Rectilinear Motion

If $s(t) =$ position, then

$s'(t) = v(t) =$ velocity

$s''(t) = a(t) =$ acceleration

$|v(t)| =$ speed

$\int v(t)dt =$ displacement

$\int |v(t)|dt =$ dist. travelled

$$\begin{pmatrix}\text{ave. veloc. from} \\ t = a \text{ to } t = b\end{pmatrix} = \frac{s(b) - s(a)}{b - a}$$

$$s(b) = s(a) + \int_a^b v(t)dt$$

1) A particle moves along the y-axis such that its velocity, for $0 \leq t \leq 8$ is given by $v(t) = t - 3\ln(t+2) + 2t\sin\left(\frac{t^2}{12}\right)$. It is known that its initial position is $y(0) = -3$.

a) Find all values of t on the interval $4 \leq t \leq 7$ for which the speed is 5.

$|v(t)| = 5$

$t = 5.621, 6.520$

b) Write an expression involving an integral for $y(t)$ and use it to find the position at $t = 3$.

$y(t) = y(0) + \int_0^t v(x)\,dx$

$y(3) = y(0) + \int_0^3 v(t)\,dt = -6.263$

1) A particle moves along the *y*-axis such that its velocity, for $0 \leq t \leq 8$ is given by $v(t) = t - 3\ln(t+2) + 2t\sin\left(\frac{t^2}{12}\right)$. It is known that its initial position is $y(0) = -3$.

c) Find all values of t on the interval $0 \leq t \leq 8$ at which the particle changes directions. Justify your answer.

$v(t) = 0$

$t = 2.341,\ 6.127$

$v(t)$ changes signs at both times

d) Is the speed increasing or decreasing at $t = 5$. Justify your answer.

$v(5) = 7.877$

$a(5) = -1.772$

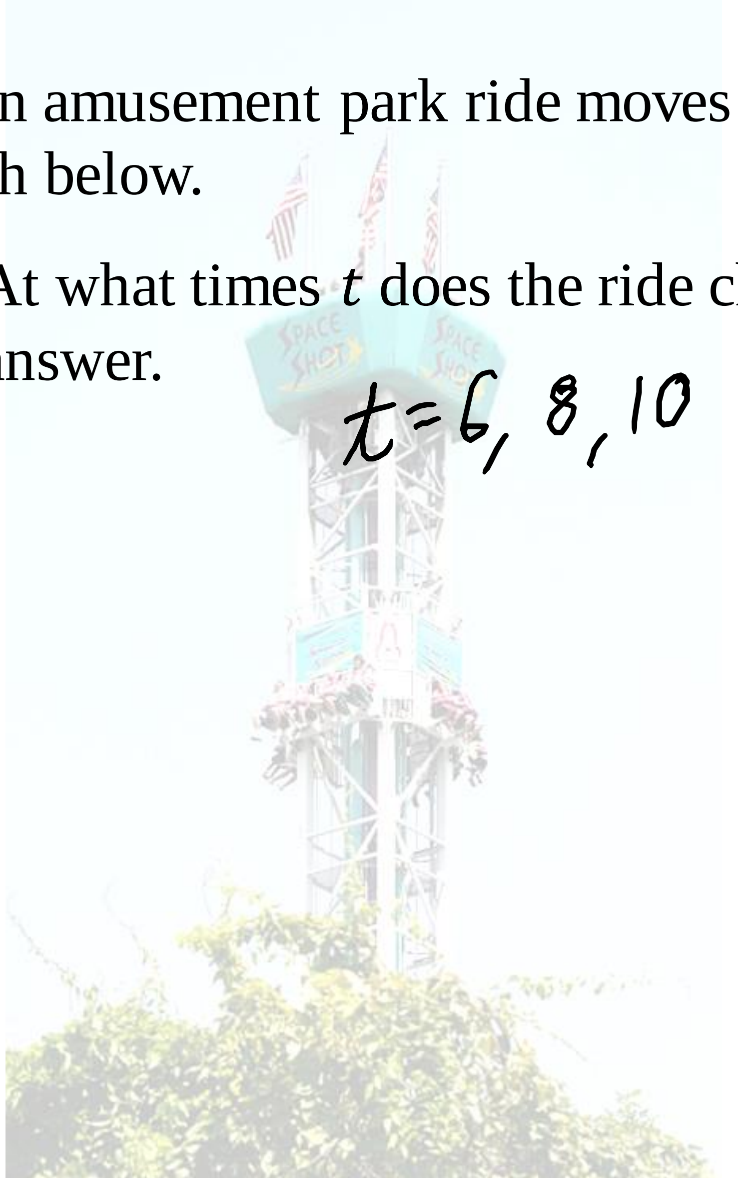
dec. because v and a diff. signs at $t = 5$

1) A particle moves along the y-axis such that its velocity, for $0 \le t \le 8$ is given by $v(t) = t - 3\ln(t+2) + 2t\sin\left(\frac{t^2}{12}\right)$. It is known that its initial position is $y(0) = -3$.

e) Find the total distance traveled by the particle on the interval $0 \le t \le 8$.

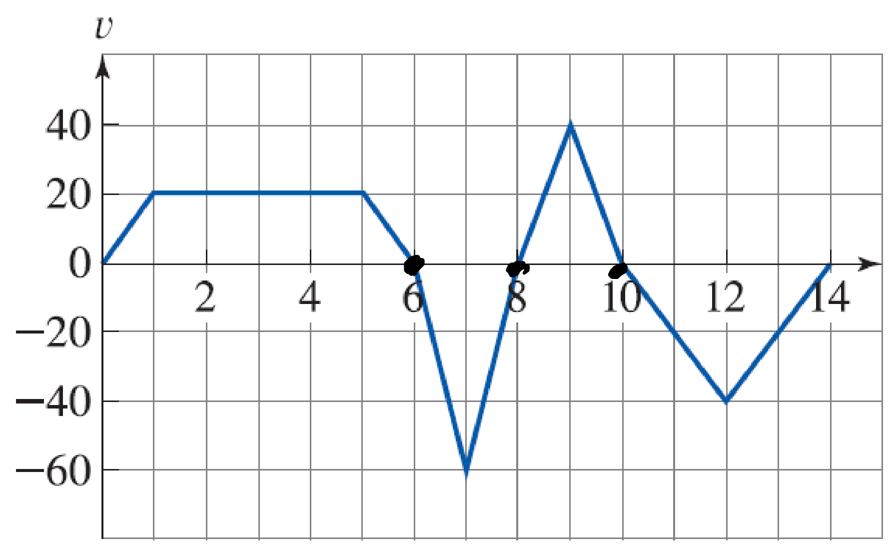
$$\int_0^8 |v(t)|\, dt = 40.287$$

2) An amusement park ride moves vertically and has velocity given in the graph below.



a) At what times t does the ride change directions? Give a reason for your answer.

$t = 6, 8, 10$ $\quad$ v changes signs



Graph of v

2) An amusement park ride moves vertically and has velocity given in the graph below.

b) If the ride starts on the ground, what is the maximum height of the ride?

$t=0$: $v(t)$ is pos. after

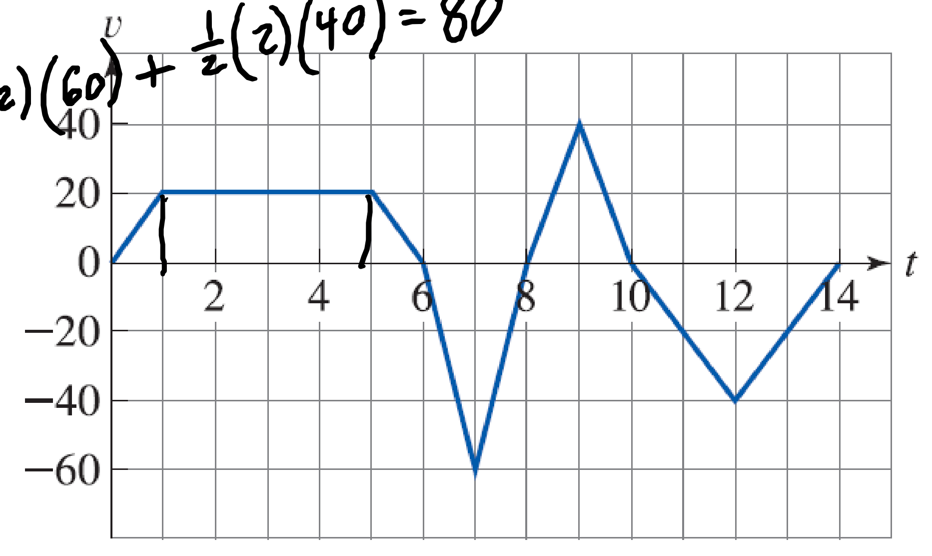
$t=6$: $y(6) = y(0) + \int_0^6 v(t)\,dt = \frac{1}{2}(1)(20) + 4(20) + \frac{1}{2}(1)(20) = 100$

$t=8$: local min.

$t=10$: $y(10) = y(0) + \int_0^{10} v(t)\,dt = 100 - \frac{1}{2}(2)(60) + \frac{1}{2}(2)(40) = 80$

$t=14$: $v(t)$ neg. before

$\boxed{\text{max. height is } 100}$

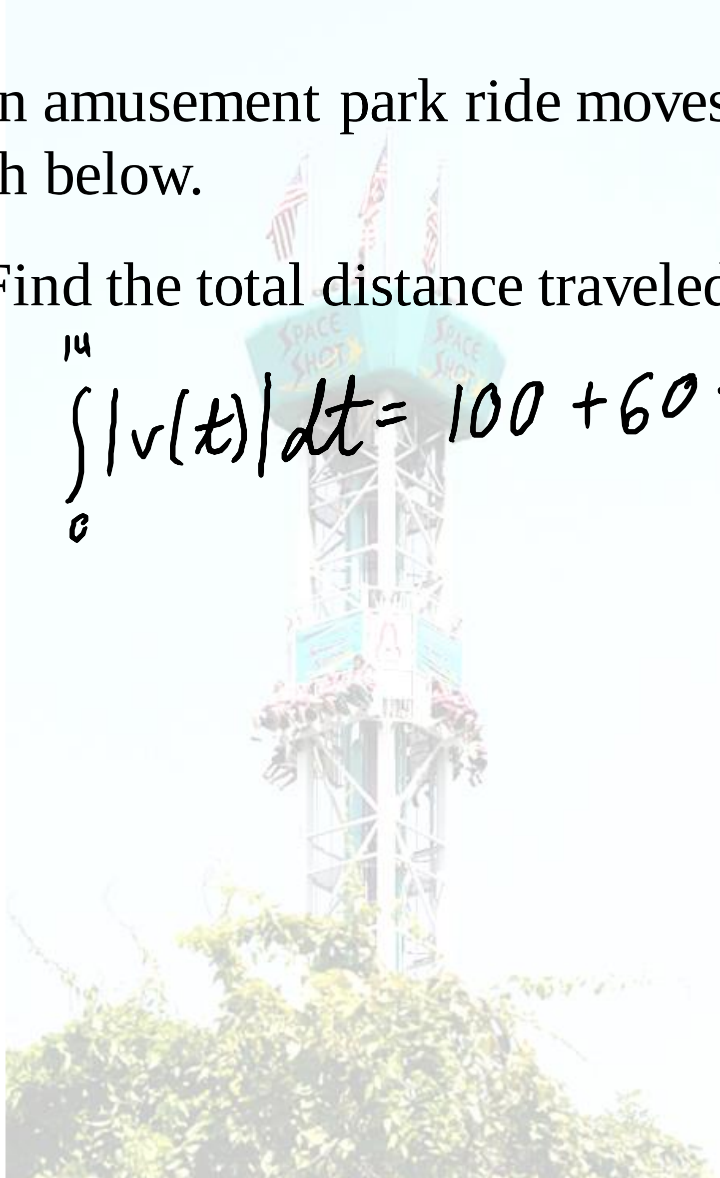


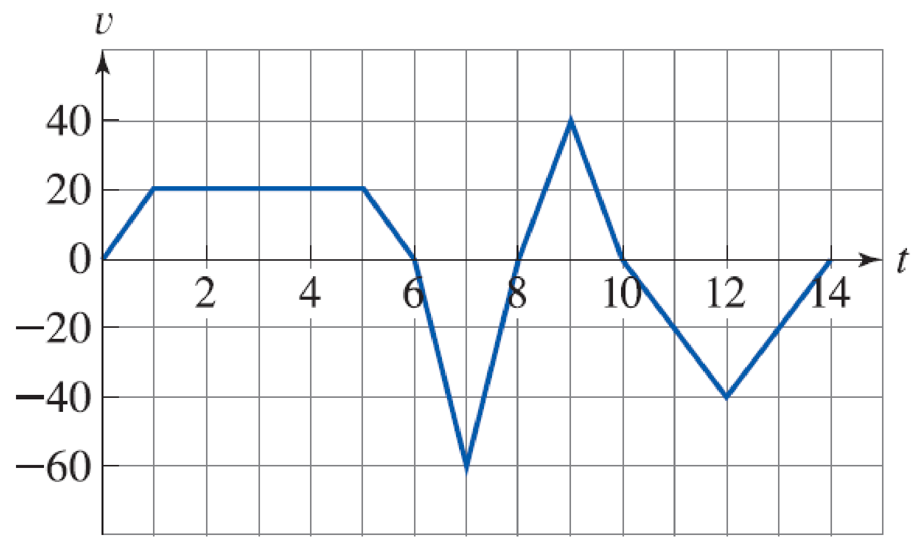
Graph of v

2) An amusement park ride moves vertically and has velocity given in the graph below.

c) Find the total distance traveled on the interval $0 \le t \le 14$.

$$\int_0^{14} |v(t)|\,dt = 100 + 60 + 40 + \frac{1}{2}(4)(40) = 280$$





Graph of v

3) A particle moves along the x-axis such that its velocity, for $0 \le t \le 10$ is given by $v(t) = t^2 - 9t + 14$. It is known that its initial position is $x(0) = 15$.

a) On what intervals is the particle moving to the left? $\rightarrow v(t) < 0$?

$v(t) = t^2 - 9t + 14 = 0$

$(t-2)(t-7) = 0$

$t = 2, 7$

v	+	−	+
0	2	7	10

$\boxed{(2, 7)}$

b) Find the position of the particle at time $t = 8$.

$$x(8) = x(0) + \int_0^8 v(t)\,dt = 15 + \int_0^8 (t^2 - 9t + 14)\,dt$$

$$= 15 + \left(\frac{1}{3}t^3 - \frac{9}{2}t^2 + 14t\right)\Big|_0^8 = 15 + \frac{1}{3}(8)^3 - \frac{9}{2}(8)^2 + 14(8) - 0$$

4) Valerie swims in a straight line. For $0 \leq t \leq 50$, her velocity is a differentiable function with values given in the table below.

a) Estimate the value of $v'(30)$. $\approx \frac{v(40)-v(20)}{40-20} = \frac{1.3-(-.7)}{20} = \frac{2}{20} = \frac{1}{10}$

sec.

b) Using correct units, explain the meaning of $v'(30)$. $= \frac{m/sec}{sec}$

At $t=30$ sec., veloc. is changing at a rate of $v'(30)$ m/sec/sec.

t (sec.)	0	8	20	40	50
$v(t)$ (m/sec.)	0	1.2	-0.7	1.3	1

4) Valerie swims in a straight line. For $0 \le t \le 50$, her velocity is a differentiable function with values given in the table below.

c) Is there a point on $8 \le t \le 40$ such that $v'(t) = \frac{1}{320}$? Justify your answer.

v(t) is cont. and diff.

$v'(t) = \frac{v(40)-v(8)}{40-8} = \frac{1.3-1.2}{32} = \frac{.1}{32} = \frac{1}{320}$

yes. by MVT

d) Is there a point on $0 \le t \le 8$ such that $v(t) = \frac{1}{2}$? Justify your answer.

v(t) is cont. because v is diff.

$v(0) < \frac{1}{2}$

$v(8) > \frac{1}{2}$

$\therefore v(t) = \frac{1}{2}$ on interval by IVT

t (sec.)	0	8	20	40	50
$v(t)$ (m/sec.)	0	1.2	-0.7	1.3	1

4) Valerie swims in a straight line. For $0 \leq t \leq 50$, her velocity is a differentiable function with values given in the table below.

e) Using correct units, explain the meaning of $\int_0^{50} |v(t)|\, dt$.

Total dist. traveled, in m, from $t = 0$ sec. to $t = 50$ sec.

f) Approximate $\int_0^{50} |v(t)|\, dt$ using a right Riemann sum with the intervals indicated in the table.

" $v(8) \cdot 8 + |v(20)| \cdot 12 + v(40) \cdot 20 + v(50) \cdot 10$

$= (1.2)(8) + (0.7)(12) + (1.3)(20) + 1(10)$

t (sec.)	0	8	20	40	50
$v(t)$ (m/sec.)	0	1.2	-0.7	1.3	1

Physics Extravaganza

Position is the place you are,
at any given time you see.
The instantaneous rate of change of that
is the velocity;
Which is direction and the speed
two parts of information,
Its instantaneous rate of change is called acceleration.

Displacement is how far you are
from your initial starting spot,
Remembering the average value
of a function is a lot,
One over *B* minus *A* times the value of the integral
from *A* to *B* of *f* of *x dx* if on an interval.

The total distance traveled
is by no means an atrocity,
the integral of absolute value of the velocity!
Another point of interest know
the integral of force is work.
Accelerations rate of change is
surge or lurch or jolt, or jerk!

Unit 6 Progress Check: MCQ Part B

- Do #2, 5-6, 10-12

Unit 8 Progress Check: MCQ Part A

- Do #2, 7-10