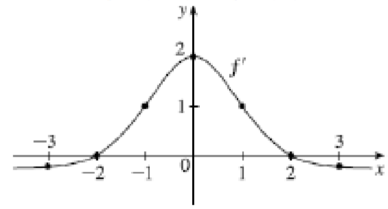
p. 188: 7-9, 11-19, 25-27, 30-34, 53-56, 67-70, 74-76

7. It appears that $f$ is an odd function, so $f'$ will be an even function – that is, $f'(-a) = f'(a)$

(a) $f'(-3) \approx -0.2$

(b) $f'(-2) \approx 0$ (c) $f'(-1) \approx 1$ (d) $f'(0) \approx 2$

(e) $f'(1) \approx 1$ (f) $f'(2) \approx 0$ (g) $f'(3) \approx -0.2$



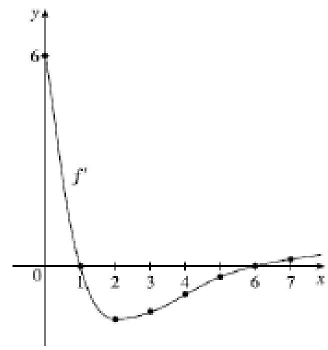

8. Your answers may vary depending on your estimates.

(a) *Note*: By estimating the slopes of tangent lines on the graph of $g$, it appears that $g'(0) \approx 6$.

(b) $g'(1) \approx 0$ (c) $g'(2) \approx -1.5$ (d) $g'(3) \approx -1.3$

(e) $g'(4) \approx -0.8$ (f) $g'(5) \approx -0.3$ (g) $g'(6) \approx 0$

(f) $g'(7) \approx 0.2$




9. (a) True. Because $f'(5)$ exists, the function must be continuous at $x = 5$ and $\lim_{x \to 5} f(x)$ must equal $f(5) = 2$.

(b) True. Using the definition of the derivative at $x = 5$, $\lim_{x \to 5} \frac{f(x) - f(5)}{x - 5} = f'(5) = 3$.

(c) True. Using the definition of the derivate at $x = 5$,

$$\lim_{h \to 0} \frac{f(5+h) - 2}{h} = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h} = f'(5) = 3.$$

(d) We cannot determine whether $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$ exists without more information about $f$. The information given does not allow us to tell whether $f$ is defined, continuous, or differentiable at $x = 3$.

11. (a)′ = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function in graph II follows the same pattern.

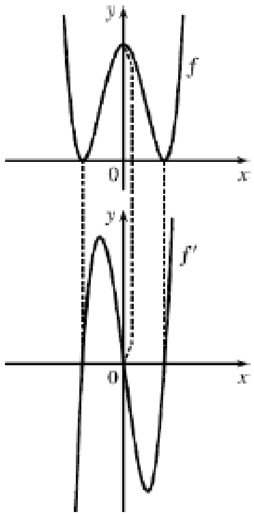
(b)′ = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

(c)′ = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

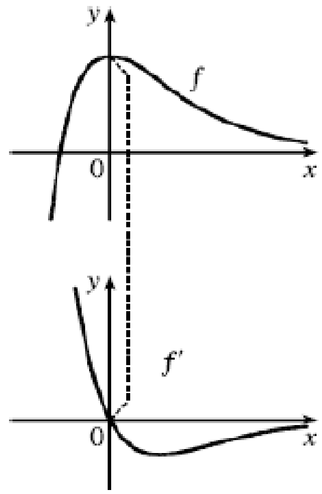
(d)′ = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

*Hints for Exercises 12-19: First plot x-intercepts on the graph of $f'$ for any horizontal tangents on the graph of f. Look for any corners on the graph of f –there will be a discontinuity on the graph of $f'$. On any interval where f has a tangent with positive/negative) slope, the graph of $f'$ will be positive/negative. If the graph of the function is linear, the graph of $f'$ will be a horizontal line whose height is of the slope of the function.*

12.




13.

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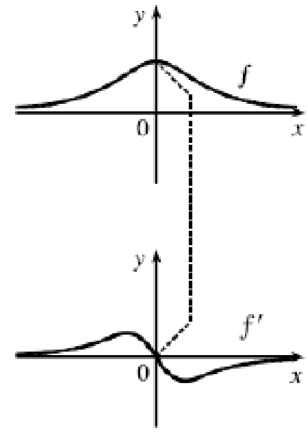


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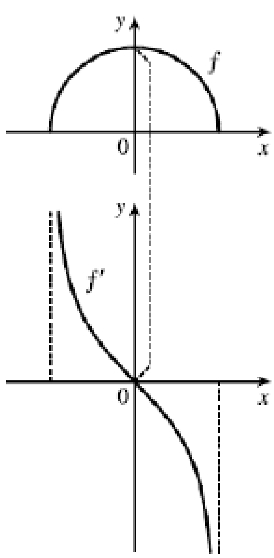


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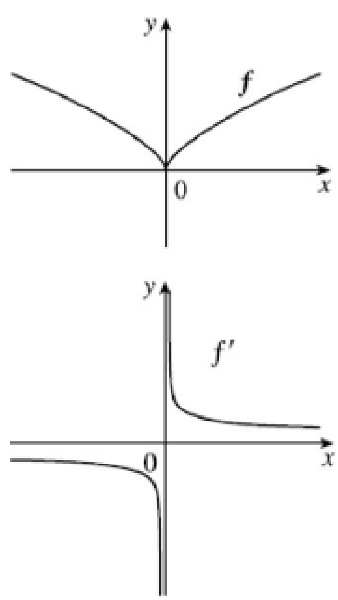


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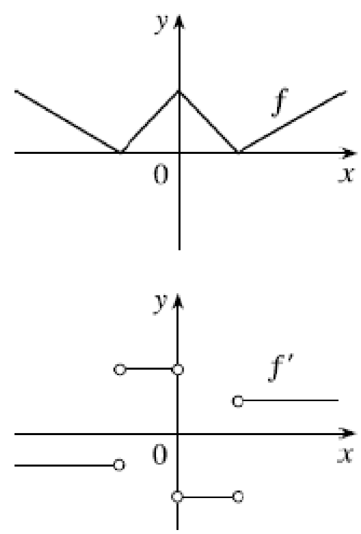

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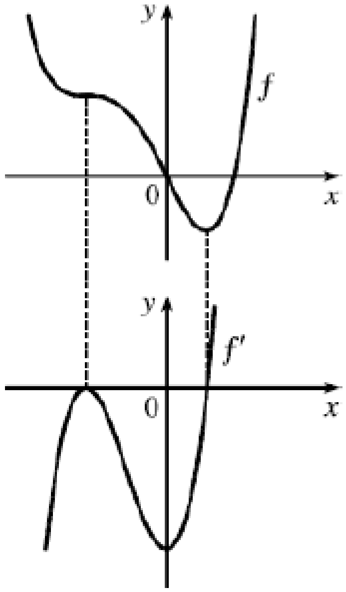


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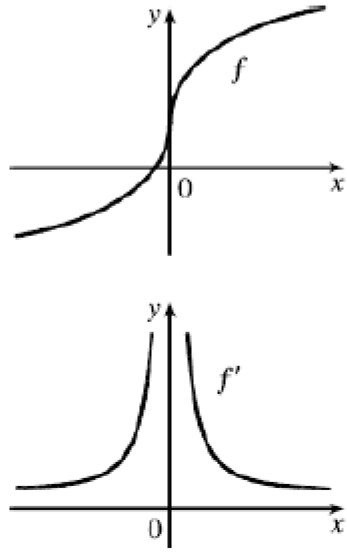
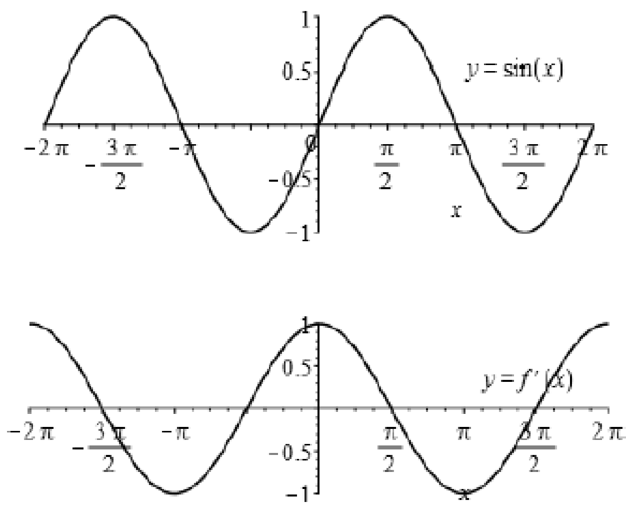


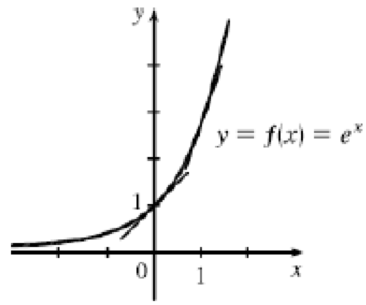

25.

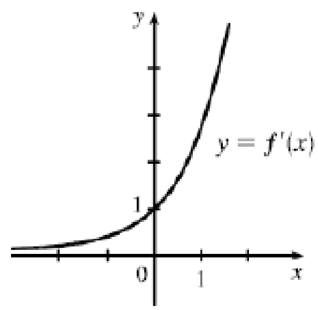
The slope at $x = -2\pi$, 0 and $2\pi$ appears to be one.
At $x = -3\pi/2, -\pi/2, 3\pi/2$, and $3\pi/2$, the slope is zero.
At $x = \pm\pi$, the slope appears to be $-1$. Therefore, we might guess that $f'(x) = \cos(x)$.



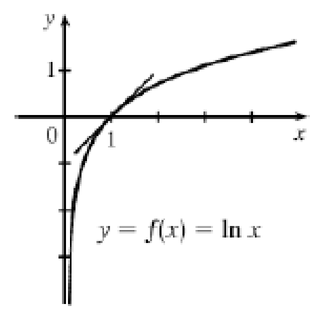
26.

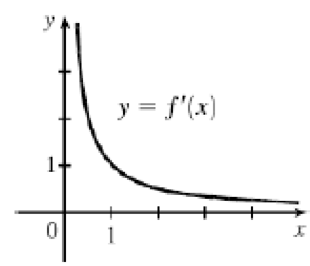
The slope at 0 appears to be 1 and the slope at 1 appears to be 2.7. As $x$ decreases, the slope gets closer to 0. Since the graphs are so similar, we might guess that $f'(x) = e^x$.




27. As $x$ increases toward 1, $f'(x)$ decreases from very large numbers to 1. As $x$ becomes large, $f'(x)$ gets closer to 0. As a guess, $f'(x) = \frac{1}{x^2}$ or $f'(x) = \frac{1}{x}$ makes sense.




30. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h) - 8 - (3x-8)}{h} = \lim_{h \to 0} \frac{3x + 3h - 8 - 3x + 8}{h}$

$= \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3 = 3$. Domain of $f' = \mathbb{R}$.

31. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{m(x+h) + b - (mx+b)}{h} = \lim_{h \to 0} \frac{mx + mh + b - mx - b}{h}$

$= \lim_{h \to 0} \frac{mh}{h} = \lim_{h \to 0} m = m$. Domain of $f$ and $f'$ is $\mathbb{R}$.

32. $f'(t)=\lim_{h\to 0}\dfrac{f(t+h)-f(t)}{h}=\lim_{h\to 0}\dfrac{2.5(t+h)^2+6(t+h)-\left(2.5t^2+6t\right)}{h}$

$=\lim_{h\to 0}\dfrac{2.5(t^2+2th+h^2+6t+6h-2.5t^2-6t}{h}=\lim_{h\to 0}\dfrac{2.5t^2+5th+2.5h^2+6h-2.5t^2}{h}$

$=\lim_{h\to 0}\dfrac{5th+2.5h^2+6h}{h}=\lim_{h\to 0}\dfrac{h(5t+2.5h+6)}{h}=\lim_{h\to 0}(5t+2.5h+6)=5t+6.$

Domain of $f$ and $f'$ is $\mathbb{R}$.

33. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{4+8(x+h)-5(x+h)^2-\left(4+8x-5x^2\right)}{h}$

$=\lim_{h\to 0}\dfrac{4+8x+8h-5\left(x^2+2xh+h^2\right)-4-8x+5x^2}{h}=\lim_{h\to 0}\dfrac{8h-5x^2-10xh-5h^2+5x^2}{h}$

$=\lim_{h\to 0}\dfrac{8h-10xh-5h^2}{h}=\lim_{h\to 0}\dfrac{h(8-10x-5h)}{h}=\lim_{h\to 0}(8-10x-5h)=8-10x.$

Domain of $f$ and $f'$ is $\mathbb{R}$.

34. $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\dfrac{(x+h)^2-2(x+h)^3-\left(x^2-2x^3\right)}{h}$

$=\lim_{h\to 0}\dfrac{x^2+2xh+h^2-2x^3-6x^2h-6xh^2-2h^3-x^2+2x^3}{h}$

$=\lim_{h\to 0}\dfrac{2xh+h^2-6x^2h-6xh^2-2h^3}{h}=\lim_{h\to 0}\dfrac{h\left(2x+h-6x^2-6xh-2h^2\right)}{h}$

$=\lim_{h\to 0}\left(2x+h-6x^2-6xh-2h^2\right)=2x-6x^2.$ Domain of $f$ and $f'$ is $\mathbb{R}$.

53. $f$ is not differentiable at $x=-4$, because the graph has a corner there, at $x=0$, because there is a discontinuity there, and at $x=2$ because there is a vertical tangent line there.

54. $f$ is not differentiable at $x=-1$, because there is a discontinuity there, and at $x=2$, because the graph has a corner there.

55. $f$ is not differentiable at $x=1$, because $f$ is not defined there.

56. $f$ is not differentiable at $x=-2$ and $x=3$, because the graph has corners there, and at $x=1$, because there is a discontinuity there.

67. If $x+y=k$ is tangent to the graph of $y=f(x)$ at $x=2$ then

$\lim_{x\to 2}\dfrac{f(x)-f(2)}{x-2}=f'(2)$ is equal to the slope of the line $x+y=k$,

which is (**B**).

68. If $f(x)=|x+5|-|3x-6|$, when $x=-1$, $f(x)=(x+5)-[-(3x-6)]=x+5+3x-6=4x-1$. In this case, $f'(x)=4$ (the slope of the line), so $f'(-1)$ is (**A**) 4.

69. (a) True; if $\lim_{h\to 0}\frac{f(5+h)-f(5x)}{h}=\infty$ then $f'(5)$ does not exist because *f* has a vertical tangent line at $x = 5$.

(b) False. If *f* had a horizontal tangent line at the point (5, 7), then $\lim_{h\to 0}\frac{f(5+h)-f(5x)}{h}$ would have to be zero.

(c) False. Because $f'(5)=\lim_{h\to 0}\frac{f(5+h)-f(5x)}{h}=\infty,\ f'(5)\neq 7$.

(d) False. Because $f'(5)=\lim_{h\to 0}\frac{f(5+h)-f(5x)}{h}=\infty,\ f'(5)\neq 0$.

70. Because the line $2x+y=13$ is tangent to $y=f(x)$ at $x = 3$, the line and *y* intersect at the point $(3, f(3))$. Therefore $f(3)=13-2(3)=7$.

The slope of the line $2x+y=13$ is –2, and this slope must also be the value of the derivative of *f* at $x=3$. So $f'(3)=-2$.

74. (a) Note that we have factored $x - a$ as the difference of cubes in the third step.

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(x)}{x-a}=\lim_{x\to a}\frac{x^{1/3}-a^{1/3}}{x-a}=\lim_{x\to a}\frac{x^{1/3}-a^{1/3}}{\left(x^{1/3}-a^{1/3}\right)\left(x^{2/3}+x^{1/3}a^{1/3}+a^{2/3}\right)}$$

$$=\lim_{x\to a}\frac{1}{\left(x^{2/3}+x^{1/3}a^{1/3}+a^{2/3}\right)}=\frac{1}{3a^{2/3}}\text{ or }\frac{1}{3}a^{-2/3}.$$

(b) $f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\sqrt[3]{h}-0}{h}=\lim_{h\to 0}\frac{1}{h^{2/3}}$. This function increases without bound, so the limit does not exist, and therefore $f'(0)$ does not exist.

(c) $\lim_{x\to 0}\left|f'(x)\right|=\lim_{x\to 0}\frac{1}{3x^{2/3}}=\infty$ and *f* is continuous at $x = 0$ (it is a root function), so *f* has a vertical tangent at $x = 0$.

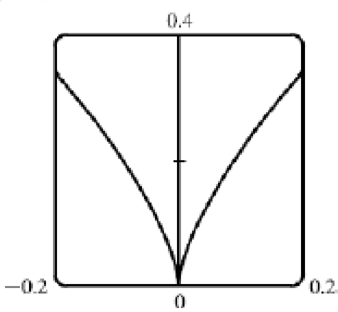
75. (a) $g'(0)=\lim_{x\to 0}\frac{g(x)-g(0)}{x-0}=\lim_{x\to 0}\frac{x^{2/3}-0}{x}=\lim_{x\to 0}\frac{1}{x^{1/3}}$, which does not exist.

(b) $$g'(a)=\lim_{x\to a}\frac{g(x)-g(a)}{x-a}=\lim_{x\to 0}\frac{x^{2/3}-a^{2/3}}{x-a}=\lim_{x\to 0}\frac{\left(x^{1/3}-a^{1/3}\right)\left(x^{1/3}+a^{1/3}\right)}{\left(x^{1/3}-a^{1/3}\right)\left(x^{2/3}+x^{1/3}a^{1/3}+a^{2/3}\right)}$$

$$=\lim_{x\to 0}\frac{x^{1/3}+a^{1/3}}{x^{2/3}+x^{1/3}a^{1/3}+a^{2/3}}=\frac{2a^{1/3}}{3a^{2/3}}=\frac{2}{3a^{1/3}}\text{ or }\frac{2}{3}a^{-1/3}$$

(c) $g(x)=x^{2/3}$ is continuous at $x = 0$ and this shows that *g* has a vertical tangent line at $x = 0$.

(d)



76. $f(x)=|x-6|=\begin{cases} x-6 & \text{if } x-6\ge 0 \\ -(x-6) & \text{if } x-6<0 \end{cases}=\begin{cases} x-6 & \text{if } x\ge 6 \\ 6-x & \text{if } x<6 \end{cases}$

So the right-hand limit is $\lim_{x\to 6^+}\frac{f(x)-f(6)}{x-6}=\lim_{x\to 6^+}\frac{|x-6|-0}{x-6}=\lim_{x\to 6^+}\frac{x-6}{x-6}=\lim_{x\to 6^+}1=1,$ and the left-hand limit is $\lim_{x\to 6^-}\frac{f(x)-f(6)}{x-6}=\lim_{x\to 6^-}\frac{|x-6|-0}{x-6}=\lim_{x\to 6^-}\frac{6-x}{x-6}=\lim_{x\to 6^-}-1=-1$. Since these limits are not equal, $f'(6)=\lim_{x\to 6}\frac{f(x)-f(6)}{x-6}$ does not exist and $f$ is not differentiable at 6.

(b) A formula for is $f'(x)=\begin{cases} 1 & \text{if } x>6 \\ -1 & \text{if } x<6 \end{cases}$. Another way of writing the formula is $f'(x)=\frac{x-6}{|x-6|}$.

A graph of the function and its derivative are shown here: