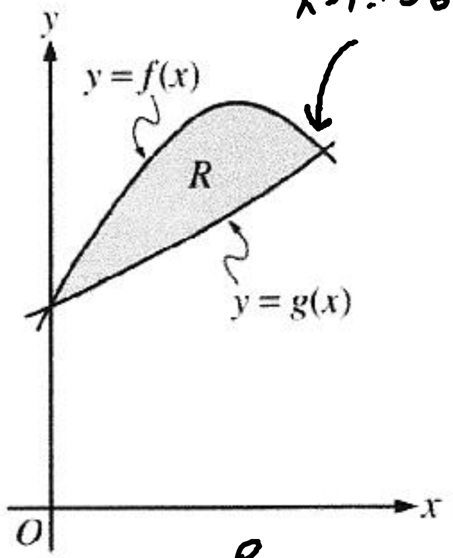
Warm up Problem

$x = 1.136 = R$



Let $f(x)$ and $g(x)$ be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of $f(x)$ and $g(x)$ as shown in the figure above. Find the area of R.

$$A = \int_0^R [f(x) - g(x)]\,dx = .429$$

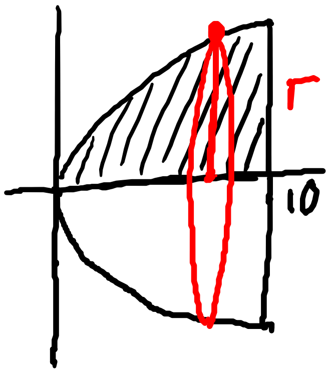
Volume

When we found areas, we broke the region up into rectangles

→The integral was like adding all the areas

→With volumes, we will cut the object into slices and then integrate the volume of each slice

Ex. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $x = 10$, and $y = 0$ about the x-axis.



$$V = \int_0^{10} \pi(\sqrt{x})^2\,dx = \int_0^{10} \pi x\,dx$$

$$= \frac{\pi}{2}x^2\Big|_0^{10}$$

$$= 50\pi$$

$r = y = \sqrt{x}$

Volume of Revolution

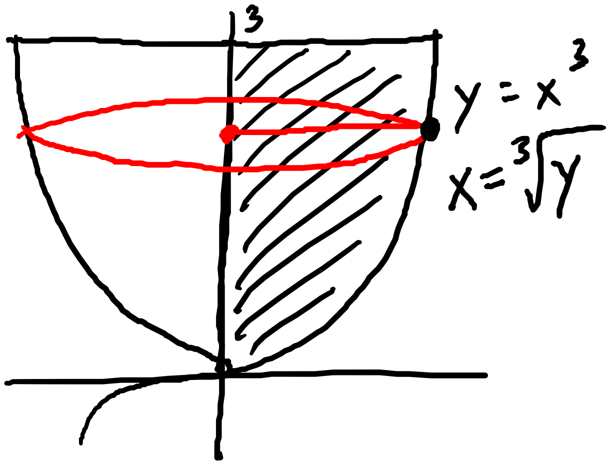
1. Draw a picture: shade the original region, draw the reflection, and draw an arbitrary disk/washer

2. The axis of integration is the same as the axis of rotation

3. Identify the radius of your disk/washer. It will include your variable of integration

4. $V = \pi \int_a^b r^2 dx$ or $V = \pi \int_a^b [R^2 - r^2] dx$

<u>Ex.</u> Region bounded by $y = x^3$, $x = 0$, $y = 3$; y-axis



$$r = x = \sqrt[3]{y}$$

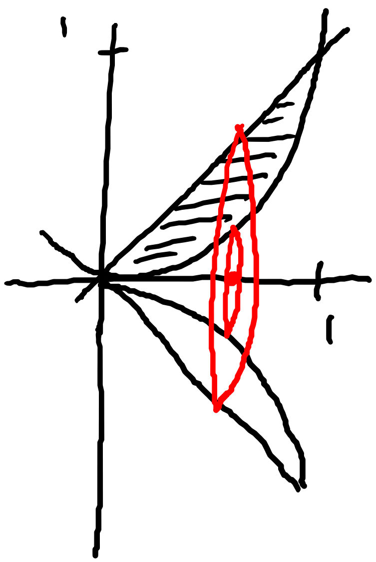
$$V = \pi \int_0^3 \left(\sqrt[3]{y}\right)^2 dy$$

$$= \pi \int_0^3 y^{2/3} dy$$

$$= \pi \frac{3}{5} y^{5/3} \Big|_0^3$$

$$= \frac{3\pi}{5}(3)^{5/3}$$

<u>Ex.</u> Region bounded by $y = x$, $y = x^2$; x-axis



$$V = \pi \int_0^1 (x)^2 - (x^2)^2 \, dx$$

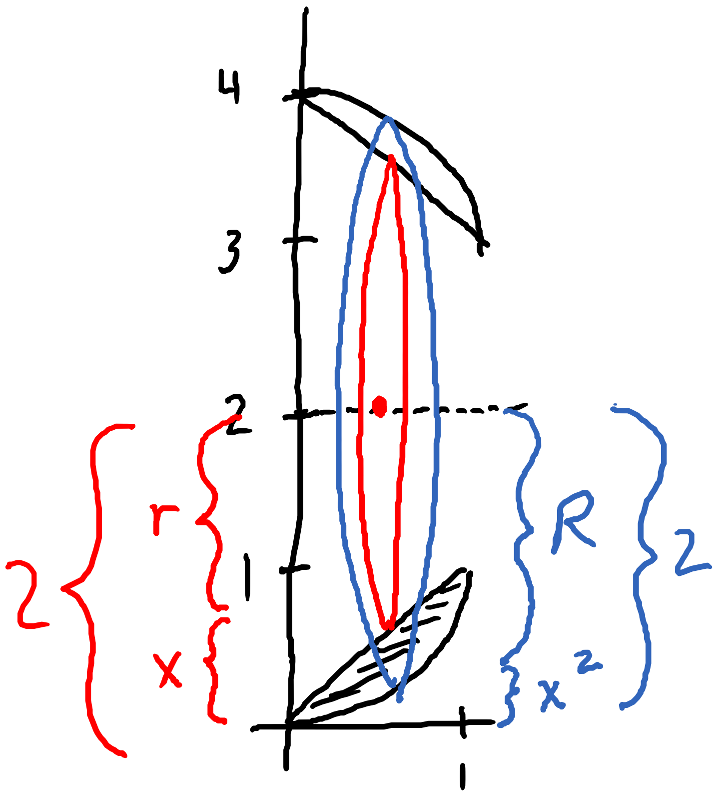
$$= \pi \int_0^1 x^2 - x^4 \, dx = \pi \left(\frac{1}{3}x^3 - \frac{1}{5}x^5 \right) \Bigg|_0^1$$

$$= \pi \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{2\pi}{15}$$

$R = x$

$r = x^2$

Ex. Region bounded by $y = x$, $y = x^2$; line $y = 2$



$$V = \pi \int_0^1 (2 - x^2)^2 - (2 - x)^2 \, dx$$

$R = 2 - x^2$

$r = 2 - x$

<u>Pract.</u> Region bounded by $y = \sqrt{x}$, $y = 3$, $x = 0$; y-axis

$$\pi \int_{0}^{3} (y^2)^2 dy = \frac{243\pi}{5}$$

<u>Pract.</u> Region bounded by $y = x^2 + 1$, $y = 2$; x-axis

$$\pi \int_{-1}^{1} [(2)^2 - (x^2 + 1)^2] dx = \frac{64\pi}{15}$$

<u>Pract.</u> Region bounded by $y = x$, $y = x^2$; line $y = -1$

$$\pi \int_{0}^{1} [(1 + x)^2 - (1 + x^2)^2] dx = \frac{7\pi}{15}$$