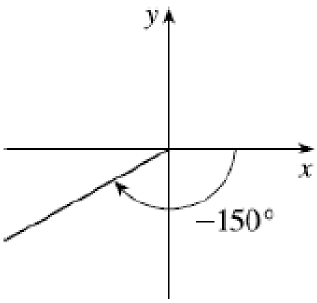
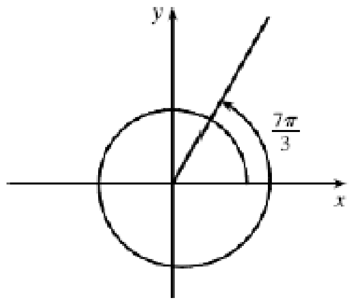
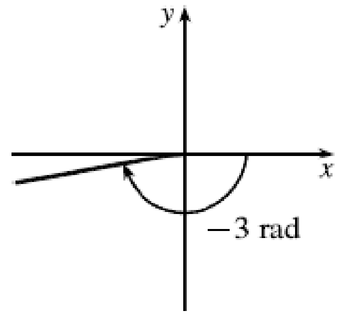
p. 37: 23-33 odd, 77-87 odd



23.



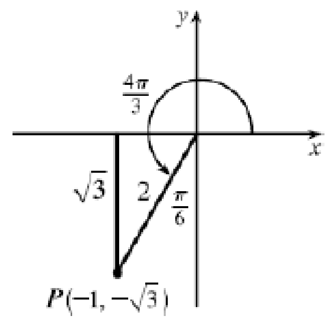
25.



27.

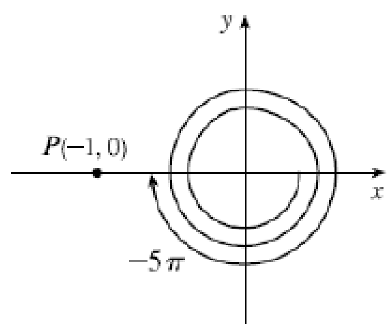
29. From the diagram, we see that appoint on the terminal side is $P\left(-1,-\sqrt{3}\right)$.
Therefore, taking $x=-1, y=-\sqrt{3}, r=2$ in the definitions of the trig ratios, we have

$$\sin\frac{4\pi}{3}=-\frac{\sqrt{3}}{2},\quad \cos\frac{4\pi}{3}=-\frac{1}{2},\quad \tan\frac{4\pi}{3}=\sqrt{3},$$
$$\csc\frac{4\pi}{3}=-\frac{2}{\sqrt{3}},\quad \sec\frac{4\pi}{3}=-2, \text{ and } \cot\frac{4\pi}{3}=\frac{1}{\sqrt{3}}.$$



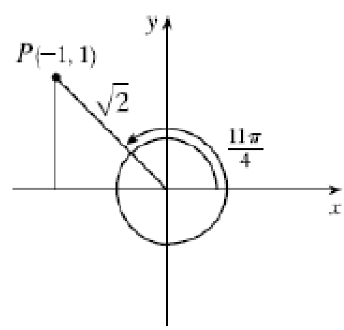
31. From the diagram, we see that appoint on the terminal side is $P(0,1)$. Therefore, taking $x=0, y=2, r=1$ in the definitions of the trig ratios, we have

$$\sin(-3\pi)=0,\quad \cos(-3\pi)=-1,\quad \tan(-3\pi)=0,$$
$$\csc(-3\pi)=\text{undefined},\quad \sec(-3\pi)=-1, \text{and} \cot(-3\pi)=\text{undefined}.$$



33. From the diagram, we see that appoint on the terminal side is $P(-1,1)$.
Therefore, taking $x=-1, y=1, r=\sqrt{2}$ in the definitions of the trig ratios, we have

$$\sin\frac{11\pi}{4}=\frac{\sqrt{2}}{2},\quad \cos\frac{11\pi}{4}=-\frac{\sqrt{2}}{2},\quad \tan\frac{11\pi}{4}=-1,$$
$$\csc\frac{11\pi}{4}=\sqrt{2},\ \sec\frac{11\pi}{4}=-\sqrt{2}, \text{ and } \cot\frac{11\pi}{4}=-1.$$



77. $2\cos x-1=0 \Rightarrow 2\cos x=1 \Rightarrow \cos x=\frac{1}{2} \Rightarrow x=\frac{\pi}{3},\frac{5\pi}{3}$

79. $2\sin^2 x=1 \Rightarrow \sin^2 x=\frac{1}{2} \Rightarrow \sin x=\pm\frac{1}{\sqrt{2}} \Rightarrow x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

81. $\sin 2x=\cos x \Rightarrow 2\sin x\cos x=\cos x \Rightarrow \cos x(2\sin x-1)=0 \Rightarrow \cos x=0 \quad x=\frac{\pi}{2},\frac{3\pi}{2}$

or $2\sin x-1=0 \Rightarrow \sin x=\frac{1}{2} \Rightarrow x=\frac{\pi}{6},\frac{5\pi}{6}$

83. $\sin x = \tan x \;\Rightarrow\; \sin x - \dfrac{\sin x}{\cos x} = 0 \;\Rightarrow\; \sin x\left(1 - \dfrac{1}{\cos x}\right) = 0 \;\Rightarrow\; \sin x = 0 \;\Rightarrow\; x = 0, \pi, 2\pi$

or $1 - \dfrac{1}{\cos x} = 0 \;\Rightarrow\; \cos x = 1 \;\Rightarrow\; x = 0, 2\pi$. So the solutions are $x = 0, \pi, 2\pi$.

85. $\cos 3x = -\sin 3x \;\Rightarrow\; \dfrac{\cos 3x}{\sin 3x} = -1 \;\Rightarrow\; \cot 3x = -1 \;\Rightarrow\; x = \dfrac{\cot^{-1}(-1)}{3} = \dfrac{\pi}{4}, \dfrac{7\pi}{12}, \dfrac{11\pi}{12}, \dfrac{5\pi}{4}, \dfrac{19\pi}{12}, \dfrac{23\pi}{12}$

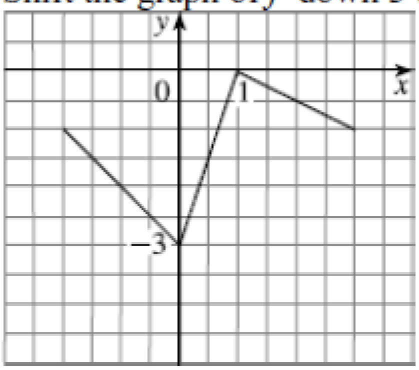
87. $\tan x = \sec x \Rightarrow \frac{\sin x}{\cos x} = \frac{1}{\cos x} \Rightarrow \sin x = 1 \Rightarrow x = \frac{\pi}{2}$

Because the original equation is undefined at $x = \frac{\pi}{2}$, there is no solution.

p. 46: 16, 57-71 odd, 76-78

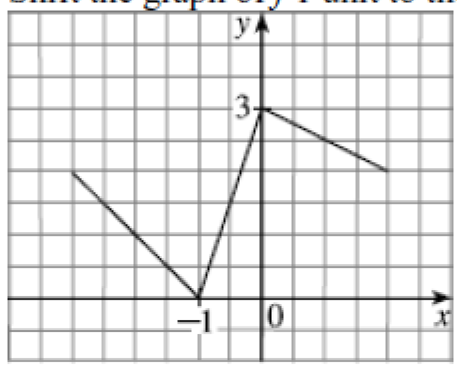
16. (a) $y = f(x) - 3$:

Shift the graph of f down 3 units:



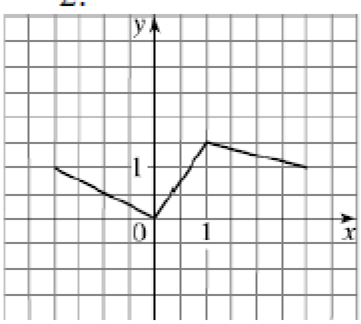
(b) $y = f(x+1)$:

Shift the graph of f 1 unit to the left:



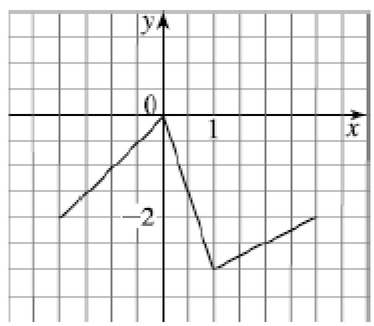
(c) $y = \frac{1}{2} f(x)$:

Shrink the graph of f vertically by a factor of 2:



(d) $y = -f(x)$:

Reflect the graph of f about the x-axis.



57. (a) $(f \circ g)(x) = (1-4x)^3 - 2 = -64x^3 + 48x^2 - 12x - 1$. Its domain is all real numbers.

(b) $(g \circ f)(x) = 1 - 4(x^3 - 2) = -4x^3 + 9$

(c) $(f \circ f)(x) = (x^3 - 2)^3 - 2 = x^9 - 6x^6 + 12x^3 - 10$. Its domain is all real numbers.

(d) $(g \circ g)(x) = 1 - 4(1 - 4x) = 16x - 3$. Its domain is all real numbers.

59. (a) $(f \circ g)(x) = \sin(x^2+1)$. Its domain is all real numbers.

(b) $(g \circ f)(x) = \sin^2 x + 1$. Its domain is all real numbers.

(c) $(f \circ f)(x) = \sin(\sin x)$. Its domain is all real numbers.

(d) $(g \circ g)(x) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2$. Its domain is all real numbers.

61. (a) $(f \circ g)(x) = \dfrac{\sin 2x}{1+\sin 2x}$. Its domain is $\{x \in \mathbb{R} \mid x \neq (\frac{3}{4}+n)\pi\}$, $n \in \mathbb{Z}$.

(b) $(g \circ f)(x) = \sin\left(\dfrac{x}{1+x}\right)$. Its domain is $\{x \in \mathbb{R} \mid x \neq -1\}$.

(c) $(f \circ f)(x) = \dfrac{\frac{x}{1+x}}{1+\frac{x}{1+x}} = \dfrac{\left(\frac{x}{1+x}\right)\cdot(1+x)}{\left(1+\frac{x}{1+x}\right)\cdot(1+x)} = \dfrac{x}{2x+1}$ Its domain is $\{x \in \mathbb{R} \mid x \neq -1, -\frac{1}{2}\}$.

(d) $(g \circ g)(x) = \sin(2\sin 2x)$. Its domain is all real numbers.

63. $(f \circ g \circ h)(x) = \left|\left(\sqrt{x}\right)^2 - 4\right| = \left||x| - 4\right|$

65. $(f \circ g \circ h)(x) = f\left(g\left(\sqrt[3]{x}\right)\right) = f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right) = \tan\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$

67. If $f(x) = x^2$, $g(x) = \cos x$. then $(f \circ g)(x) = (\cos x)^2 = \cos^2 x = F(x)$.

69. If $f(x) = \sqrt[3]{x}$, $g(x) = \dfrac{x}{1+x}$, then $(f \circ g)(x) = \sqrt[3]{\dfrac{x}{1+x}} = F(x)$.

71. If $f(t) = \dfrac{t}{1+t}$, $g(t) = \tan t$, then $(f \circ g)(t) = f(\tan t) = \dfrac{\tan t}{1+\tan t} = F(t)$.

76. (a) $f(g(1)) = f(6) = 5$

(b) $g(f(1)) = g(3) = 2$

(c) $f(f(1)) = f(3) = 4$

(d) $g(g(1)) = g(6) = 3$

(e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$

(f) $(f \circ g)(6) = f(g(6)) = f(3) = 4$

77. (a) $f(g(2)) = f(5) = 4$

(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6) =$ undefined

(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

78. To find a particular value of $f(g(x))$, say for $x = 0$, we note from the graph that $g(0) \approx 2.8$ and $f(2.8) \approx -0.5$. Thus $f(g(0)) \approx f(2.8) \approx -0.5$. The other values listed in the table are obtained in a similar fashion.

x	$g(x)$	$f(g(x))$
–5	–0.2	–4
–4	1.2	–3.3
–3	2.2	–1.7
–2	2.8	–0.5
–1	3	–0.2

x	$g(x)$	$f(g(x))$
0	2.8	–0.5
1	2.2	–1.7
2	1.2	–3.3
3	–0.2	–4
4	–1.9	–2.2
5	–4.1	1.9