Warm up Problems

1. $\int_1^2 \frac{t}{1+3t^2}\,dt$

2. $\int \frac{\sin\sqrt{y}}{\sqrt{y}}\,dy$

3. $\int \tan x\,dx$

Integration by Parts

To find our formula, we use the product rule (don't write this down):

$$\int d(u \cdot v) = \int u\, dv + \int v\, du$$

$$uv = \int u\, dv + \int v\, du$$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

- We pick u and dv
- After using formula, you still have x's

Ex. $\int x \cos x \, dx$ $= x \sin x - \int \sin x \, dx$

$\int u \, dv = uv - \int v \, du$

$u = x$	$dv = \cos x \, dx$
$du = dx$	$v = \sin x$

$= x \sin x - (-\cos x) + C$

When picking u, go in LIATE order:

Logarithm

Inverse Trigonometric

Algebraic (Polynomial) x ← u

Trigonometric $\cos x$

Exponent

Ex. $\int x \sec^2 x \, dx$

$u = x$	$dv = \sec^2 x \, dx$
$du = dx$	$v = \tan x$

$= x \tan x - \int \tan x \, dx$

$= x \tan x - \ln|\sec x| + C$

L
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E

$uv - \int v \, du$

Ex. $\int x^2 e^x dx$

$u = x^2 \qquad dv = e^x dx$

$du = 2x\,dx \qquad v = e^x$

$= x^2 e^x - \int 2x e^x dx$

$u = 2x \qquad dv = e^x dx$

$du = 2dx \qquad v = e^x$

$= x^2 e^x - \left[2x e^x - \int e^x \cdot 2 dx\right]$

$= x^2 e^x - 2x e^x + 2e^x + C$

1. $\int xe^{-x}dx$

$-xe^{-x} - e^{-x} + c$

2. $\int x \ln x \, dx$

$\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$

3. $\int x^2 \sin x \, dx$

$-x^2 \cos x + 2x \sin x + 2 \cos x + c$

4. $\int x \cos(3x) \, dx$

$\frac{1}{3}x \sin(3x) + \frac{1}{9}\cos(3x) + c$

Ex. $\int \ln x \cdot 1 \, dx$

$u = \ln x \qquad dv = 1\,dx$

$du = \frac{1}{x}dx \qquad v = x$

$= x \ln x - \int x \cdot \frac{1}{x} dx$

$= x \ln x - \int 1 \, dx$

$$\int \ln x \, dx = x \ln x - x + C$$

T-shirt design ideas are due next class.