Warm up Problem

Let $v(t) = 5t - t^2$ be the velocity of a particle moving along the x-axis.

1) Find the total distance travelled by the particle from $t = 0$ to $t = 7$.

2) If $x(0) = 1$, find $x(7)$.

FTOC

Thm. Fundamental Theorem of Calculus

If $f(x)$ is a continuous function on $[a, b]$, and if $F'(x) = f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

→ The integral of the rate of change gives the total change.

$$\int_a^b g'(x)dx = g(b) - g(a)$$

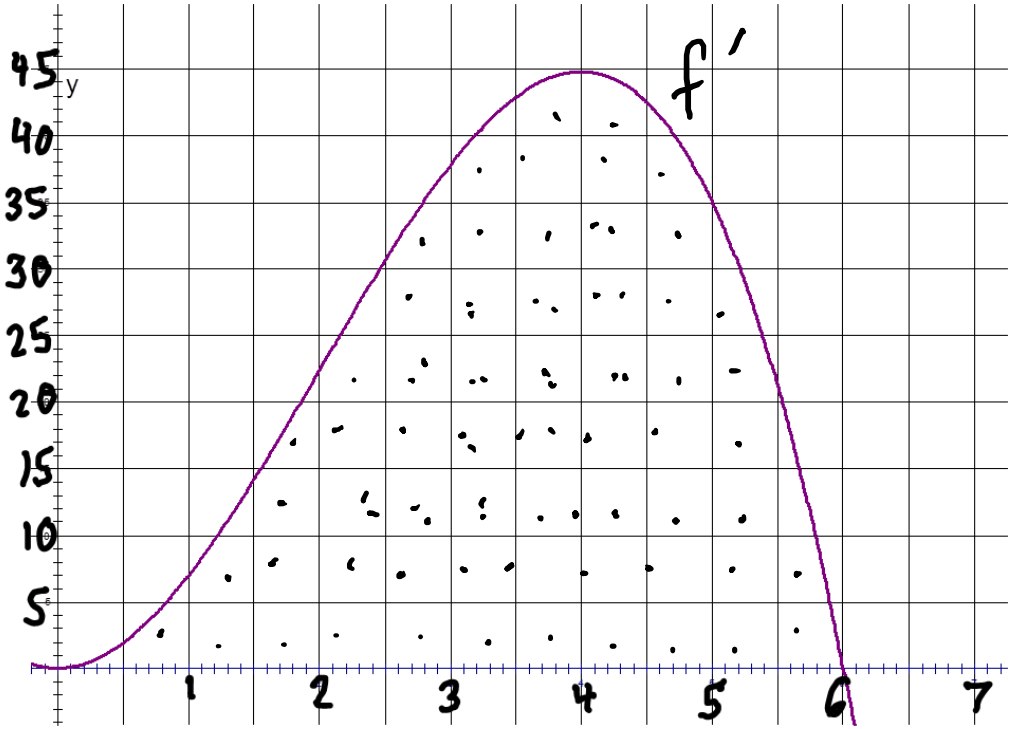
→ Ending value is the starting value plus the integral of the rate.

$$g(b) = g(a) + \int_a^b g'(x)dx$$

Ex. The rate at which people enter Sea World is given by $E(t) = \frac{15600}{t^2 - 24t + 160}$. How many people entered the park during park hours, 9am to 5pm? (Assume t is hours since midnight.)

$$\int_9^{17} E(t)\,dt = 6004.270$$

Ex. Let f be a differentiable function for all x, with graph of f' shown below. What is a good approximation for the total change of f on [0,6]?



$$\int_0^6 f'(x)\,dx$$

$$= 60\left(\tfrac{1}{2}\cdot 5\right)$$

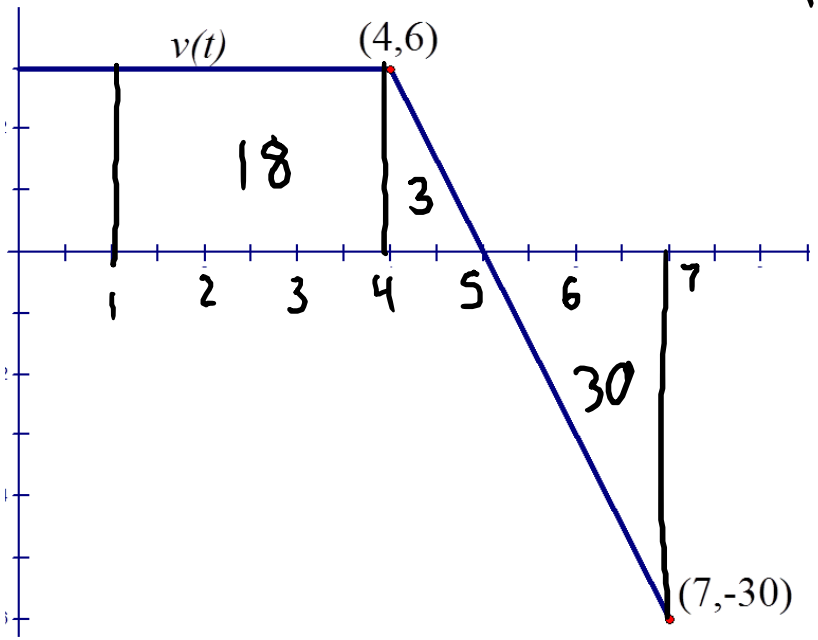
$$= 150$$

Ex. For a particle moving along the x-axis, you are given the graph of the velocity below. Assume $x(1) = 10$.

a) Find the distance travelled on [1,7]. $= \int_1^7 |v(t)|\,dt = 21 + 30 = 51$

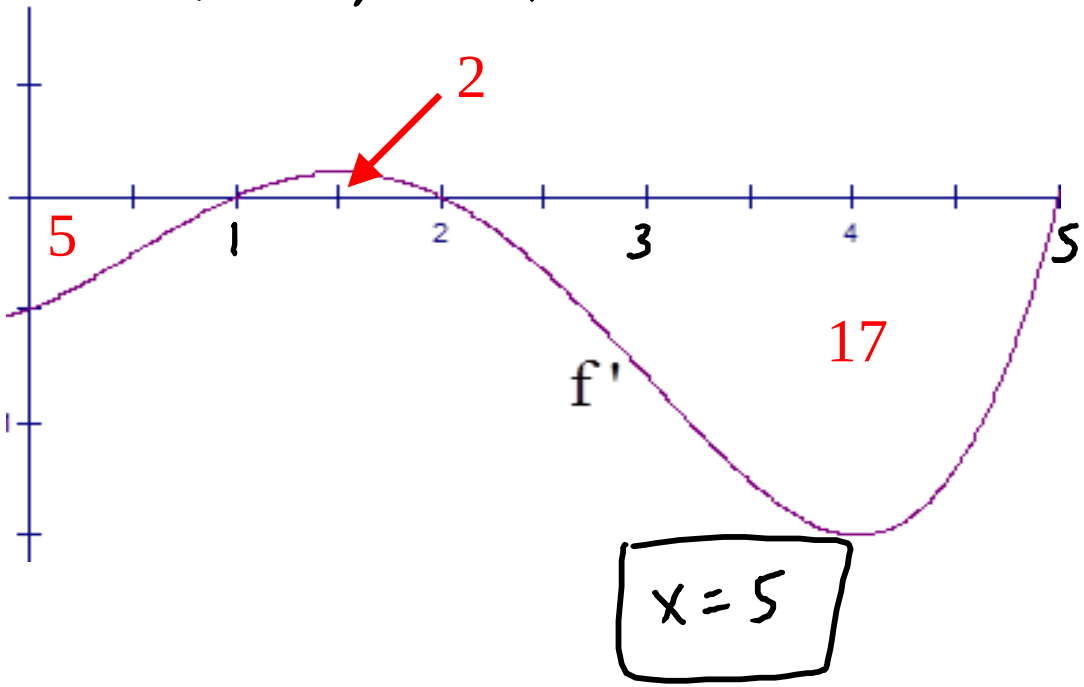
b) Find $x(7)$. $= x(1) + \int_1^7 v(t)\,dt$

$= 10 + (21 - 30) = 1$



Ex. The graph of f' is given below, with areas indicated. If $f(0) = 9$, find the x-coordinate of the point where $f(x)$ has an absolute minimum on $[0,5]$. Justify your answer.

$x=0,\ x=5,\ x=1,\ x=2$



$f(0) = 9$

$f(1) = f(0) + \int_0^1 f'(x)\,dx$

$= 9 + (-5) = 4$

~~$f(2)$~~ = local max.

$f(5) = f(0) + \int_0^5 f'(x)\,dx$

$= 9 + (-5 + 2 - 17) = -11$

You must check ALL candidates.

Unit 6 Progress Check: MCQ Part A

- Only do 1-2, 4, 13-15