Warm up Problems

1. If $y - 5x^{10} - \ln(xy) = 2\sin x$, find $\frac{dy}{dx}$.

2. Find the equation of the line tangent to $x^3 + y^3 = 6xy$ at (3,3).

Motion Problems

If $s(t) =$ position, then

$s'(t) = v(t) =$ velocity

$s''(t) = a(t) =$ acceleration

$|v(t)| =$ speed

$$\begin{pmatrix}\text{ave. veloc. from} \\ t = a \text{ to } t = b\end{pmatrix} = \frac{s(b) - s(a)}{b - a}$$

Ex. A particle moves along the x-axis such that its position, for $t \geq 0$, is given by $x(t) = t^3 - 6t^2 + 9t + 3$ (t is measured in minutes and x in meters).

a) Find the velocity at any time t.

$v(t) = 3t^2 - 12t + 9$

b) Find the velocity at time $t = 4$.

$v(4) = 3(4)^2 - 12(4) + 9 = 9$

c) Find all times when the particle is at rest.

$3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$

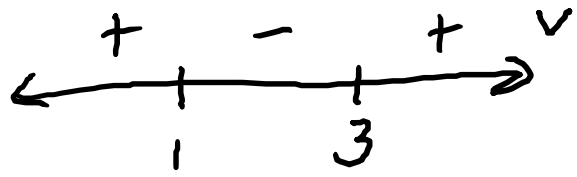
$\rightarrow t = 1, t = 3$

Ex. A particle moves along the x-axis such that its position, for $t \geq 0$, is given by $x(t) = t^3 - 6t^2 + 9t + 3$ (t is measured in minutes and x in meters).

d) When is the particle moving to the right? Justify your answer.

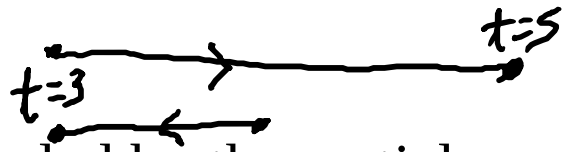
$v(t) = 3(t-1)(t-3) = 0$

$t = 1, \; t = 3$

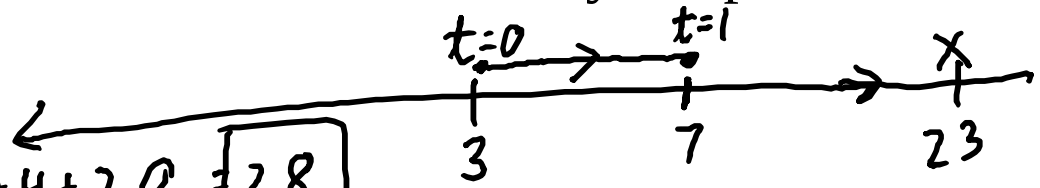


$[0,1) \quad (3,\infty)$

$v(t)$ is pos.



e) Find the total distance traveled by the particle over the interval [0,5].



$x(0) = 3$

$x(1) = 1 - 6 + 9 + 3 = 7$

$x(3) = 27 - 54 + 27 + 3 = 3$

$x(5) = 125 - 150 + 45 + 3 = 23$

$dist = 4 + 4 + 20 = \boxed{28}$

Ex. A particle moves along the x-axis such that its position, for $t \geq 0$, is given by $x(t) = t^3 - 6t^2 + 9t + 3$ (t is measured in minutes and x in meters).

$v(t) = 3t^2 - 12t + 9$

f) Find the acceleration at any time t.

$a(t) = 6t - 12$

g) Find the acceleration at time $t = 4$.

$a(4) = 6(4) - 12 = 12$

h) Is the speed increasing or decreasing at time $t = 4$? Justify your answer.

$v(4) = 9$ Speed inc. because $v(4)$ and $a(4)$ are same sign

Ex. A particle moving along the y-axis has velocity given by $v(t) = \ln(t+1)\sin(t+e^{-t})$ for $0 \le t \le 10$.

a) Find the times at which the particle changes directions. Justify your answer.

$v(t) = 0$

$t = 3.096, 6.281, 9.425$

$v(t)$ changes signs at these point

b) Find $v(4)$.

-1.237

c) Find $a(4)$.

-1.164

d) Is the speed increasing or decreasing at time $t = 4$? Justify your answer.

inc., $v(4)$ and $a(4)$ same signs