Warm up Problems

1. $\frac{d}{dx}\left[\frac{2x^3}{5x^2-12}\right] = \frac{(5x^2-12)(6x^2)-(2x^3)(10x)}{(5x^2-12)^2}$

2. $\frac{d}{dx}[2^x(5x^2-\sqrt{e})] = 2^x(10x+0)+(5x^2-\sqrt{e})\cdot 2^x \ln 2$

3. Find the equation of the line tangent to $f(x) = e^x(x^2-5)$ at $x = 1$.

$f(1) = e^1(1-5) = -4e$

$f'(x) = e^x(2x)+(x^2-5)e^x$

$f'(1) = 2e^1 - 4e^1 = -2e$

$y + 4e = -2e(x-1)$

Trigonometry Review:

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

Ex. Prove $\frac{d}{dx}\tan x = \sec^2 x$

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\cos x(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

✓

Ex. If $y = x^2 \sin x$, find y'.

$$y' = x^2(\cos x) + \sin x(2x)$$

Ex. Find an equation of the line tangent to $y = \csc x$ at $x = \frac{\pi}{4}$.

$y' = -\csc x \cot x$

$y'\left(\frac{\pi}{4}\right) = -\csc\left(\frac{\pi}{4}\right)\cot\left(\frac{\pi}{4}\right)$

$= -\left(\frac{2}{\sqrt{2}}\right)(1) = -\sqrt{2}$

$y = \csc\left(\frac{\pi}{4}\right) = \sqrt{2}$

$y - \sqrt{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$

Ex. On the interval $[0,2\pi]$, find all values of x where $f(x) = \frac{\sec x}{1+\tan x}$ has a horizontal tangent line.

$$f'(x) = \frac{(1+\tan x)(\sec x \tan x) - \sec x(\sec^2 x)}{(1+\tan x)^2} = 0$$

$$(1+\tan x)(\sec x \tan x) - \sec x(\sec^2 x) = 0$$

$$\sec x\left[(1+\tan x)\tan x - \sec^2 x\right] = 0$$

$$\sec x(\tan x + \underbrace{\tan^2 x - \sec^2 x}_{-1}) = 0$$

$$\sec x(\tan x - 1) = 0$$

$\sec x = 0$ ✗

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

Pract. $\frac{d}{d\theta} e^{\theta} \cos\theta = e^{\theta}(-\sin\theta) + \cos\theta\, e^{\theta}$

Pract. $\frac{d}{dx} \frac{x}{2-\tan x} = \frac{(2-\tan x)\cdot 1 - x(-\sec^2 x)}{(2-\tan x)^2}$