Warm up Problems

1. $\int (5x^3 - 4\sin x)dx$

2. $\int \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

3. $\int 7e^x dx$

More With Integrals

Ex. $\int e^{3x}dx = \frac{1}{3}e^{3x} + C$

Ex. $\int (2x-1)^{10}dx = \frac{1}{2}\cdot\frac{1}{11}(2x-1)^{11} + C$

Ex. $\int \frac{t^2 - 1}{t} dt$

Thm. Fundamental Theorem of Calculus

If $f(x)$ is a continuous function on $[a, b]$, and if $F'(x) = f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

- $F(x)$ is an antiderivative of $f(x)$.
- Find an antiderivative, then plug in the endpoints

$$\underline{\text{Ex.}} \int_3^5 2x\,dx = x^2 \Big|_3^5 = 5^2 - 3^2 = 16$$

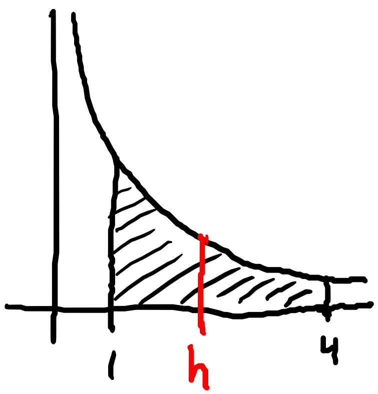
$$\underline{\text{Ex.}} \int_0^{\pi/2} \sin x \, dx = -\cos x \Big|_0^{\pi/2} = \left(-\cos\frac{\pi}{2}\right) - \left(-\cos 0\right) = 1$$

Ex. $\int_0^2 e^x dx = e^x \Big|_0^2 = e^2 - e^0 = e^2 - 1$

Pract. $\int_1^2 x^4 dx = \frac{1}{5}x^5 \Big|_1^2 = \frac{1}{5}(2)^5 - \frac{1}{5}(1)^5 = \frac{31}{5}$

- Don't write "$+c$" on definite integrals
- We could use a calculator to get the answer, but this way we get the exact answer, not just a decimal approximation

Ex. Let R be the region bounded by $y = \frac{1}{x}$, the x-axis, $x = 1$, and $x = 4$. Find a value for h so that the line $x = h$ splits R into two regions of equal area.



$$\int_1^h \frac{1}{x}\,dx = \frac{1}{2}\int_1^4 \frac{1}{x}\,dx$$

$$\ln|x|\Big|_1^h = \frac{1}{2}\ln|x|\Big|_1^4$$

$$\ln h - \cancel{\ln 1} = \frac{1}{2}\left(\ln 4 - \cancel{\ln 1}\right)$$

$$\ln h = \frac{1}{2}\ln 4$$

$$\ln h = \ln\left(4^{1/2}\right)$$

$$\boxed{h = 2}$$