Related Rates

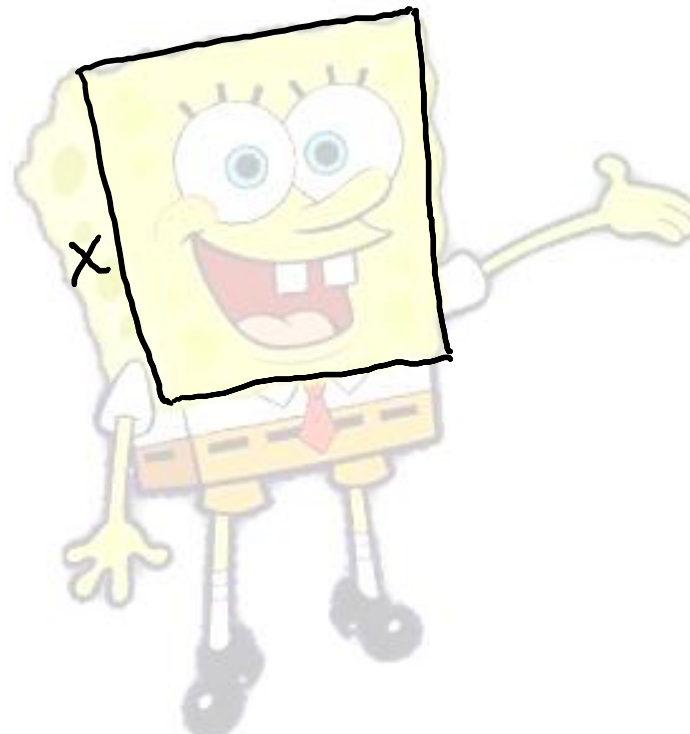
<u>Def.</u> A <u>rate</u> is the change of a quantity with respect to time.

Ex. Spongebob Square**face** is growing so that his edge is increasing at a rate of .7 cm/sec. How fast is the area of his face changing when the edge is 3 cm. long?

$x = 3$

$\frac{dx}{dt} = .7$

$\frac{dA}{dt} = ?$



$A = x^2$

$\frac{dA}{dt} = 2x \frac{dx}{dt}$

$= 2(3)(.7)$

$= \boxed{4.2 \text{ cm}^2/\text{sec}}$

Strategy for Related Rates
"PGWEDA"

P) Draw a picture

Change = variable

Fixed = number

G) Identify given information, including rates (derivatives)

All variables must be in the picture

W) Identify what you want to find

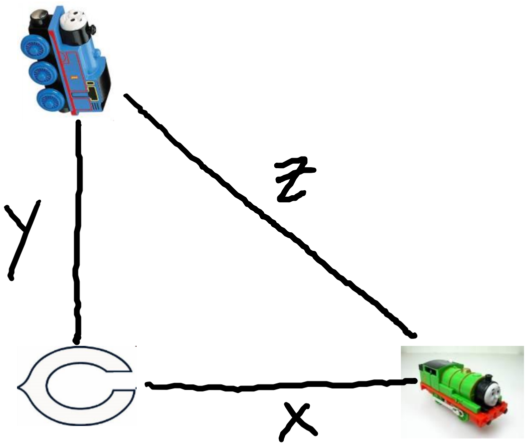
Use the picture

E) Find an equation to relate the variables

D) Take the derivative with respect to time

A) Plug in values to get your answer

Ex. At noon, two trains leave Carlsbad. Thomas travels north at 65 mph, Percy travels east at 70 mph. Find the rate at which the distance between the trains is changing at 2pm.



$$\frac{dy}{dt} = 65$$

$$\frac{dx}{dt} = 70$$

$$\frac{dz}{dt} = ?$$

at 2pm.

$y = 130$

$x = 140$

$$130^2 + 140^2 = z^2$$

$$z = 191.050$$

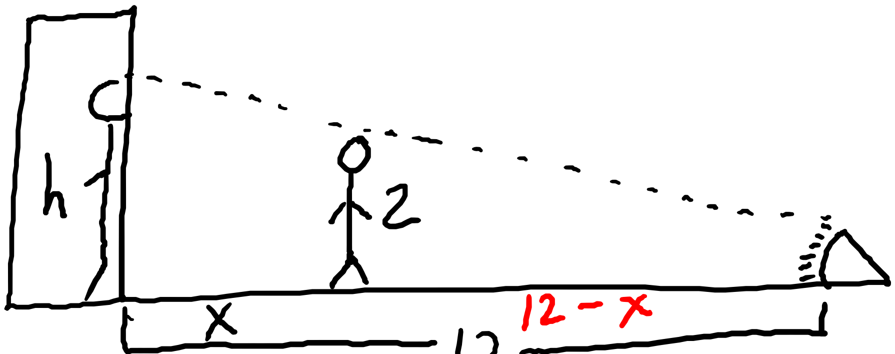
$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2(140)(70) + 2(130)(65) = 2(191.050)\frac{dz}{dt}$$

$$\frac{dz}{dt} = 95.525 \text{ mph}$$

<u>Ex.</u> A spotlight on the ground shines on a building 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?



$x = 4$

$\frac{dx}{dt} = -1.6$

$\frac{dh}{dt} = ?$

$$\frac{h}{12} = \frac{2}{12-x}$$

$$h = 24(12-x)^{-1}$$

$$\frac{dh}{dt} = -24(12-x)^{-2}(-1)\frac{dx}{dt}$$

$$= \frac{-24}{(12-4)^2}(-1)(-1.6)$$

$$= -.6 \text{ m/sec.}$$