1.6 and 1.7

p. 73: 21-29 odd, 34-36, 39-43 odd, 53-63, 70-72, 77-79 odd, 94, 101-113 odd

21. The function f is not one-to-one because $2 \neq 6$ but $f(2) = f(6) = 2$.

23. Because there are horizontal lines that intersect the graph in more than one point, this function is not one-to-one.

25. No horizontal line intersects the graph more than once, so this function is one-to-one.

27. The graph of $f(x) = 2x - 3$ is a line with slope 2. It passes the horizontal line test, so f is one-to-one.

29. If $g(x) = 1 - \sin x$, then $g(0) = g(\pi) = 1$, so g is not one-to-one.

34. Observe that f is a one-to-one function (f is an increasing function). By inspection, $f(1) = 3$, so $f^{-1}(3) = 1$. Because f is one-to-one, $f\left(f^{-1}(2)\right) = 2$.

35. Because g is an increasing function, g is one-to-one. By inspection $g(0) = 4$, so $g^{-1}(4) = 0$.

36. (a) The function f is one-to-one because it passes the horizontal line test.

(b) The domain of f^{-1} is the range of f which is $[-1, 3]$. The range of f^{-1} is the domain of f which is $[-3, 3]$.

(c) $f^{-1}(2) = 0$

(d) Because $f(-1.7) \approx 0$, $f^{-1}(0) \approx -1.7$.

39. $y = f(x) = 1 + \sqrt{2+3x} \Rightarrow y - 1 = \sqrt{2+3x} \Rightarrow (y-1)^2 = 2 + 3x \Rightarrow (y-1)^2 - 2 = 3x \Rightarrow$
$x = \frac{1}{3}(y-1)^2 - \frac{2}{3}$. So $f^{-1}(x) = \frac{1}{3}(x-1)^2 - \frac{2}{3}$. The domain of f^{-1} is $x \geq 1$.

41. $y = f(x) = e^{2x-1} \Rightarrow \ln y = 2x - 1 \Rightarrow 1 + \ln y = 2x \Rightarrow x = \frac{1}{2}(1 + \ln y)$. So, $f^{-1}(x) = \dfrac{1}{2}(1 + \ln x)$.

43. $y = f(x) = \ln(x+3) \Rightarrow x + 3 = e^y \Rightarrow x = e^y - 3$. So $f^{-1}(x) = e^x - 3$.

53. (a) $\log_2 32 = \log_2 2^5 = 5$.

(b) $\log_8 2 = \log_8 8^{1/3} = \dfrac{1}{3}$.

54. (a) $\log_5 \dfrac{1}{125} = \log_5 \dfrac{1}{5^3} = \log_5 5^{-3} = -3$.

(b) $\ln\left(1/e^2\right) = \ln e^{-2} = -2$.

55. (a) $\log_{10} 40 + \log_{10} 2.5 = \log_{10}\left[(40)(2.5)\right] = \log_{10} 100 = \log_{10} 10^2 = 2$

(b) $\log_8 60 - \log_8 3 - \log_8 5 = \log_8 \frac{60}{3} - \log_8 5 = \log_8 20 - \log_8 5 = \log_8 \frac{20}{5} = \log_4 = \log_8 8^{2/3} = \dfrac{2}{3}$

56. (a) $e^{-\ln 2} = \dfrac{1}{e^{\ln 2}} = \dfrac{1}{2}$ (b) $e^{\ln 2\left(\ln e^3\right)} = e^{\ln 3} = 3$

57. $\ln 10 + 2\ln 5 = \ln 10 + \ln 5^2 = \ln[(10)(25)] = \ln 250$

58. $\ln b + 2\ln c - 3\ln d = \ln b + \ln c^2 - \ln d^3 = \ln bc^2 - \ln d^3 = \ln\dfrac{bc^2}{d^3}$

59. $\frac{1}{3}\ln(x+3)^2 + \frac{1}{2}\left[\ln x - \ln\left(x^2+3x+2\right)^2\right] = \ln\left[(x+2)^3\right]^{1/3} + \frac{1}{2}\ln\dfrac{x}{\left(x^2+3x+2\right)^2} = \ln(x+2) + \ln\dfrac{\sqrt{x}}{\left(x^2+3x+2\right)^2}$

$$= \ln\frac{(x+2)\sqrt{x}}{(x+1)(x+2)} = \ln\frac{\sqrt{x}}{x+1}$$

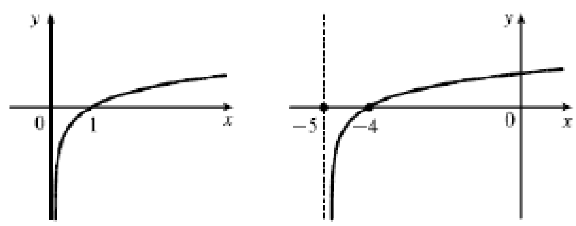
60. $\ln(1+e^{2x}) - \ln(1+e^{-2x}) = \ln\left(\frac{1+e^{2x}}{1+e^{-2x}}\right) = \ln\left(\frac{(1+e^{2x})e^{2x}}{(1+e^{-2x})e^{2x}}\right) = \ln\left(\frac{(1+e^{2x})e^{2x}}{e^{2x}+1}\right) = \ln(e^{2x}) = 2x$

61. $(\ln p)(\log_p e)\left(\sqrt{e}\right)^{\ln p} = (\ln p)\cdot\dfrac{\ln e}{\ln p}\cdot\left(e^{\frac{1}{2}}\right)^{\ln p} = \ln e\cdot\left(e^{\ln p}\right)^{\frac{1}{2}} = 1\cdot p^{\frac{1}{2}} = \sqrt{p}$

62. $\log_5 10 = \dfrac{\ln 10}{\ln 5} \approx 1.430677$

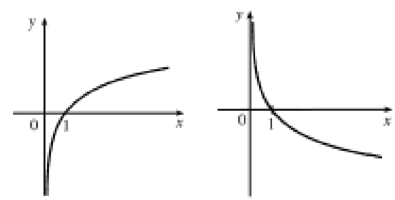
63. $\log_3 57 = \dfrac{\ln 57}{\ln 3} \approx 3.680144$

70. Shift the graph of $y = \log_{10} x$ five units to the left to obtain the graph of $y = \log_{10}(x+5)$. Note the vertical asymptote of $x = -5$.



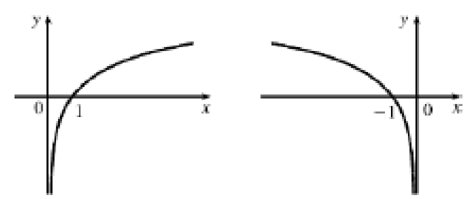
$y = \log_{10} x$ $\quad$ $y = \log_{10}(x+5)$

71. Reflect the graph of $y = \ln x$ about the x-axis to obtain the graph of $y = -\ln x$.



$y = \ln x$ $\quad$ $y = -\ln x$

72. Reflect the graph of $y = \ln x$ about the y-axis to obtain the graph of $y = \ln(-x)$.



$y = \ln x$ $\quad$ $y = \ln(-x)$

77. $e^{7-4x} = 6 \iff 7-4x = \ln 6 \iff 7 - \ln 6 = 4x \iff x = \frac{1}{4}(7-\ln 6)$

79. $\ln\left(x^2-1\right) = 3 \iff x^2 - 1 = e^3 \iff x^2 = e^3 + 1 \iff x = \pm\sqrt{e^3+1}$

94. (a) We need $e^x - 3 > 0 \iff e^x > 3 \iff x > \ln 3$. Thus the domain of $f(x) = \ln\left(e^x - 3\right)$ is $(\ln 3, \infty)$.

(b) $y = \ln\left(e^x - 3\right) \Rightarrow e^y = e^x - 3 \Rightarrow e^x = e^y + 3 \Rightarrow x = \ln\left(e^y + 3\right)$, so $f^{-1}(x) = \ln\left(e^x + 3\right)$.

Now $e^x + 3 > 0 \Rightarrow e^x > -3$, which is true for any real x, so the domain of f^{-1} is $\mathbb{R}$.

101. $\sin^{-1}(0.5)=\frac{\pi}{6}$ because $\sin\frac{\pi}{6}=0.5$ and $\frac{\pi}{6}$ is in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

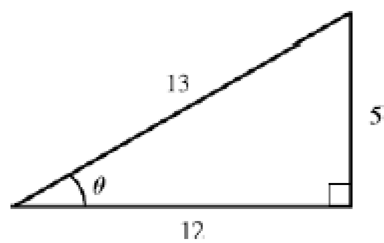
103. $\arctan(-1)=-\frac{\pi}{4}$

105. $\arcsin 1=\frac{\pi}{2}$

107. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

109. $\sec^{-1}2=\frac{\pi}{3}$

111. Let $\theta=\sin^{-1}\left(\frac{5}{13}\right)$ (see figure to the right).



Then $\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right)=\cos 2\theta=\cos^2\theta-\sin^2\theta$

$$=\left(\frac{12}{13}\right)^2-\left(\frac{5}{13}\right)^2=\frac{144}{169}-\frac{25}{169}=\frac{119}{169}$$

113. $\tan\left(\sin^{-1}x\right)=\tan\left(\frac{\pi}{2}\right)=\frac{\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}}$ is undefined because $\cos\frac{\pi}{2}=0$.

p. 78: 5-10, 13

5. $x=1.127$

6. The functions $-2\cos\left(x-\frac{\pi}{4}\right)$ and $3x+8$ do not intersect on the interval $[0,2\pi)$.

7. The only solution of $\cos x=e^{2x-1}$ on the interval $[0,2\pi)$ is $(0.448, 0.901)$.

8. The only solution of $\log_{1/2}x=\tan x$ on the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is $(0.614, 0.704)$.

9. The solutions of $\sin x+\cos x=1$ in $[-2\pi,2\pi]$ are $(-2\pi,1),(-\frac{3\pi}{2},1),(0,1),(\frac{\pi}{2},0)$, and $(2\pi,1)$.

10. The solutions of $\frac{1}{\sin x}-2=3$ in the interval $[-2\pi,2\pi]$ are $x=-6.082,-3.343,0.201$, and $x=2.940$.

13. (a)A quadratic function could intersect $y=\sin x$ in at most 3 points.

(b) A cubic function could intersect $y=\sin x$ in at most 3 points.

(c) An nth degree polynomial could intersect $y=\sin x$ in at most 3 points.

(d) A linear function could intersect $y=\sin x$ in infinitely many points.