and Exponents

Today (and all of next chapter), we will be creating a list of derivative formulas.

→Keep a running list in the front of your notebook so that you have easy access to it.

i. $\frac{d}{dx}[x^n] = nx^{n-1}$ ← Power Rule

ii. $\frac{d}{dx}[x] = 1$

iii. $\frac{d}{dx}[c] = 0$

iv. $\frac{d}{dx}[cf(x)] = cf'(x)$

v. $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

vi. $\frac{d}{dx}[a^x] = a^x \ln a$

vii. $\frac{d}{dx}[e^x] = e^x$

<u>Ex.</u> $y = \sqrt{3}x^7 - \frac{x^5}{5} + \pi$

$\frac{1}{5}x^5$

$y' = \sqrt{3}\,(7x^6) - \frac{1}{5}(5x^4) + 0$

$y' = 7\sqrt{3}\,x^6 - x^4$

Ex. $f(x) = 5\sqrt{x} - \frac{10}{x^2} + \frac{1}{2\sqrt{x}}$

$$= 5x^{1/2} - 10x^{-2} + \frac{1}{2}x^{-1/2}$$

$$f'(x) = \frac{5}{2}x^{-1/2} + 20x^{-3} - \frac{1}{4}x^{-3/2}$$

<u>Ex.</u> $y = 2x^{\sqrt{2}} + x + 5^{3/2}$

$y' = 2\sqrt{2}\,x^{\sqrt{2}-1} + 1 + 0$

Ex. $f(x) = \frac{x^5 - 3x^2 + 2x - 1}{x^2} = \frac{x^5}{x^2} - \frac{3x^2}{x^2} + \frac{2x}{x^2} - \frac{1}{x^2}$

$= x^3 - 3 + 2x^{-1} - x^{-2}$

$f'(x) = 3x^2 - 2x^{-2} + 2x^{-3}$

Power Rule

Power Rule!
Power Rule!
When you have *A* times an *X* to the *B*,
1 over *X* is just,
1 over *X* is Just,
you know you always use:
Power Rule!
Power Rule!
X to the minus 1.
X to the minus 1:
Power Rule.
Power Rule!
Power Rule!
Fo Polynomials, Foo!
Look *A B X* to the *B* minus one,

is the derivative.
Wake up you’re naked at School.
La la la, la la la la la la la la la la la la la. Power Rule!
Power Rule.
La la la, la la la la la la la la la la la la la. Power Rule!

Derivatives of constants are always a slope of zero.
Derivatives of constants are always a slope of zero.
Square root is the one half power,
Square Root is the one half power,
You have nothing to fear, Oh, Elephant shoes.
You have nothing to fear, Oh! how can you lose?
For all polynomials you can forget all your troubles,
For all polynomials you can forget all your troubles,
cause everyone knows you use:
cause everyone knows you use:

Ex. $y = 7^x$

$y' = 7^x \ln 7$

Ex. Find the instantaneous velocity of $s(t) = 4^t + t^4$ at the point $t = 2$.

$v(t) = 4^t \ln 4 + 4t^3$

$v(2) = 4^2 \ln 4 + 4(2)^3$

$= 16 \ln 4 + 32$

Ex. $y = e^{x}$

$y' = e^{x}$

Ex. $f(x) = e^{10x} = (e^{10})^{x}$

$f'(x) = (e^{10})^{x} \cdot \ln(e^{10})$

$= 10e^{10x}$

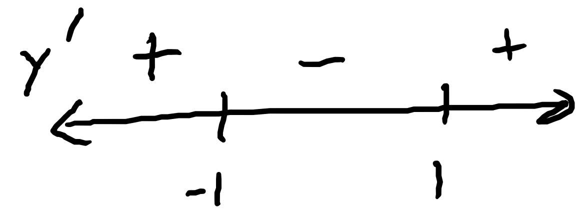
<u>Ex.</u> For what values of x is the graph of $y = x^5 - 5x$ both increasing and concave up?

$f' > 0$ $\qquad$ $f'' > 0$

$y' = 5x^4 - 5 = 0$

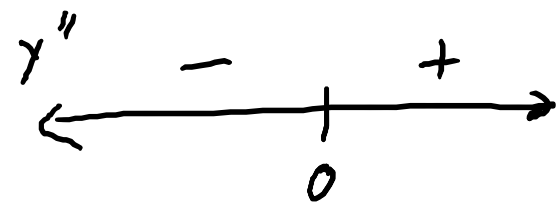
$5x^4 = 5$

$x^4 = 1$ $\rightarrow$ $x = \pm 1$



$y'' = 20x^3 = 0$

$x^3 = 0$ $\rightarrow$ $x = 0$



$\boxed{x > 1}$

$\underline{\text{Ex.}}$ Consider $f(x) = \begin{cases} |x-1|+2, & x < 1 \\ ax^2+bx, & x \geq 1 \end{cases}$

a) If $a = 2$ and $b = 3$, is f continuous at $x = 1$?

$$f(x) = \begin{cases} |x-1|+2 & x < 1 \\ 2x^2+3x & x \geq 1 \end{cases}$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2x^2+3x) = 5$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (|x-1|+2) = 2$$

$\boxed{\text{no}}$

$$f(x) = \begin{cases} |x-1|+2, & x<1 \\ ax^2+bx, & x \geq 1 \end{cases}$$

$-(x-1)+2$

$-x+3$

b) Find values for a and b so that f is differentiable at $x = 1$.

$$f'(x) = \begin{cases} -1 & x<1 \\ 2ax+b & x>1 \end{cases}$$

f and f' are cont.

f cont.?

$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} (ax^2+bx) = a+b$

$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} (|x-1|+2) = 2$

$a+b=2$

f′ cont.?

$\lim_{x\to1^+} f'(x) = \lim_{x\to1^+} (2ax+b) = 2a+b$

$\lim_{x\to1^-} f'(x) = \lim_{x\to1^-} (-1) = -1$

$2a+b=-1$

$$\begin{array}{r} 2a+b=-1 \\ -\quad a+b=2 \\ \hline a=-3 \end{array}$$

$-3+b=2$

$b=5$

Unit 2 Progress Check: MCQ Part A

- Do them all

Unit 2 Progress Check: MCQ Part B

- Do #1-3