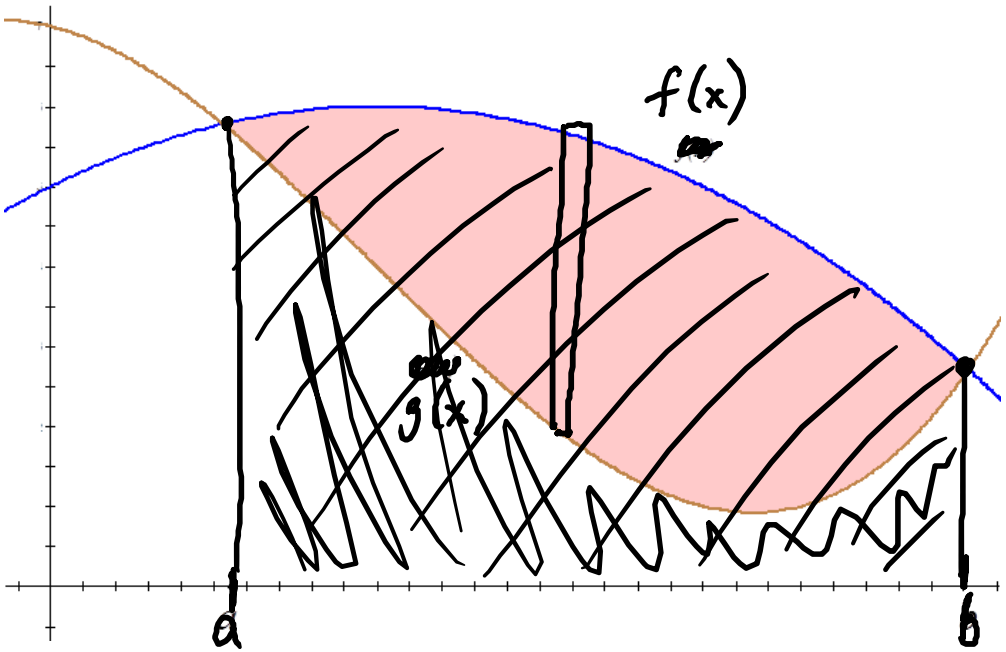
Area Between Curves



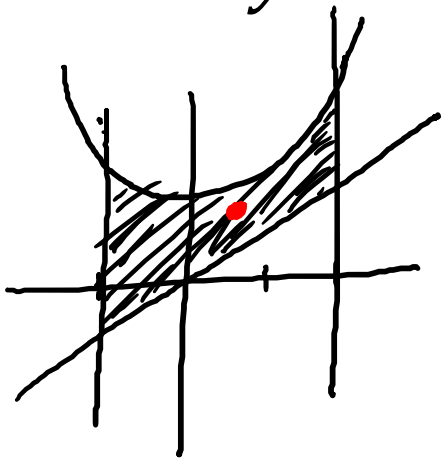
$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

The area between the curves can be found with:

$$A = \int_a^b [f(x) - g(x)]dx$$

$$A = \int_a^b [y_{top} - y_{bottom}]dx$$

Ex. Find the area of the region bounded by $y = x^2 + 1$, $y = x$, $x = -1$, and $x = 2$.

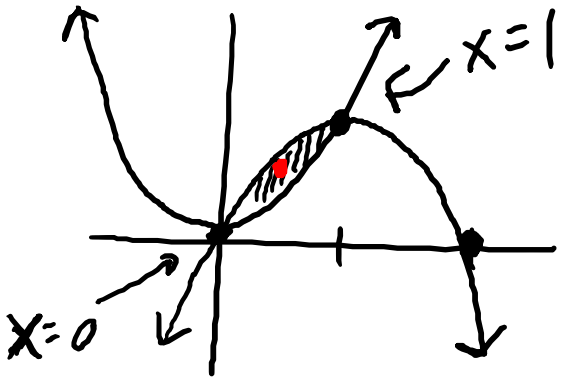


$$A = \int_{-1}^{2} \left[(x^2 + 1) - x \right] dx$$

$$= \frac{1}{3}x^3 + x - \frac{1}{2}x^2 \Big|_{-1}^{2}$$

$$= \left(\frac{8}{3} + 2 - 2 \right) - \left(-\frac{1}{3} - 1 - \frac{1}{2} \right)$$

Ex. Find the area of the region bounded by $y = x^2$ and $y = 2x - x^2$.



$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$x = 0 \quad x = 1$

$= x(2-x)$

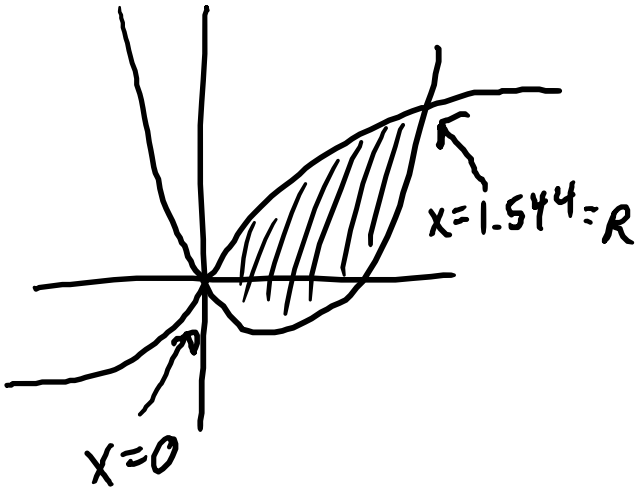
$$A = \int_0^1 (2x - x^2) - x^2 \, dx$$

$$= \int_0^1 (2x - 2x^2) \, dx = x^2 - \frac{2}{3}x^3 \Big|_0^1$$

$$= \left(1 - \frac{2}{3}\right) - 0 = \boxed{\frac{1}{3}}$$

Ex. Find the area of the region bounded by

$y = x^2 - x$ and $y = \frac{x}{\sqrt{x^2+1}}$.



$$A = \int_0^R \frac{x}{\sqrt{x^2+1}} - (x^2 - x)\,dx$$

$$\approx .805$$

Pract. Find the area of the region bounded by

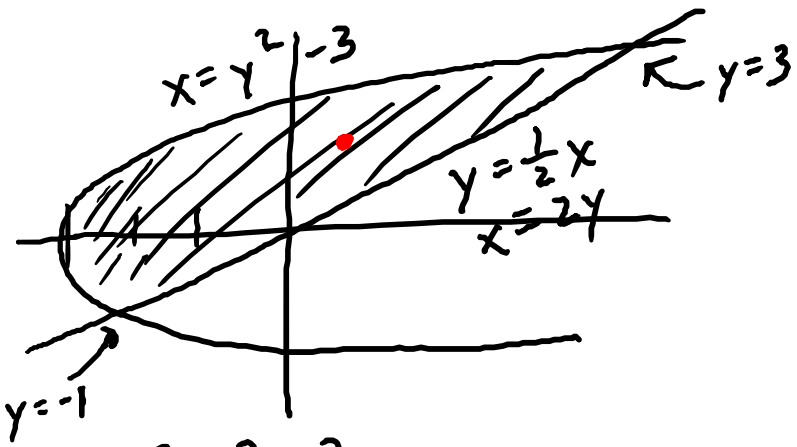
a) $y = 2 - x^2$ and $y = x$ [No calculators]

$$\int_{-2}^{1} [(2 - x^2) - x]dx = \frac{9}{2}$$

b) $y = e^x - 1$ and $y = 3x$ [Calculators OK]

$$\int_{0}^{1.904} [3x - (e^x - 1)]dx = 1.629$$

Ex. Find the area of the region bounded by $x = y^2 - 3$ and $y = \frac{1}{2}x$.



$$y^2 - 3 = 2y$$
$$y^2 - 2y - 3 = 0$$
$$(y-3)(y+1) = 0$$
$$y = 3 \quad y = -1$$

$$A = \int_{-1}^{3} 2y - (y^2 - 3)\, dy$$
$$= y^2 - \frac{1}{3}y^3 + 3y \Big|_{-1}^{3}$$
$$= (9 - 9 + 9) - \left(1 + \frac{1}{3} - 3\right)$$