New seats today, you may sit where you wish.

Not the back of any column!

- Blue part is out of 42
- Green part is out of 58

→Total of 100 points possible

Velocity

<u>General Concepts</u>

- Slope is the steepness of a line
  - Bigger slope = steeper line
- Velocity has direction (pos./neg.) and magnitude
  - Speed has only magnitude
  - Speed = |velocity|
- The tangent line of a curve passes through the graph at exactly one point.
- The secant line of a curve passes through the graph at two points.

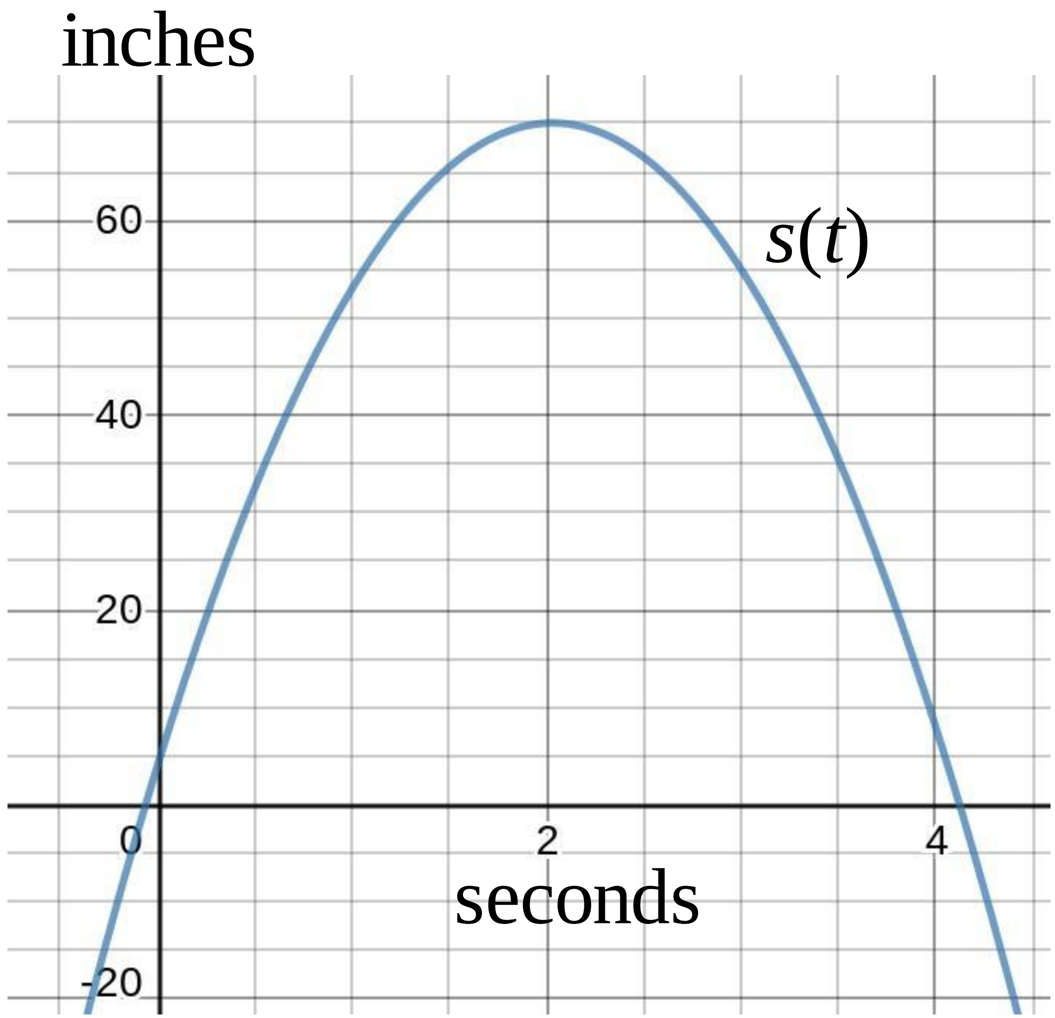
- Evaluating $\lim_{h \to 0} \frac{(3+h)^2-9}{h}$ $= \frac{0}{0}$

  – Numerically (table of values)

  – Algebraically (simplify and reduce)

Goal: Given the position of an object, find its velocity at a specific instant.

→ Consider tossing a ball into the air. If we were given an equation for the height of the ball, we want to find the velocity at one moment.

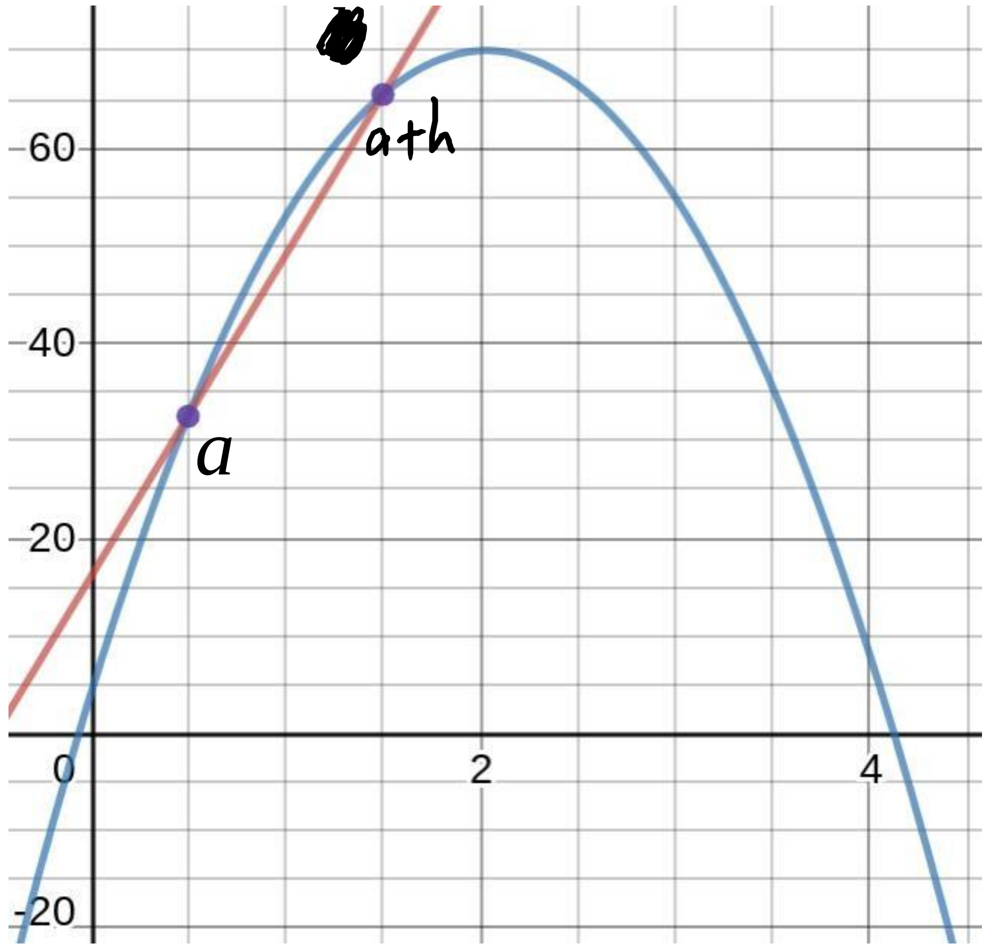

inches
60
40
20
0
2
4
-20
seconds
$s(t)$

Since velocity is the change of distance over time, we can approximate it by the change of height (s) divided by the change of time (t). This is called the <u>average velocity</u>:

$$v \approx \frac{\Delta s}{\Delta t}$$

$$\boxed{\text{(ave. veloc. from } t = a \text{ to } t = b) = \frac{s(b) - s(a)}{b - a}}$$

This is the slope of the secant line.



$$\frac{s(b)-s(a)}{b-a}$$

$$\frac{s(a+h)-s(a)}{h}$$

This will give us an approximation of the velocity (over an interval), but we want the velocity at one instant.

→ Instead of the slope between two points, we want the slope of the graph at one point.

→We can get a more precise answer if we let b get closer to a.

→Here's what it looks like.

Consider the interval from a to $a+h$, let's find the slope of the secant line:

$$\text{ave. veloc.} = \frac{s(a+h)-s(a)}{h}$$

If we let $h \to 0$, we will get the instantaneous velocity at $t = a$:

$$\boxed{(\text{inst. veloc. at } t = a) = \lim_{h\to 0} \frac{s(a+h)-s(a)}{h}}$$

This is the "slope" at $t = a$.

Ex. Find the average velocity of $s(t) = t^3$ on the interval $[0,2]$, then find the instantaneous velocity at $t = 1$.

$$\text{ave. veloc} = \frac{s(2)-s(0)}{2-0} = \frac{2^3-0^3}{2-0} = \frac{8}{2} = \boxed{4}$$

$$\text{inst. veloc} = \lim_{h\to 0} \frac{s(1+h)-s(1)}{h} = \lim_{h\to 0} \frac{(1+h)^3-1^3}{h}$$

$$= \lim_{h\to 0} \frac{1+3h+3h^2+h^3-1}{h} = \lim_{h\to 0} \frac{3h+3h^2+h^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3+3h+h^2)}{\cancel{h}} = \lim_{h\to 0} (3+3h+h^2) = \boxed{3}$$

Pract. Find the average velocity of $s(t)$ on the interval $[0,2]$, then find the instantaneous velocity at $t = 1$.

a) $s(t) = t^2 - 1$

b) $s(t) = e^t$

ave. veloc. $= \dfrac{s(2)-s(0)}{2-0} = \dfrac{3-(-1)}{2} = \boxed{2}$

inst. veloc. $= \lim_{h\to 0} \dfrac{s(1+h)-s(1)}{h} = \lim_{h\to 0} \dfrac{[(1+h)^2-1]-0}{h}$

$= \lim_{h\to 0} \dfrac{1+2h+h^2-1}{h} = \lim_{h\to 0} \dfrac{2h+h^2}{h} = \lim_{h\to 0} \dfrac{h(2+h)}{h}$

$= \lim_{h\to 0} (2+h) = \boxed{2}$

b) $s(t) = e^t$

ave. veloc. $= \dfrac{s(2)-s(0)}{2-0} = \dfrac{e^2 - e^0}{2} = 3.195$

inst. veloc. $= \lim_{h \to 0} \dfrac{s(1+h)-s(1)}{h} = \lim_{h \to 0} \dfrac{e^{1+h} - e^1}{h} = 2.718$