<u>3.1</u>

p. 173: 14-17 (NC on 16), 22-25 (NC on 22), 34-37, 41-42, 47-48 (NC), 57-58, 68-71

14. If $y = f(x)$, then $f'(1) = \lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$ which is option (**C**).

15. The average rate of change of $f(x) = x^2 - 2x$ on the interval $[-1,2]$ is

$$\frac{f(2)-f(-1)}{2-(-1)} = \frac{(2^2-2\cdot 2)-((-1)^2-2\cdot(-1))}{3} = \frac{0-(1+2)}{3} = -\frac{3}{3} = -1,$$ which is choice (**B**).

16. The tangent line has equation $y = f'(0)(x-0)+f(0)) = \left(\lim_{x\to 0}\dfrac{f(x)-f(0)}{x-0}\right)(x-0)+f(0)$

$$= \left(\lim_{x\to 0}\frac{x^3-0}{x-0}\right)(x-0)+0 = \left(\lim_{x\to 0}\frac{x^3}{x}\right)\cdot x = \lim_{x\to 0} x^2\cdot x = 0$$ or $y = 0$.

17. The tangent line has equation $y = f'(0)(x-0)+f(0) = \left(\lim_{x\to 0}\dfrac{f(x)-f(0)}{x-0}\right)(x-0)+f(0)$

$$= \left(\lim_{x\to 0}\frac{x^{1/3}-0}{x-0}\right)\cdot(x)+0 = \left(\lim_{x\to 0}\frac{x^{1/3}}{x}\right)\cdot x = \left(\lim_{x\to 0} x^{-2/3}\cdot x\right) = \left(\lim_{x\to 0} x^{1/3}\right) = 0$$

22. With $f(x) = 4x - 3x^2$ and the point $(2,-4)$, the slope of the tangent line is

$$m = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{4x-3x^2-(-4)}{x-2} = \lim_{x\to 2}\frac{4x-3x^2+4}{x-2} = \lim_{x\to 2}\frac{-(3x+2)(x-2)}{x-2}$$
$$= \lim_{x\to 2} -(3x+2) = -8;$$

The equation of the tangent line is $y = -8(x-2)-4$ or $y = -8x+12$.

23. With $f(x) = x^3 - 3x + 1$ and the point $(2,3)$, the slope of the tangent line is

$$m = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{x^3-3x+1-(3)}{x-2} = \lim_{x\to 2}\frac{x^3-3x-2}{x-2} = \lim_{x\to 2}\frac{(x+1)^2(x-2)}{x-2}$$
$$= \lim_{x\to 2} -(3x+2) = 9;$$ The equation of the tangent line is $y = 9(x-2)+3$ or $y = 9x+15$.

$=\lim_{x\to 2}(x+1)^2 = 9.$ The equation of the tangent line is $y = 9(x-2)+3$ or $y = 9x+15$.

24. With $f(x) = \sqrt{x}$ and the point $(1,1)$, the slope of the tangent line is

$$m = \lim_{x\to 1}\frac{f(x)-f(1)}{x-1} = \lim_{x\to 1}\frac{\sqrt{x}-\sqrt{1}}{x-1} = \lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1}\frac{1}{\sqrt{x}+1} = \frac{1}{2}.$$

The equation of the tangent line is $y = \frac{1}{2}(x-1)+1$ or $y = \frac{1}{2}x+\frac{1}{2}$.

25. With $f(x)=\dfrac{2x+1}{x+2}$ and the point $(1,1)$, the slope of the tangent line is

$$m=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{\frac{2x+1}{x+2}-1}{x-1}=\lim_{x\to 1}\frac{\frac{2x+1-(x+2)}{x+2}}{x-1}=\lim_{x\to 1}\frac{x-1}{(x-1)(x+2)}$$

$=\lim_{x\to 1}\dfrac{1}{x+2}=\dfrac{1}{1+2}=\dfrac{1}{3}$; The equation of the tangent line is $y=\frac{1}{3}(x-1)+1$ or $y=\frac{1}{3}x+\frac{2}{3}$.

34. $g'(0)$ is the only negative value. The slope at $x=4$ is smaller than the slope at $x=2$ and both are smaller than the slope at $x=-2$, Thus $g'(0)<0<g'(4)<g'(2)<g'(-2)$.

35. (a) On [20, 60]: $\dfrac{f(60)-f(20)}{60-20}=\dfrac{700-300}{40}=\dfrac{400}{40}=10$

(b) Pick any interval that has the same y-value at its endpoints. [0, 57] is such an interval since $f(0)=f(57)=600$.

(c) On [40, 60]: $\dfrac{f(60)-f(40)}{60-40}=\dfrac{700-200}{20}=\dfrac{500}{20}=25$

On [40, 70]: $\dfrac{f(70)-f(40)}{70-40}=\dfrac{900-200}{30}=\dfrac{700}{30}=23\frac{1}{3}$

Since 25 > 23⅓, the average rate of change on [40, 60] is larger.

36. (a) The tangent line at $x=50$ appears to pass through the points $(40,f(40))$ and $(50,f(50))$, so

$f'(50)=\dfrac{200-400}{40-50}=\dfrac{-200}{-10}=20.$

(b) The tangent line at $x=10$ is steeper than the tangent line at $x=30$, so it is larger in magnitude, but less in numerical value; that is, $f'(10)<f'(30)$.

(c) The slope of the tangent line at $x=60$, $f'(60)$, is greater than the slope of the line through

$(40,f(40))$ and $(80,f(80))$. So yes, $f'(60)>\dfrac{f(80)-f(40)}{80-40}$.

37. Since $g(5)=-3$, the point $(5,-3)$ is on the graph of g. Since $g'(5)=4$, the slope of the tangent line at $x=5$ is 4. Then the equation of the tangent line at this point is $y=4(x-5)-3$ or $y=4x-23$.

41. For the tangent line $y=4x-5$: when $x=2$, $y=4(2)-5=3$ and its slope is 4 (the coefficient of x). At the point of tangency, these values are shared with the curve $y=f(x)$; that is, $f(2)=3$ and $f'(2)=4$.

42. Since $(4,3)$ is on $y=f(x)$, $f(4)=3$. The slope of the tangent line between $(0,2)$ and $(4,3)$ is $\dfrac{1}{4}$ so

$f'(4)=\dfrac{1}{4}.$

47. Using Definition 2 with $f(x)=3x^2-x^3$ and $a=1$:

$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{\left[3(1+h)^2-\left(1+h\right)^3\right)\right]-2}{h}$$

$$=\lim_{h\to 0}\frac{\left(3+6h+3h^2\right)-\left(1+3h+3h^2+h^3\right)-2}{h}=\lim_{h\to 0}\frac{3h-h^3}{h}=\lim_{h\to 0}\frac{h\left(3-h^2\right)}{h}$$

$$=\lim_{h\to 0}\left(3-h^2\right)=3-0-3.$$

The tangent line is $y=3(x-1)+2$ or $y=3x-1$.

48. With $g(x)=x^4-2$ and $a=1$, $g'(1)=\lim_{x\to 1}\frac{g(x)-g(1)}{x-1}=\lim_{x\to 1}\frac{\left(x^4-2\right)-(-1)}{x-1}=\lim_{x\to 1}\frac{x^4-1}{x-1}$

$$=\lim_{x\to 1}\frac{\left(x^2+1\right)\left(x^2-1\right)}{x-1}=\lim_{x\to 1}\frac{\left(x^2+1\right)\left(x+1\right)\left(x-1\right)}{x-1}=\lim_{x\to 1}\left(x^2+1\right)\left(x+1\right)=2(2)=4.$$

The tangent line is $y=4(x-1)-1$ or $y=4x-5$.

57. $\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}=f'(9)$, where $f(x)=\sqrt{x}$ and $a=9$.

58. $\lim_{h\to 0}\frac{e^{-2+h}-e^{-2}}{h}=f'(-2)$, where $f(x)=e^x$ and $a=-2$, or $\lim_{h\to 0}\frac{e^{-2+h}-e^{-2}}{h}=f'(2)$, where $f(x)=e^{-x}$ and $a=2$.

68. (a) (i) [2008, 2010]: $\frac{N(2010)-N(2008)}{2010-2008}=\frac{12{,}440-8{,}569}{2}=\frac{3871}{2}=1935.5$ locations/year

(ii) [2010, 2012]: $\frac{N(2012)-N(2010)}{2012-2010}=\frac{16{,}680-12{,}440}{2}=\frac{4240}{2}=2120$ locations/year

(b) [2014, 2016]: $\frac{N(2016)-N(2014)}{2016-2014}=\frac{18{,}066-16{,}858}{2}=\frac{1208}{2}=604$ locations/year

69. (a) [2005, 2010]: $\frac{87{,}032-84{,}077}{2010-2005}=\frac{2955}{5}=591$ thousands of barrels per day per year. This means that oil consumption rose by an average of 591 thousands of barrels per day each year from 2005 to 2010.

(b) [2015, 2010]: $\frac{93{,}770-87{,}302}{2015-2010}=\frac{6468}{5}=1293.6$

An estimate of the instantaneous rate of change in 2013 is 1293.6 thousands of barrels per day per year.

70. (a) (i) [4, 11]: $\frac{V(11)-V(4)}{11-4}=\frac{9.4-54}{7}=\frac{-43.6}{7}=-6.23\frac{\text{RNA copies/mL}}{\text{day}}.$

(ii) [8, 11]: $\frac{V(11)-V(8)}{11-8}=\frac{9.4-18}{3}=\frac{-8.6}{3}=-2.87\frac{\text{RNA copies/mL}}{\text{day}}.$

(iii) [11, 15]: $\frac{V(15)-V(11)}{15-11}=\frac{5.2-9.4}{4}=\frac{-4.2}{4}=-1.05\frac{\text{RNA copies/mL}}{\text{day}}.$

(iv) [11, 22]: $\frac{V(22)-V(11)}{22-11}=\frac{3.6-9.4}{11}=\frac{-5.8}{1}=-0.58\frac{\text{RNA copies/mL}}{\text{day}}.$

(b) An estimate of $V'(11)$ is the average of the answers from part (a)(ii) and (iii).

$$V'(11) \approx \tfrac{1}{2}\left[-2.87 + (-1.05)\right] = -1.96\ \frac{\text{RNA copies/mL}}{\text{day}}.$$

$V'(11)$ measures the instantaneous rate of change of patients 303's viral load 11 days after ABT-538 treatment begins.

71. (a) (i) $\dfrac{\Delta C}{\Delta x} = \dfrac{C(105) - C(100)}{105 - 100} = \dfrac{6601.25 - 6500}{5} = \$20.25/\text{unit}.$

(ii) $\dfrac{\Delta C}{\Delta x} = \dfrac{C(101) - C(100)}{101 - 100} = \dfrac{6520.05 - 6500}{1} = \$20.05/\text{unit}.$

(b) $\dfrac{C(100+h) - C(100)}{h} = \dfrac{\left[5000 + 10(100+h) + 0.05(100+h)^2\right] - 6500}{h} = \dfrac{20h + 0.05h^2}{h}$

$= 20.0.05h,\ h \neq 0.$ So the instantaneous rate of change is

$$\lim_{h \to 0} \frac{C(100+h) - C(100)}{h} = \lim_{h \to 0} (20 + 0.05h) = \$20/\text{unit}.$$