Warm up Problems

1. $\int_{1}^{5} \frac{1}{x}\, dx$

2. $\int_{0}^{\pi} \cos x \, dx$

3. $\int dt$

Differential Equations

<u>Def.</u> A <u>differential equation</u> is an equation that involves a function and some of its derivatives.

<u>Ex.</u> $y'' = 3y' - 5y^2$

Ex. Verify that $x = 5$ is a solution to $3x - 2 = 13$.

$3(5) - 2 = 13$ ✓

Ex. Verify that $y = e^{2x}$ is a solution to $y'' - 3y' + 2y = 0$

$y' = e^{2x} \cdot 2$

$y'' = e^{2x} \cdot 4$

$4e^{2x} - 3(2e^{2x}) + 2e^{2x} = 0$

$e^{2x}(4 - 6 + 2) = 0$ ✓

Ex. If $y' = 6x^2 - 5$, find the general solution.

$$y = 2x^3 - 5x + C$$

A <u>separable</u> equation can be written

$$\frac{dy}{dx} = f(x)g(y)$$

→To solve, we treat $\frac{dy}{dx}$ as a fraction.

→Put the y's on the left with dy, put x's on the right with dx, and integrate each side.

<u>Ex.</u> Let $\frac{dP}{dt} = \frac{t^2}{2P^3}$, find the general solution.

$$\int P^3\,dP = \int \tfrac{1}{2}t^2\,dt$$

$$\tfrac{1}{4}P^4 = \tfrac{1}{6}t^3 + C$$

$$P^4 = \tfrac{2}{3}t^3 + D$$

$$P = \pm\sqrt[4]{\tfrac{2}{3}t^3 + D}$$

$$\tfrac{1}{4}P^4 = \tfrac{1}{6}t^3$$

$$P^4 = \tfrac{2}{3}t^3$$

$$P = \sqrt[4]{\tfrac{2}{3}t^3} + C$$

Pract.

1) $\dfrac{dy}{dx} = x^3 + \sec^2 x$

$\int dy = \int (x^3 + \sec^2 x)\,dx$

$y = \frac{1}{4}x^4 + \tan x + C$

2) $\dfrac{dR}{dt} = \dfrac{t^2}{R}$

$\int R\,dR = \int t^2\,dt$

$\frac{1}{2}R^2 = \frac{1}{3}t^3 + C$

$R^2 = \frac{2}{3}t^3 + D$

$R = \pm\sqrt{\frac{2}{3}t^3 + D}$

3) $\dfrac{dy}{dx} = 12y^2$

y^{-2}

$\frac{1}{y^2}\,dy = 12\,dx$

$-y^{-1} = 12x + C$

$\frac{1}{y} = -12x + D$

$y = \frac{1}{-12x + D}$

Def. An initial value problem (IVP) is a differential equation and a value of the solution.

Ex. Solve the IVP $\underbrace{\frac{dy}{dx} = \frac{1}{x}}$ if $y(1) = 3$.

$$\int dy = \int \frac{1}{x}\, dx$$

$$y = \ln|x| + C \quad \leftarrow \text{general solution}$$

$$y(1) = \ln|1| + C = 3$$

$$C = 3$$

$$y = \ln|x| + 3 \quad \leftarrow \text{particular solution}$$

Ex. Solve the IVP $\frac{dP}{dt} = 3P$ if $P(0) = 5$.

$$\int \frac{1}{P}\,dP = \int 3\,dt$$

$$e^{\ln|P|} = e^{3t + c}$$

$$|P| = e^{3t} \cdot e^{c}$$

$$P = De^{3t}$$

$$P(0) = De^{0} = 5$$

$$D = 5$$

$$\boxed{P = 5e^{3t}}$$

Important: A solution must be a function that is differentiable over the largest interval containing the initial value and satisfying the original differential equation.

Ex. Find the solution to $\frac{dy}{dx} = \frac{x}{y}$ if $y(5) = -3$

$\frac{dy}{dx} = \frac{x}{y}$

$\int y\,dy = \int x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$y^2 = x^2 + D$

$y = \pm\sqrt{x^2+D}$

$-3 = \pm\sqrt{5^2+D}$

$9 = 25 + D$

$D = -16$

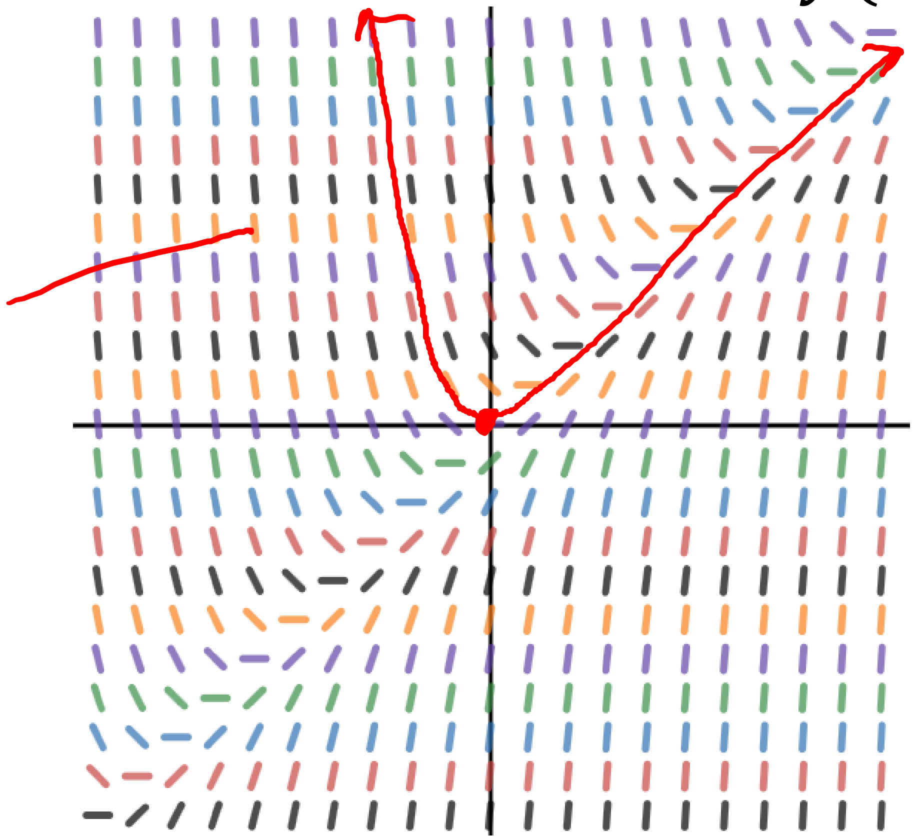
$\boxed{y = -\sqrt{x^2-16}}$

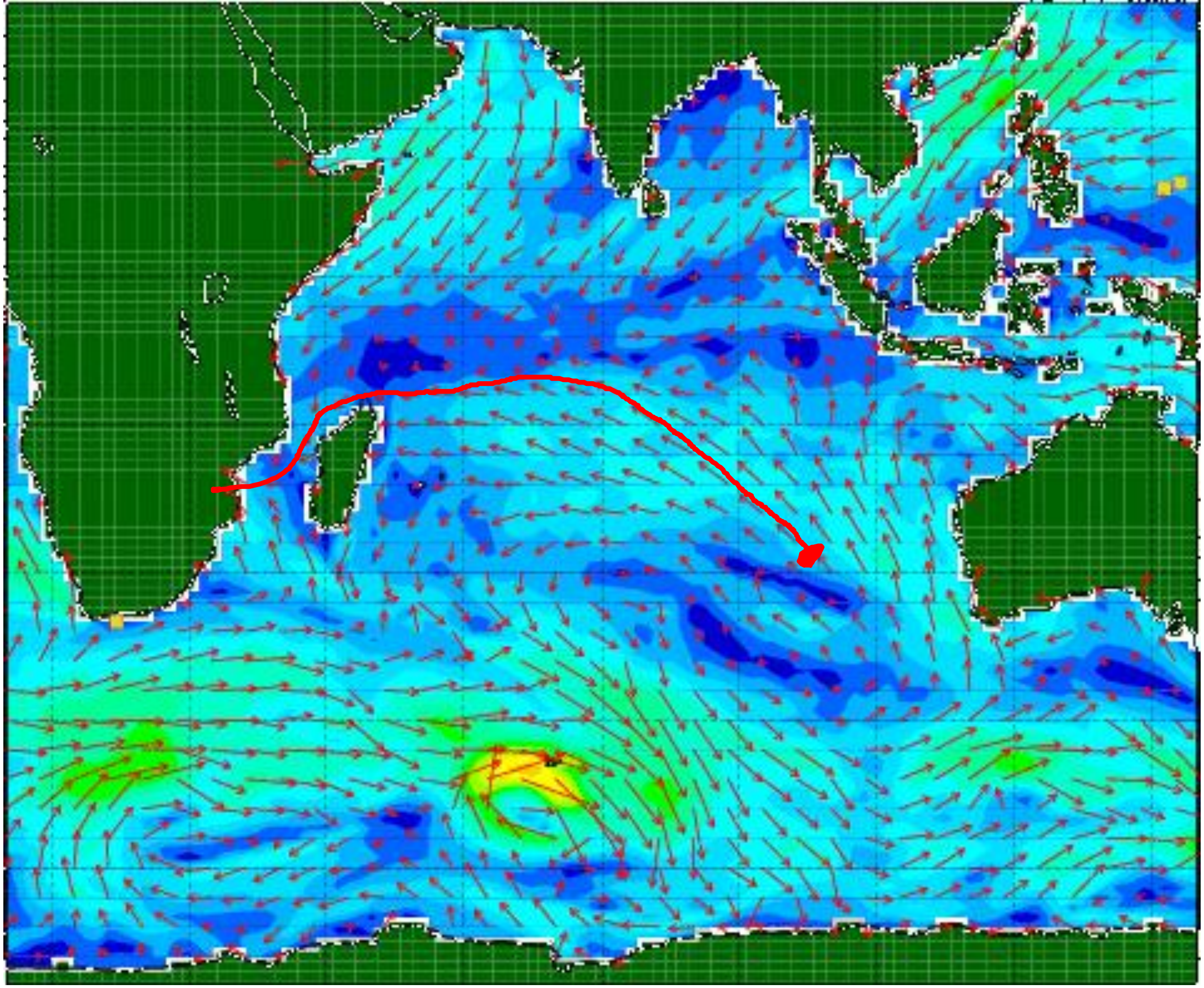
$\cancel{(-\infty, -4]} \cup (4, \infty)$

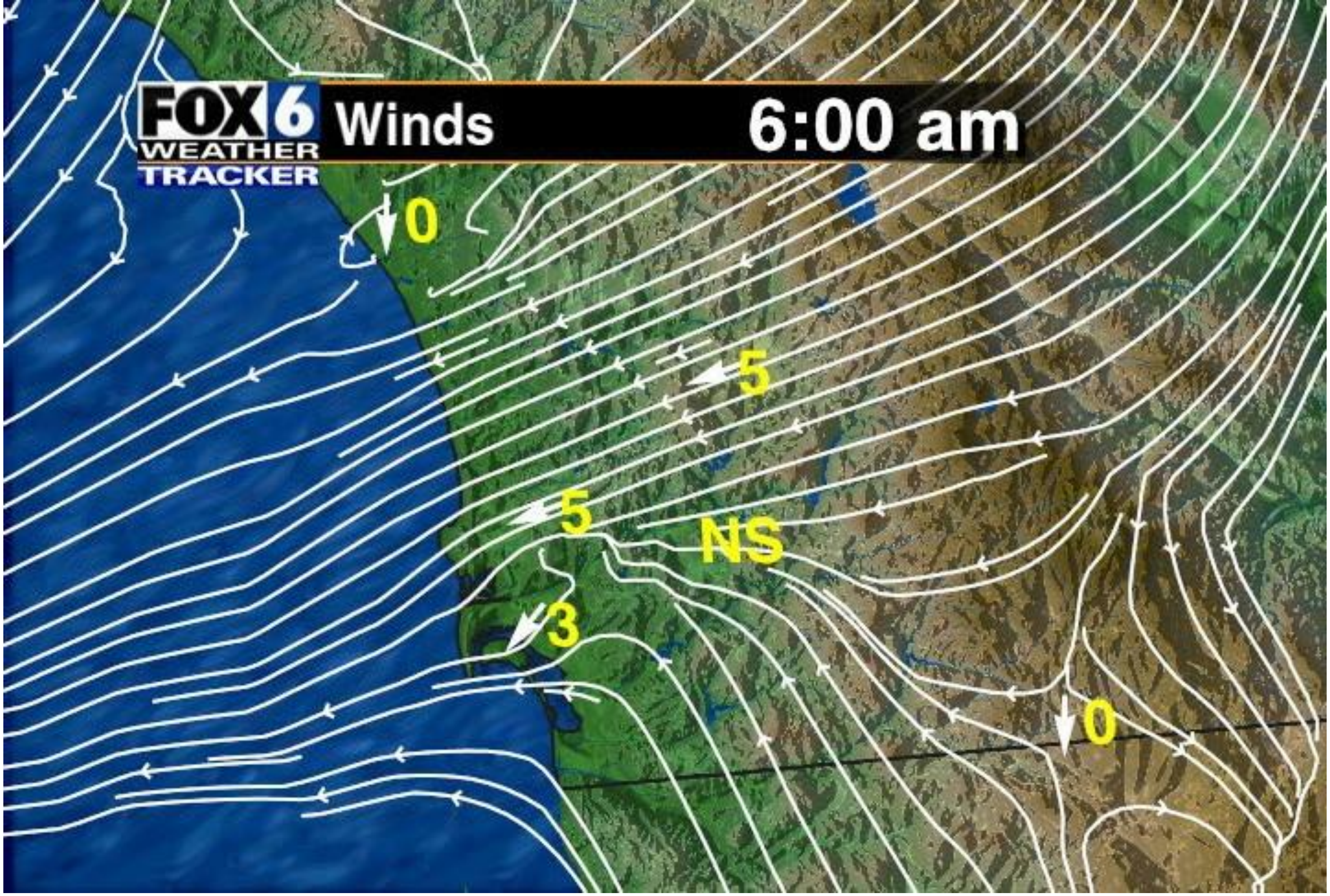
Slope Fields

Def. A slope field (or direction field) is a diagram that shows the slope of a solution at several points.

Ex. Draw a slope field for $\frac{dy}{dx} = x - y$, then sketch a solution that satisfies $y(0) = 0$.






FOX 6
WEATHER TRACKER
Winds
6:00 am
0
5
5
NS
3
0