Warm up Problems

$\cancel{(-\infty, 0)}$ $(0, \infty)$

Solve the IVP $\frac{dy}{dx} = \frac{y+1}{x}$, $y(1) = 2$, then sketch the solution.

$$\frac{1}{y+1}\,dy = \frac{1}{x}\,dx$$

$$\ln|y+1| = (\ln|x| + C)$$

$$e^{\ln|y+1|} = e^{\ln|x|} \cdot e^{C}$$

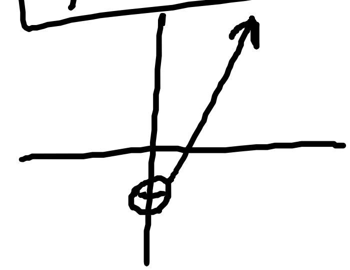
$$y+1 = D|x|$$

$$y = D|x| - 1$$

$$2 = D|1| - 1$$

$$D = 3$$

$$\boxed{y = 3|x| - 1}$$



1. $\int_1^x (2t-3)\,dt = t^2 - 3t \Big|_1^x = (x^2-3x)-(1-3) = x^2-3x+2$

2. $\frac{d}{dx}\left[\int_1^x (2t-3)\,dt\right] = \frac{d}{dx}(x^2-3x+2) = 2x-3$

3. $\int_{10}^x f'(t)\,dt = f(t)\Big|_{10}^x = f(x)-f(10)$

4. $\frac{d}{dx}\int_{10}^x f'(t)\,dt = \frac{d}{dx}\left[f(x)-f(10)\right] = f'(x)$

Second FTOC

$$F(x) = \int_a^x f(t)\,dt$$ is called an integral function.

$$\boxed{F'(x) = f(x)}$$

$$\boxed{\frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

<u>Ex.</u> Let $F(x) = \int_3^x te^{-t}dt$ $\longrightarrow$ $F'(x) = xe^{-x}$

a) Find $F(5)$ $= \int_3^5 te^{-t}dt = .159$

b) Find $F'(5) = 5e^{-5} = .034$

c) Find a value of x where $F(x) = 0$ $\quad x = 3$

d) Find $F''(5) = -.027$

Ex. $\frac{d}{dx}\int_x^2 \left(\frac{1}{t} - \sin t\right) dt = -\frac{d}{dx}\int_2^x \left(\frac{1}{t} - \sin t\right) dt$

$= -\left(\frac{1}{x} - \sin x\right)$

Ex. $\frac{d}{dx}\int_3^{5x} \cos^2 t \, dt = \cos^2(5x) \cdot 5$

<u>Ex.</u> $$\frac{d}{dx}\int_{5x}^{x^2} e^t \sin t \, dt$$

$$= e^{x^2} \sin(x^2) \cdot 2x - e^{5x} \sin(5x) \cdot 5$$

Ex. Let f be the continuous function whose graph is shown.

Let g be the function $g(x) = \int_1^x f(t)\,dt$

a) Find $g(2)$ and $g(-2)$.

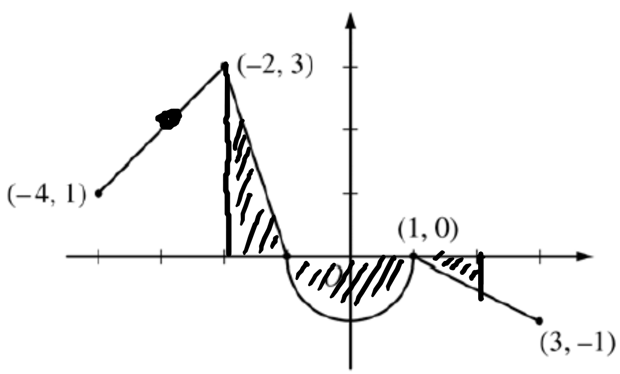
$$g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$$

$$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt = -\left[\frac{1}{2}(1)(3) - \frac{1}{2}\pi(1)^2\right] = \frac{\pi}{2} - \frac{3}{2}$$

b) Find $g'(-3)$ and $g''(-3)$.

$$g'(x) = f(x) \qquad g''(x) = f'(x)$$

$$g'(-3) = f(-3) = 2 \qquad g''(-3) = f'(-3) = 1$$



Graph of f

$$g(x) = \int_1^x f(t)\,dt$$

$g' = f$

c) Find the x-coord. of each point where g has a horiz. tangent line. Classify each point and justify your answer.

$\left.\begin{array}{l} g' = 0 \\ f = 0 \end{array}\right\}$ $x = -1 \rightarrow$ local max, f goes pos. to neg.

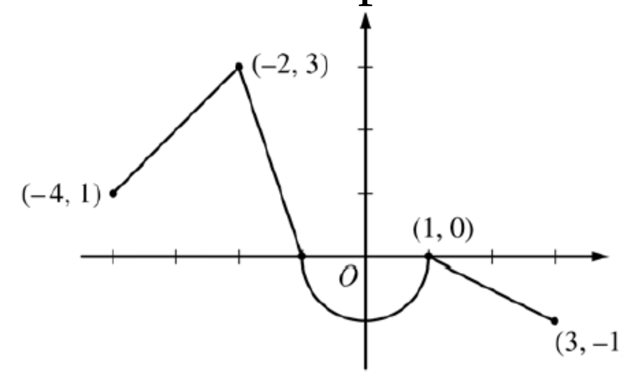
$x = 1 \rightarrow$ neither, f doesn't change signs.

d) Find the x-coord. of each point where g has an inflection pt. and justify your answer.

$x = -2$ slope of f changes signs

$x = 0$

$x = 1$



Graph of f

Unit 6 Progress Check: MCQ Part A

- Do #8-12

Unit 6 Progress Check: MCQ Part B

- Do #1, 4

Unit 7 Progress Check: MCQ

- Do #8-10, 12-13