

- Blue part is out of 27
  - Green part is out of 73
- Total of 100 points possible

Let's talk about calculus shirts.

# Substitution

Right now, we can only integrate a small number of functions...in this chapter we will learn two methods that will expand that list.

Ex. If  $f(x) = (x^{10} - 4)^4$ , find  $f'(x)$ .

$$f'(x) = 4(x^{10} - 4)^3 \cdot 10x^9$$

$$\underline{\text{Ex.}} \int \sin(12x) dx = \int \sin u \cdot \frac{1}{12} du$$

$$\begin{aligned} u &= 12x \\ du &= 12 dx \\ \frac{1}{12} du &= dx \end{aligned}$$

$$= \frac{1}{12} \int \sin u du$$

$$= -\frac{1}{12} \cos u + C$$

$$= -\frac{1}{12} \cos(12x) + C$$

---

$$\underline{\text{Ex.}} \int x(x^2 - 5)^4 dx$$

$$\begin{aligned} u &= x^2 - 5 \\ du &= 2x dx \\ \frac{1}{2} du &= x dx \end{aligned}$$

$$= \int u^4 \frac{1}{2} du = \frac{1}{10} u^5 + C$$

$$= \frac{1}{10} (x^2 - 5)^5 + C$$

Substitution is a result of the chain rule

- $u$  will be the stuff on the inside
  - its derivative will always show up on the outside
- All  $x$ 's must change to  $u$ 's before we can integrate

$$\underline{\text{Ex.}} \int \sqrt{\cos x} \sin x dx = \int u^{1/2} (-1) du$$

$$\begin{array}{l} u = \cos x \\ du = -\sin x dx \\ -du = \sin x dx \end{array}$$

$$\begin{aligned} &= -\frac{2}{3} u^{3/2} + C \\ &= -\frac{2}{3} (\cos x)^{3/2} + C \end{aligned}$$

$$\underline{\text{Ex.}} \int \frac{(\ln x)^4}{x} dx = \int (\ln x)^4 \cdot \frac{1}{x} dx = \int u^4 du$$

$$\begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array}$$

$$\begin{aligned} &= \frac{1}{5} u^5 + C \\ &= \frac{1}{5} (\ln x)^5 + C \end{aligned}$$

$$\underline{\text{Ex.}} \int \tan^5 x \sec^2 x dx = \int (\tan x)^5 \underline{\sec^2 x dx}$$

$$\boxed{\begin{array}{l} u = \tan x \\ du = \underline{\sec^2 x dx} \end{array}}$$

$$= \int u^5 du = \frac{1}{6} u^6 + C$$

$$= \frac{1}{6} \tan^6 x + C$$

$$\underline{\text{Ex.}} \int (x^3 - 2)^2 dx = \int (x^6 - 4x^3 + 4) dx$$

$$= \frac{1}{7} x^7 - x^4 + 4x + C$$

$$1. \int \frac{1}{(1+2x)^3} dx$$

$$-\frac{1}{4}(1+2x)^{-2} + c$$

$$2. \int \sec^2(3x) dx$$

$$\frac{1}{3}\tan(3x) + c$$

$$3. \int x^2 \sqrt{x^3+1} dx$$

$$\frac{2}{9}(x^3+1)^{3/2} + c$$

$$4. \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

$$-2 \cos \sqrt{x} + c$$