Warm up Problems

1. $\frac{d}{dx}(\ln x)^2$

2. $\frac{d}{dy}y^2 \tan^{-1} y$

3. $\frac{d}{dt}e^{\ln\left(\frac{1}{t}\right)}$

Implicit Differentiation

<u>Explicit Functions</u> → $y = f(x)$

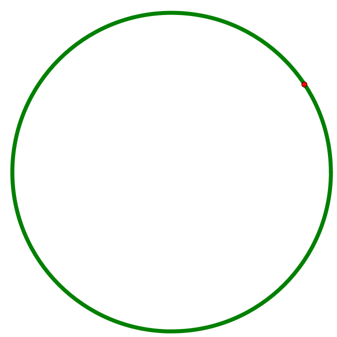
$$y = \sin x + e^x$$

$$y = x^5 + 3x^2 + 2$$

Find the slope of the graph at the point

<u>Implicit Functions</u> → implies $y = f(x)$

$$x^2 + y^2 = 9$$



Find the slope of the graph at the point

Ex. Differentiate

1. $y = x$

$y' = 1$

2. $y = x^2$

$y' = 2x$

3. $y = (2x - 1)^2$

$y' = 2(2x-1) \cdot 2$

4. $y = \left(f(x)\right)^2$

$y' = 2f(x) \cdot f'(x)$

5. $x = y^2$

$1 = 2y\,y'$

To differentiate an implicit function, we differentiate term-by-term:

- Take the derivative of x-function as usual.
- The derivative of y-function gets multiplied by y'.
- If x's and y's are in the same term, use product rule.

After differentiating, solve for y'.

Ex. If $\cos x + y^2 - y = x$, find $\frac{dy}{dx}$.

$$-\sin x + 2yy' - 1y' = 1$$

$$2yy' - y' = 1 + \sin x$$

$$y'(2y-1) = 1 + \sin x$$

$$y' = \frac{1 + \sin x}{2y - 1}$$

<u>Ex.</u> $\ln y + \underbrace{x^2 y^4} + e^x = 5$

$$\frac{1}{y}y' + \underbrace{x^2 \cdot 4y^3 \cdot y' + y^4 \cdot 2x} + e^x = 0$$

$$\frac{1}{y}y' + 4x^2y^3y' = -2xy^4 - e^x$$

$$y'\left(\frac{1}{y} + 4x^2y^3\right) = -2xy^4 - e^x$$

$$y' = \frac{-2xy^4 - e^x}{\frac{1}{y} + 4x^2y^3}$$

Pract. $x^2 + y^2 = 16$

$$y' = -\frac{x}{y}$$

Pract. $x^3 y + y^3 = -10$

$$y' = \frac{-3x^2 y}{x^3 + 3y^2}$$

Ex. Find the slope of the line tangent to
$y = x + \cos(xy)$ at the point where $x = 0$.

$y' = 1 - \sin(xy)(xy' + y \cdot 1)$

$y' = 1 - \sin(0 \cdot 1)(0y' + 1)$

$y' = 1$

$y = 0 + \cos(0 \cdot y)$

$y = 1$

Ex. If $\cos x + y^2 - y = x$, find $\frac{d^2y}{dx^2}$. $= y''$

$$y' = \frac{1+\sin x}{2y-1}$$

$$y'' = \frac{(2y-1)(\cos x) - (1+\sin x)(2y')}{(2y-1)^2}$$

Ex. Find the coordinates of any point on $x^2 + y^2 = 16$ where the tangent line has the slope of -1.

$$y' = \frac{-x}{y} = -1$$

$$-x = -y$$

$$y = x$$

$$x^2 + y^2 = 16$$

$$x^2 + x^2 = 16$$

$$2x^2 = 16$$

$$x^2 = 8$$

$$x = \pm\sqrt{8}$$

$$\boxed{(\sqrt{8}, \sqrt{8}) \quad (-\sqrt{8}, -\sqrt{8})}$$

Ex. Let $\cancel{f(x)}\ y = x^3 + x$. If $g(x) = f^{-1}(x)$ and $f(2) = 10$, find $g'(10)$.

$(2, 10)$

$\underline{g}$

$x = y^3 + y$ $\quad (10, 2)$

$1 = 3y^2 y' + 1y'$

$1 = 3(2)^2 y' + y'$

$1 = 13y'$

$y' = \frac{1}{13}$