Warm up Problems

Let $f(x) = x^3 - 3x + 1$.

1) Find and classify all critical points.

2) Find all inflection points.

Graph of a Function, Part 2

Second Derivative Test

If p is a critical point of $f(x)$ and $f''(p) < 0$, then p is a local maximum.

If p is a critical point of $f(x)$ and $f''(p) > 0$, then p is a local minimum.

Ex. Find and classify all critical points of

$f(x) = x^3 - 5x^2 + 3x - 1.$

$f'(x) = 3x^2 - 10x + 3$

$= (3x-1)(x-3) = 0$

$x = \frac{1}{3} \quad x = 3$

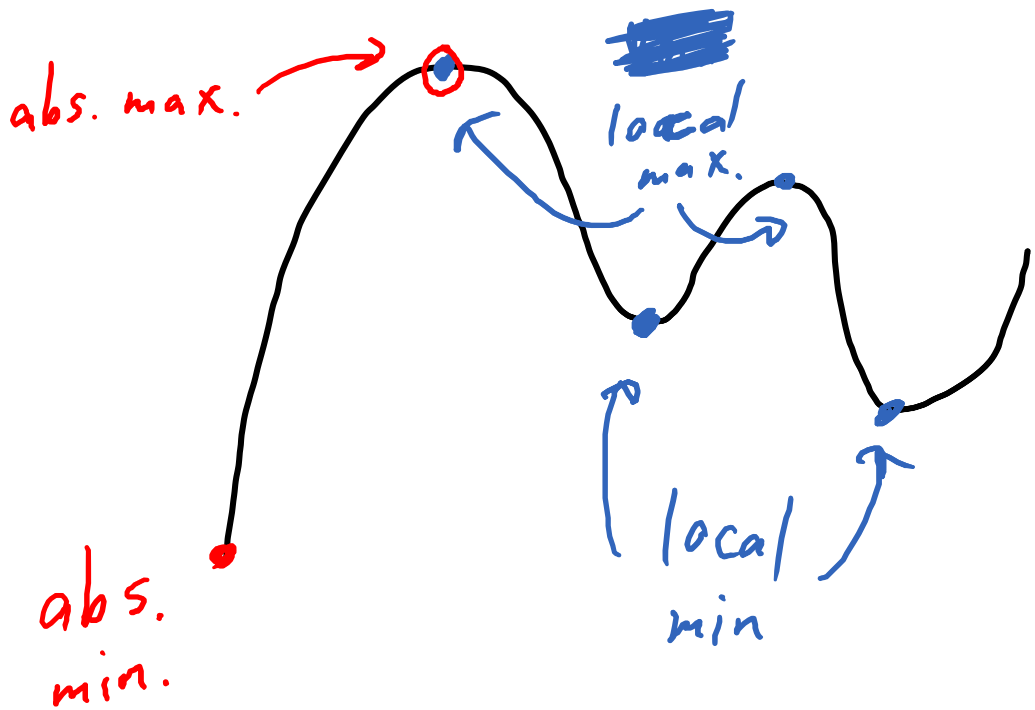
$f''(x) = 6x - 10$

$f''\left(\frac{1}{3}\right) = 6\left(\frac{1}{3}\right) - 10 = -8$ local max

$f''(3) = 6(3) - 10 = 8$ local min.

Def. The absolute maximum (global max) value of a function on an interval is the largest value that the function attains.

Def. The absolute minimum (global min) value of a function on an interval is the smallest value that the function attains.


abs. max.
local max.
local min
abs. min.

Thm. The absolute max. and min. will occur at one of the following:

$$\left.\begin{array}{l}\bullet\ \text{the point } p \text{ where } f'(p) = 0 \\ \bullet\ \text{the point } p \text{ where } f'(p) \text{ is undef.}\end{array}\right\}\ \text{critical points}$$

- an endpoint of the interval

Ex. Find the absolute max. and min. values of $f(x) = x^3 - 3x^2 + 1$ on $\left[-\frac{1}{2}, 4\right]$.

no abs. max.

$f'(x) = 3x^2 - 6x$

$= 3x(x-2) = 0$

$x = 0 \qquad x = 2$

$f\left(-\frac{1}{2}\right) = .125$

$f(0) = 1$

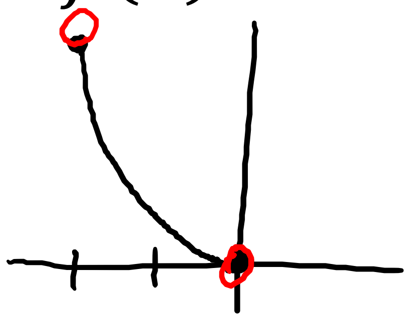
$f(2) = -3$

$f(4) = 17$

~~abs. max. value is 17~~

abs. min. value is -3

Ex. Find the x-coordinate of all local max./min. and absolute max./min. of $f(x) = x^2$ for $-2 \leq x \leq 0$ by graphing.



local max.: none

local min.: none

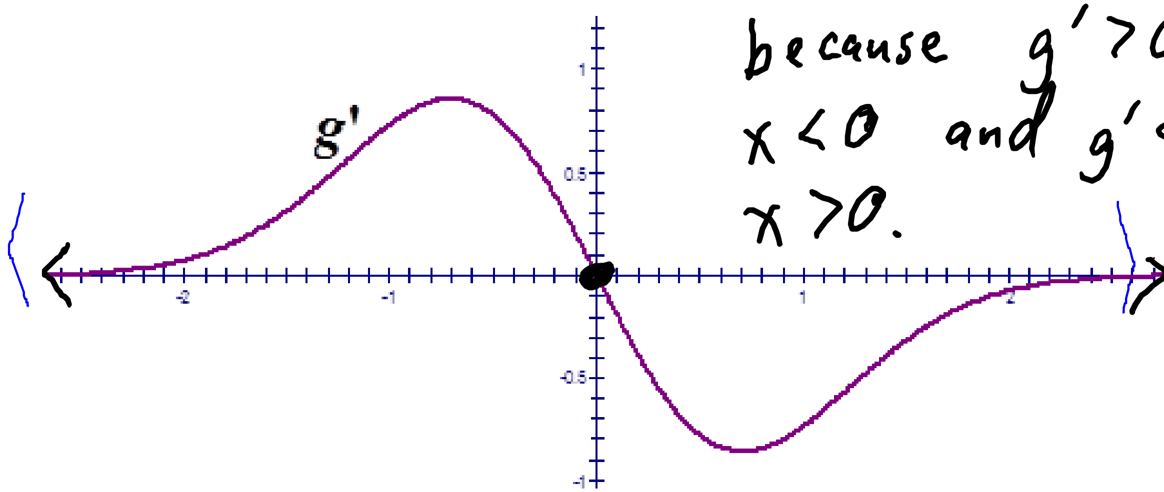
abs. max.: $\cancel{x=-2}$ none

abs. min: $\cancel{x=0}$ none

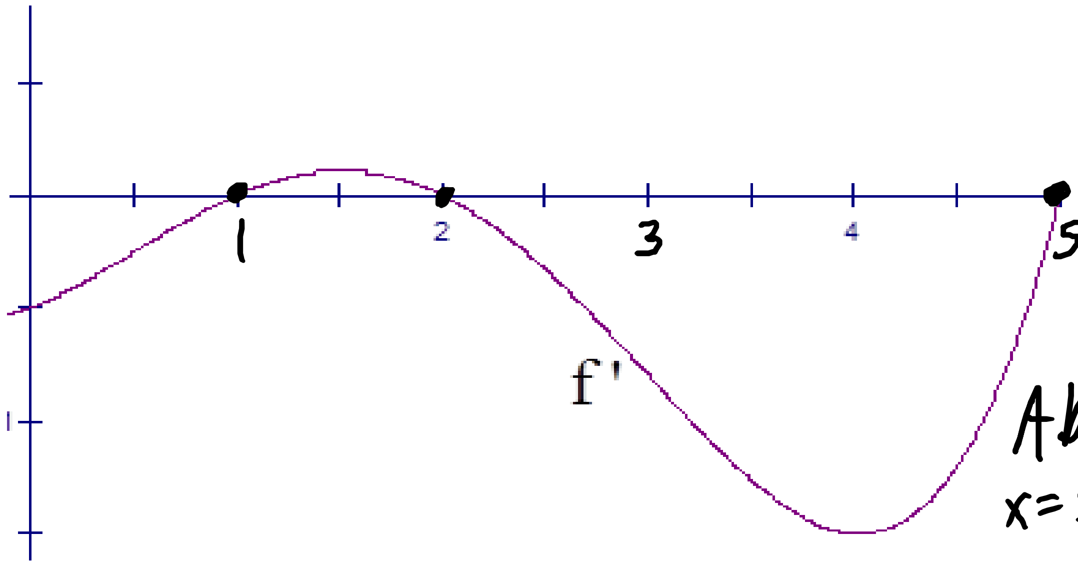
→What about open intervals?

Ex. Find the x-coordinate of the absolute maximum of $g(x)$. Justify your answer.

$x=0$ is abs. max. because $g'>0$ for all $x<0$ and $g'<0$ for all $x>0$.



Ex. Find the x-coordinate of the absolute minimum of $f(x)$ on $[0,5]$. Justify your answer.



~~$x=0$~~ : f' neg. after

$x=1$:

~~$x=2$~~ : local max.

$x=5$:

Abs. min. occurs at $x=5$ because f dec., inc., and then decreases a lot.

You must check ALL candidates.