Warm up Problems

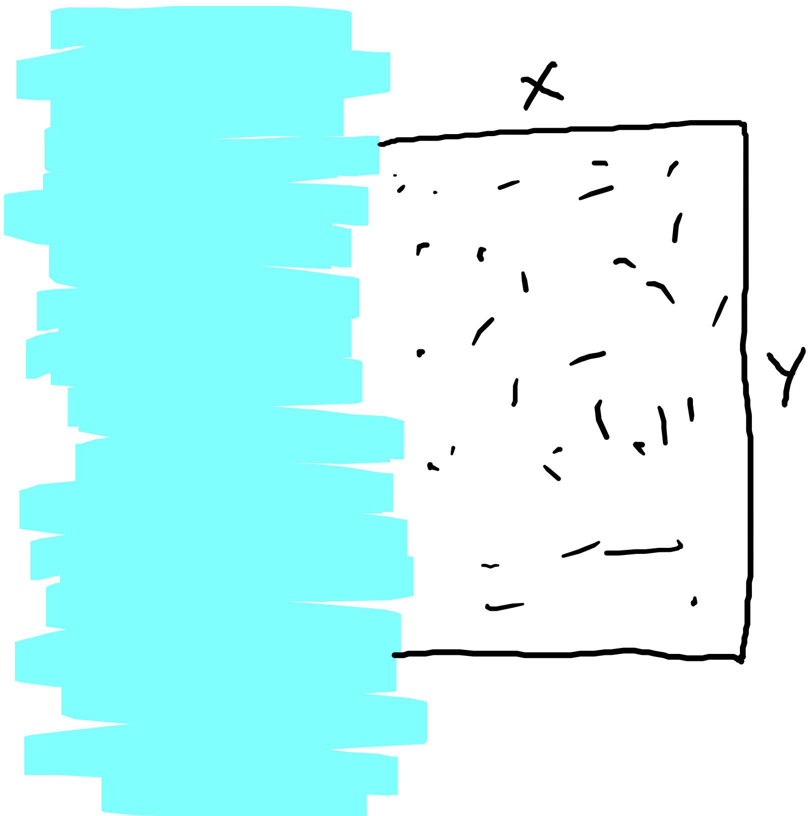
1. Find and classify all critical points of $f(x) = 4x^3 - 9x^2 - 12x + 3$

2. Find the absolute max./min. values of $f(x)$ on the interval $[-1,4]$.

Optimization

Ex. Cletus has 240 ft. of fencing and wants to enclose a rectangular field that borders a straight river. If he needs no fence along the river, find the largest area that the field can be.

$$2x + y = 240 \longrightarrow y = 240 - 2x$$



$$A = xy$$

$$A = x(240 - 2x)$$

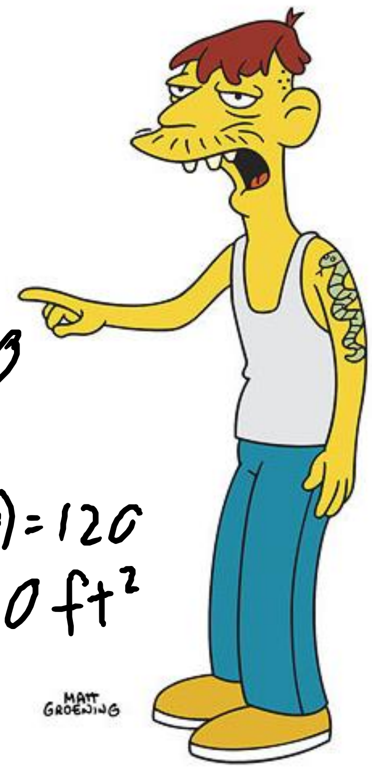
$$A = 240x - 2x^2$$

$$A' = 240 - 4x = 0$$

$$x = 60$$

$$y = 240 - 2(60) = 120$$

$$A = (60)(120) = 7200 \text{ ft}^2$$



Strategy for Optimization

1) Draw a picture, if appropriate

2) Write down given information, including an equation

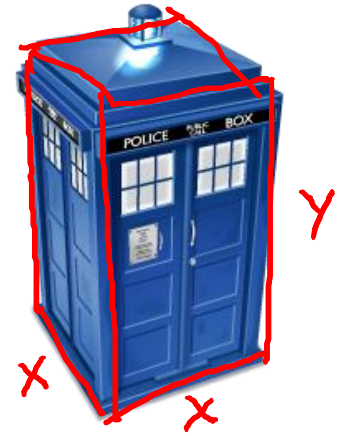
3) Find the function to be optimized

4) Substitute to get one variable

5) Take the derivative

6) Set equal to zero and solve

Ex. The TARDIS has a square base and has a volume of 1000 m^3. The Daleks have blasted all of the walls, and the Doctor wants to rebuild it as a convertible – no roof. Find the dimensions that will minimize the materials for the remaining 5 walls. (Assume it is not bigger on the inside.)



$$S = 4xy + x^2 = 4x\left(\frac{1000}{x^2}\right) + x^2 = 4000x^{-1} + x^2$$

$$S' = -4000x^{-2} + 2x = \frac{-4000}{x^2} + 2x \cdot \frac{x^2}{x^2}$$

$$= \frac{-4000 + 2x^3}{x^2} = 0 \longrightarrow -4000 + 2x^3 = 0$$

$$x^3 = 2000$$

$$x = 12.599$$

$$y = \frac{1000}{(12.599)^2} = 6.300$$

$$1000 = x^2 y$$

$$y = \frac{1000}{x^2}$$

$$\boxed{12.599\text{ m} \times 12.599\text{ m} \times 6.300\text{ m}}$$

<u>Pract.</u> Sherlock has discovered a closed cylinder at a crime scene. He determines that it has a surface area of 108 cm^2. What are the dimensions of such a cylinder that has the largest volume?

$$V = \pi r^2 h = \pi r^2 \left(\frac{108 - 2\pi r^2}{2\pi r}\right) = 54r - \pi r^3$$

$$V' = 54 - 3\pi r^2 = 0$$

$$3\pi r^2 = 54$$

$$r^2 = \frac{54}{3\pi}$$

$$\boxed{\begin{aligned} r &= 2.394 \text{ cm} \\ h &= \frac{108 - 2\pi(2.394)^2}{2\pi(2.394)} = 4.787 \text{ cm} \end{aligned}}$$

$$V = \pi r^2 h$$

$$S = 2\pi r h + 2\pi r^2$$

$$108 = 2\pi r h + 2\pi r^2$$

$$\frac{108 - 2\pi r^2}{2\pi r} = h$$