- Blue part is out of 41
- Green part is out of 62

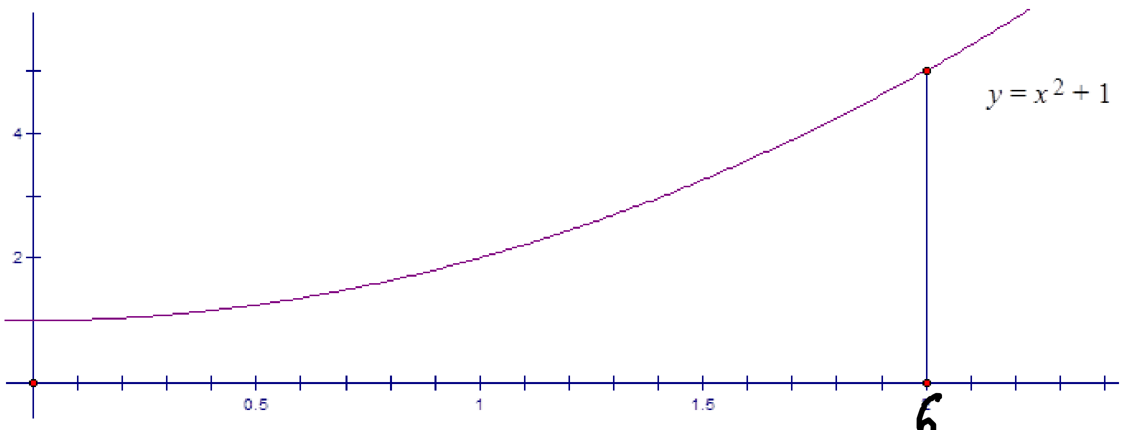
→Total of 103 points possible

→Grade is out of 100

Riemann Sums

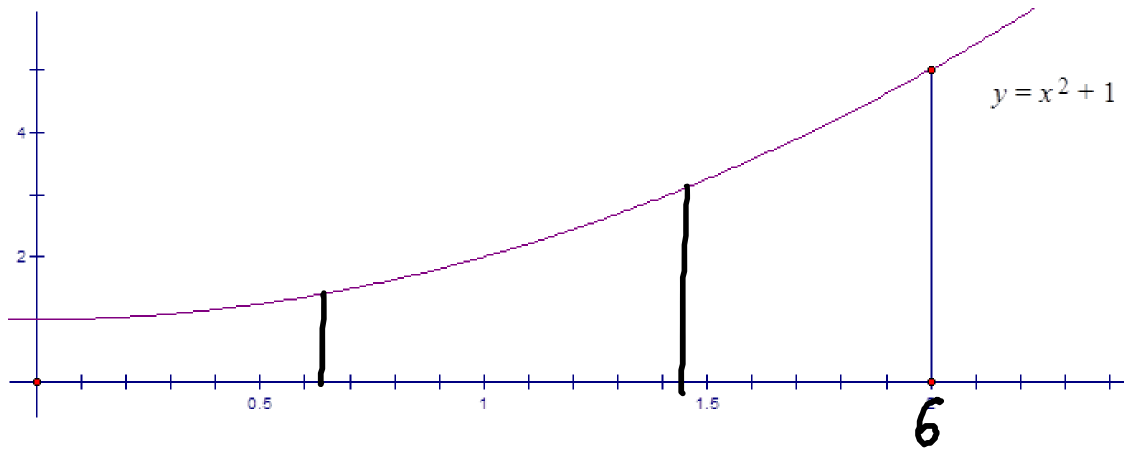
Goal: Calculate the area under a given curve.

Ex. Find the area under $y = x^2 + 1$ on the interval $[0,6]$.



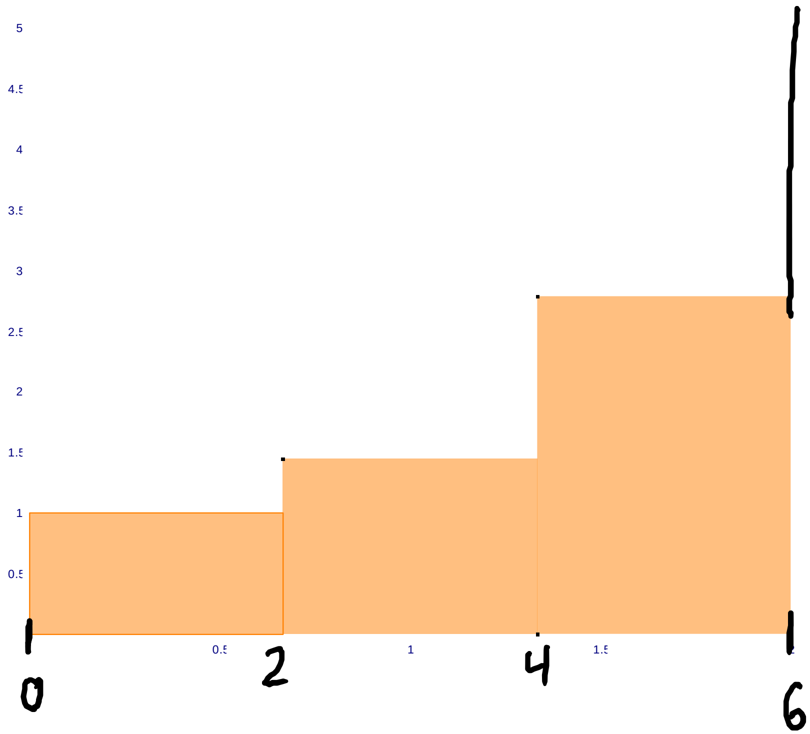
→ This means that we want the area going down to the x-axis.

To approximate this area, we will split the region into subintervals, create rectangles, and add the areas of the rectangles.



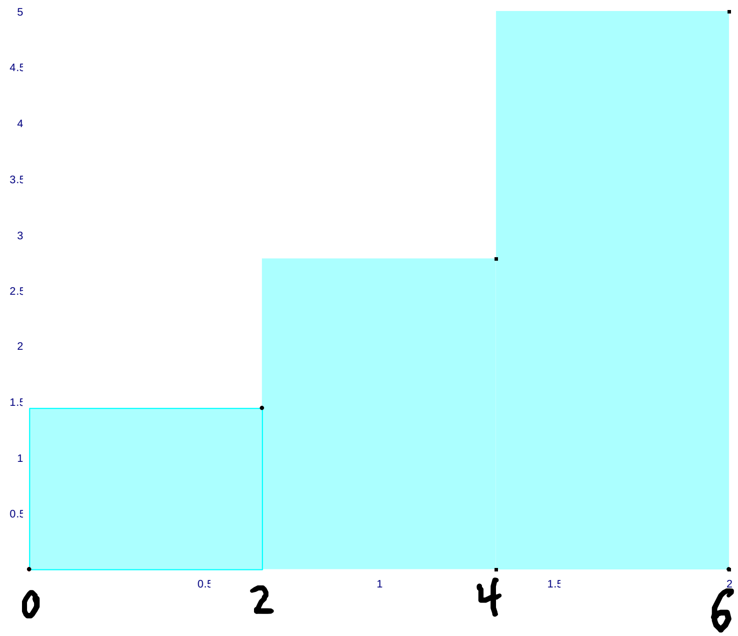
→This is called Riemann sums.

Ex. Approx. the area under $y = x^2 + 1$ on $[0,6]$ using a left-hand Riemann sum with 3 subintervals.



$$A = 2f(0) + 2f(2) + 2f(4)$$
$$= 2(1) + 2(5) + 2(17)$$
$$= 46$$

Ex. Approx. the area under $y = x^2 + 1$ on [0,6] using a right-hand Riemann sum with 3 subintervals.



$$A = 2f(2) + 2f(4) + 2f(6)$$
$$= 2(5) + 2(17) + 2(37)$$
$$= 118$$

Ex. For each of the previous examples, did we get an overestimate or an underestimate of the true value? Why?

$$\left.\begin{array}{l} \text{LHS} \longrightarrow \text{under} \\ \text{RHS} \longrightarrow \text{over} \end{array}\right\} f \text{ is inc.}$$

Ex. The table below gives selected values of $f(x)$. Use these values and a left-hand Riemann sum to approximate the area under the function on the interval $0 \le x \le 12$.

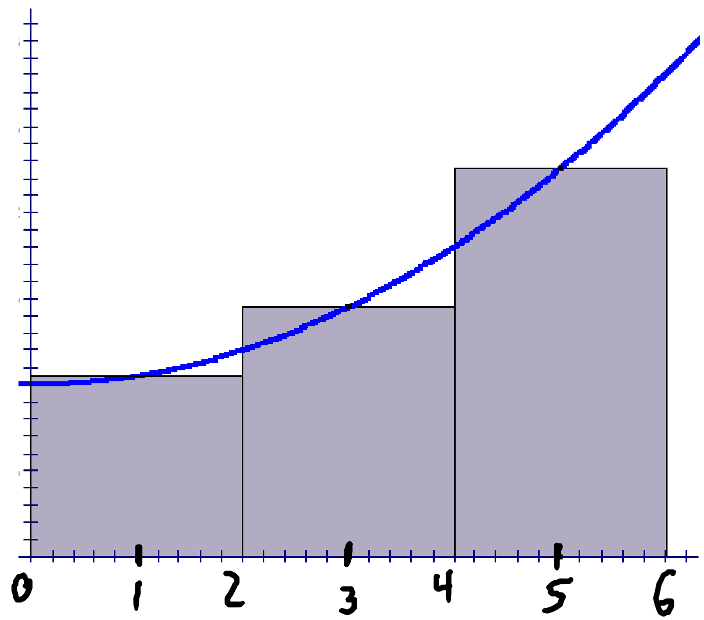
x	0	2	3	6	8	9	12
$f(x)$	0	.25	.48	.68	.84	.95	1

$$A = 2f(0) + 1f(2) + 3f(3) + 2f(6) + 1f(8) + 3f(9)$$

$$= 6.74$$

We can get a better approximation by using the midpoint:

<u>Ex.</u> Approx. the area under $y = x^2 + 1$ on [0,6] using a midpoint Riemann sum with 3 subintervals.

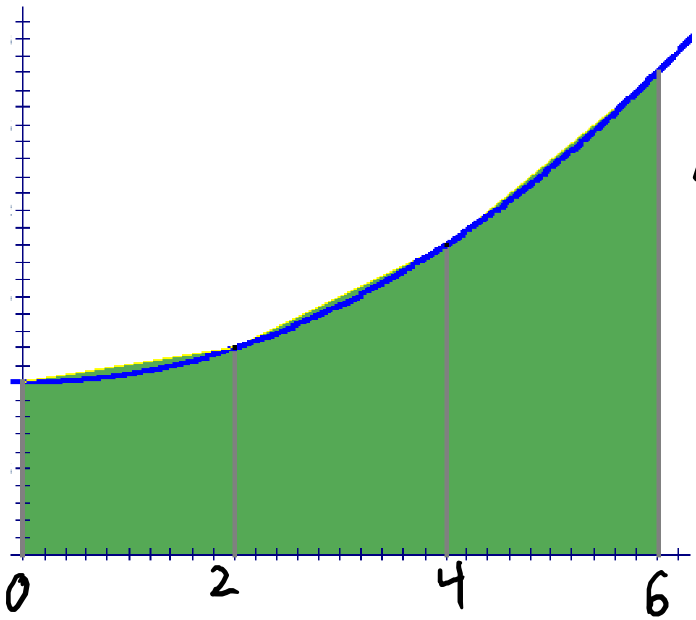


$$A = 2f(1) + 2f(3) + 2f(5)$$
$$= 2(2) + 2(10) + 2(26)$$
$$= 76$$

We could also use trapezoids:

$$A = \frac{1}{2}(h_1 + h_2)b$$

Ex. Approx. the area under $y = x^2 + 1$ on [0,6] using a trapezoidal Riemann sum with 3 subintervals.



$$A = \frac{1}{2}\left[f(0) + f(2)\right]\cdot 2 + \frac{1}{2}\left[f(2) + f(4)\right]\cdot 2 + \frac{1}{2}\left[f(4) + f(6)\right]\cdot 2$$

$$= (1 + 5) + (5 + 17) + (17 + 37)$$

$$= 82$$

Ex. For each of the previous examples, did we get an overestimate or an underestimate of the true value? Why?

mid → under
trap → over

f conc. up.

Ex. Given values in the table, approx. the area under $f(x)$ on [0,8] using midpoint and trapezoidal Riemann sum with 4 subintervals of equal width.

x	0	1	2	3	4	5	6	7	8
$f(x)$	-1	0	3	4	7	9	14	16	20

mid $= 2f(1) + 2f(3) + 2f(5) + 2f(7) = 58$

trap $= \frac{1}{2}[f(0)+f(2)]2 + \frac{1}{2}[f(2)+f(4)]\cdot 2 + \frac{1}{2}[f(4)+f(6)]\cdot 2 + \frac{1}{2}[f(6)+f(8)]\cdot 2$

$= 67$