# Warm up Problems

1. If $f(x) = \frac{1}{2}x - 3$, find $f'(x)$. $= \frac{1}{2}$

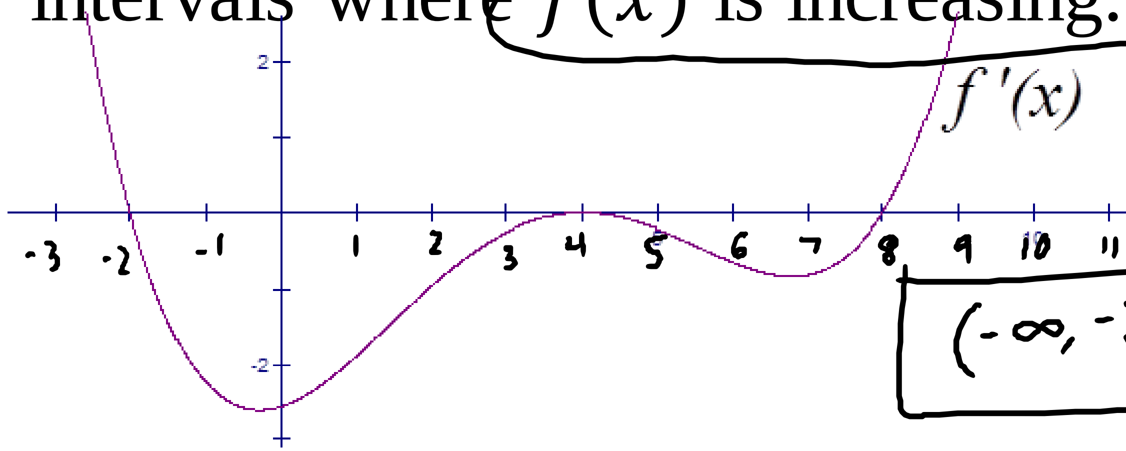
2. If $f(x) = 3x^2 - 2$, find $f'(x)$. $= 6x$

3. Given the graph of $f'(x)$, find all intervals where $f(x)$ is increasing.

→ $f$ pos. slope

$\Downarrow$

$f'$ pos. height




$\boxed{(-\infty, -2) \quad (8, \infty)}$

# Interpreting the Derivative

New Notation: If $y = f(x)$, then

$$f'(x) = \frac{dy}{dx} = (\text{derivative of } y \text{ with respect to } x)$$

$$f'(a) = \left.\frac{dy}{dx}\right|_{x=a} \qquad\qquad \frac{d}{dx}(x^3) = 3x^2$$

$\frac{dy}{dx}$ is a noun, $\frac{d}{dx}$ is a verb

Ex. The cost $C$, in dollars, of building a new Avengers facility that has area $A$ square feet is given by $C = f(A)$. What are the units of $f'(A)$? $= \$/\text{sq.ft.}$

$$f'(A) = \frac{dC}{dA} = \frac{\$}{\text{sq. ft.}}$$

sq. ft.

Ex. The cost, in dollars, for the seven dwarves to extract $T$ tons of ore from their mine is given by $M = f(T)$. What does $f'(2000) = 100$ mean?

tons $/ton

$$f'(T) = \frac{dM}{dT} = \frac{\$}{\text{tons}}$$

After 2000 tons have been removed, cost is changing at a rate of \$100/ton.

Ex. Suppose $P = f(t)$ is the population of Springfield, in millions, $t$ years since 1990. Explain $f'(15) = -2$.

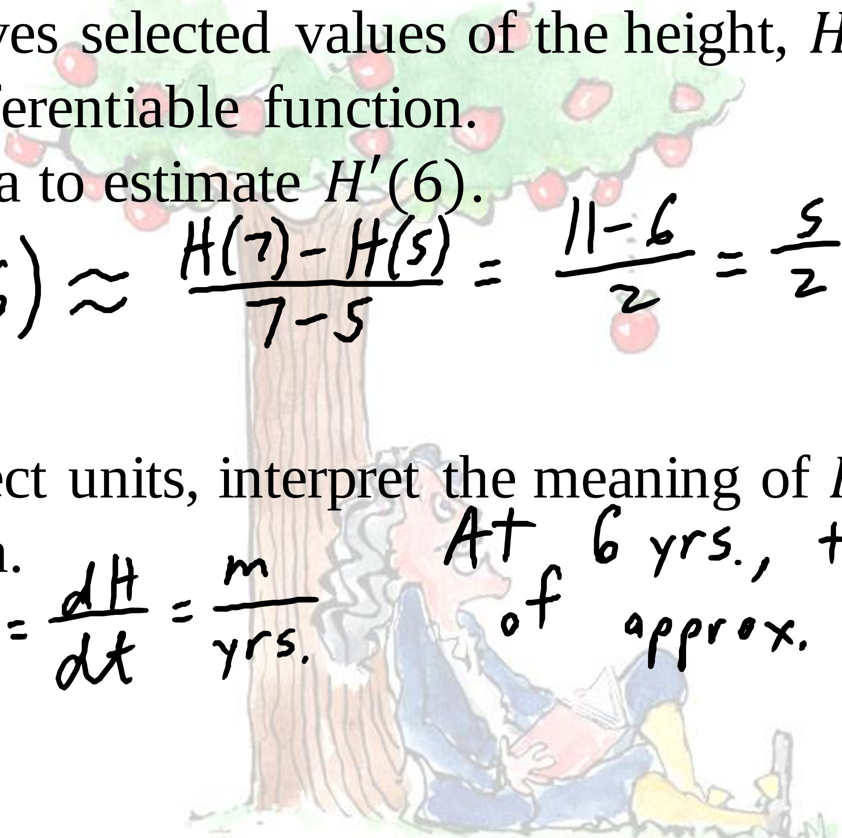
yrs.

$$f'(t) = \frac{dP}{dt} = \frac{\text{mill pop.}}{\text{yrs}}$$

In 2005, pop. changing at rate of -2 mill. people / year.

In 2005, pop. decreasing at a rate of 2 mill. people./year.

| $t$ (years) | 2 | 3 | 5 | 7 | 10 |
|---|---|---|---|---|---|
| $H(t)$ (meters) | 1.5 | 2 | 6 | 11 | 15 |



<u>Ex.</u> The table gives selected values of the height, $H$, of a tree at time $t$, where $H$ is a differentiable function.

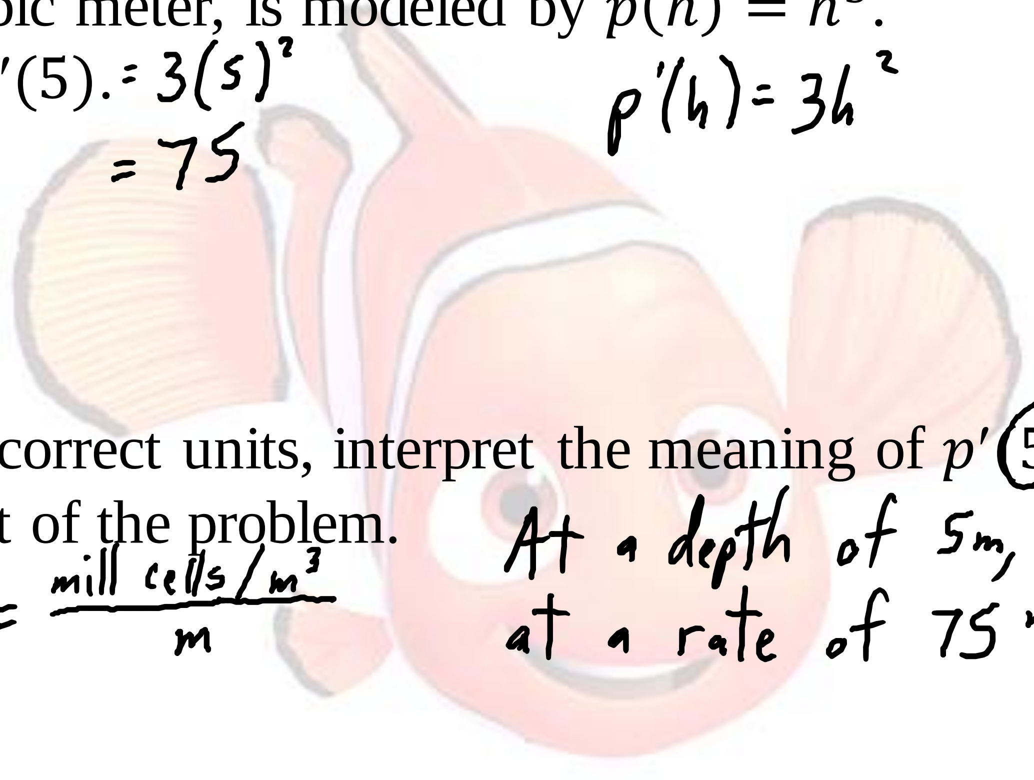
a) Use the data to estimate $H'(6)$.

$$H'(6) \approx \frac{H(7)-H(5)}{7-5} = \frac{11-6}{2} = \frac{5}{2}$$

b) Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

$$H'(t) = \frac{dH}{dt} = \frac{m}{yrs.}$$

At 6 yrs., tree grows at a rate of approx. $\frac{5}{2}$ m/yrs.



Ex. Researchers are investigating plankton cells in a sea. At a depth of $h$ meters, the density of plankton cells, in millions of cells per cubic meter, is modeled by $p(h) = h^3$.

a) Find $p'(5)$. $= 3(5)^2$

$= 75$

$p'(h) = 3h^2$

m

b) Using correct units, interpret the meaning of $p'(5)$ in the context of the problem.

$p'(h) = \frac{dp}{dh} = \frac{\text{mill cells}/m^3}{m}$

At a depth of 5m, density changes at a rate of 75 mill cells/$m^3$ per m.

**Differentiabul**

*f* of *x* plus *h* minus *f* of *x*
all over *h* as *h* drops to zero is the formula to find the derivative in other words state the instantaneous rate.

*f* of *x* plus *h* minus *f* of *x*
all over *h* as *h* drops to zero is the formula to find the derivative to find the slope at one point.

Infinitesimals *dy* over *dx*, why he wrote it I can’t say, Leibniz just liked it better that way.

So, *f* of *x* plus *h* minus *f* of *x*
all over *h* as *h* drops to zero is the formula to find the derivative, with this I will have to learn to cope! Leibniz found the limit of the slope.

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