p. 203: 11-39 odd, 45-49 odd, 61-65, 70-76, 86-87, 91-92

11. $f(x)=e^5$ is a constant function, so its derivative is 0, that is $f'(x)=0$.

13. $g(x)=\frac{7}{4}x^2-3x+12 \Rightarrow g'(x)=\frac{7}{4}(2x)-3(1)+0=\frac{7}{2}x-3$

15. $f(t)=1.4^5-2.5t^2+6.7 \Rightarrow f'(t)=1.4\left(5t^4\right)-2.5(2t)+0=7t^4-5t$

17. $H(u)=(3u-1)(u+2)=3u^2+5u-2 \Rightarrow H'(u)=3(2u)+5(1)-0=6u+5$

19. $B(y)=cy^{-6} \Rightarrow B'(y)=c\left(-6y^{-7}\right)=-6cy^{-7}$

21. $y=x^{5/3}-x^{2/3} \Rightarrow y'=\frac{5}{3}x^{2/3}-\frac{2}{3}x^{-1/3}$

23. $h(t)=\sqrt[4]{t}-4e^t=t^{1/4}-4e^t \Rightarrow h'(t)=\frac{1}{4}t^{-3/4}-4\left(e^t\right)=\frac{1}{4}t^{-3/4}-4e^t$

25. $y=\sqrt[3]{x}(2+x)=2x^{1/3}+x^{4/3} \Rightarrow y'=2\left(\frac{1}{3}x^{-2/3}\right)+\frac{4}{3}x^{1/3}=\frac{2}{3}x^{-2/3}+\frac{4}{3}x^{1/3}$ or $\dfrac{1}{3\sqrt[3]{x^2}}+\frac{4}{3}\sqrt[3]{x}$

27. $S(R)=4\pi R^2 \Rightarrow S'(R)=r\pi(2R)=8\pi R$

29. $y=\dfrac{\sqrt{x}+x}{x^2}=\dfrac{\sqrt{x}}{x^2}+\dfrac{x}{x^2}=x^{-3/2}+x^{-1} \Rightarrow y'=-\frac{3}{2}x^{-5/2}+\left(-1x^{-2}\right)=-\frac{3}{2}x^{-5/2}-x^{-2}$

31. $G(t)=\sqrt{5t}+\dfrac{\sqrt{7}}{t}=\sqrt{5}t^{1/2}+\sqrt{7}t^{-1} \Rightarrow G'(t)=\sqrt{5}\left(\frac{1}{2}t^{-1/2}\right)+\sqrt{7}\left(-1t^{-2}\right)=\dfrac{\sqrt{5}}{2\sqrt{t}}-\dfrac{\sqrt{7}}{t^2}$

33. $k(r)=e^r+r^e \Rightarrow k'(r)=e^r+er^{e-1}$

35. $F(r)=\dfrac{A+Bz+Cz^2}{z^2}=\dfrac{A}{z^2}+\dfrac{Bz}{z^2}+\dfrac{Cz^2}{z^2}=Az^{-2}+Bz^{-1}+C \Rightarrow$

$F'(r)=A\left(-2z^{-3}\right)+B\left(-1z^{-2}\right)+0=-2Az^{-3}-Bz^{-2}=-\dfrac{2A}{z^3}-\dfrac{B}{z^2}$ or $-\dfrac{2A+Bz}{z^3}$

37. $D(t)=\dfrac{1+16t^2}{(4t)^3}=\dfrac{1+16t^2}{64t^3}=\frac{1}{64}t^{-3}+\frac{1}{4}t^{-1} \Rightarrow$

39. $y=e^{x+1}+1=e^xe^1+1=e\cdot e^x+1 \Rightarrow y'=e\cdot e^x+0=e^{x+1}$

45. $y=2x^3-x^2+2 \Rightarrow y'=6x^2-2x$. At $(1,3)$, $y'=6(1)^2-2(1)=4$ and an equation of the tangent line is $y=4(x-1)+3$ or $y=4x-1$.

47. $y=x+\dfrac{2}{x}=x+2x^{-1} \Rightarrow y'=1-2x^{-2}$. At $(2,3)$, $y'=1-2(2)^{-2}=\frac{1}{2}$ and an equation of the tangent line is $y=\frac{1}{2}(x-2)+3$ or $y=\frac{1}{2}x+2$.

49. $y=x^4+2e^x \Rightarrow y'=4x^3+2e^x$. At $(0,2)$, $y'=2$, and an equation of the tangent line is $y=2(x-0)+2$ or $y=2x+2$. The slope of the normal line is $-\frac{1}{2}$ (the negative reciprocal of 2) and an equation of the normal line is $y=-\frac{1}{2}(x-0)+2$ or $y=-\frac{1}{2}x+2$.

61. (a) $s=t^3-3t \Rightarrow v(t)=s'(t)=3t^2-3 \Rightarrow a(t)=v'(t)=6t$

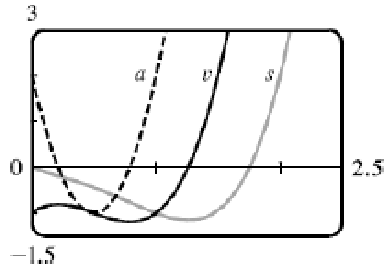
(b) $a(2)=6(2)=12\text{ m/s}^2$

(c) $v(t)=3t^2-3=0$ when $t^2=1$, that is, $t=1$ $[t\ge 0]$ and $a(1)=6\text{ m/s}^2$.

62. (a) $s = t^4 - 2t^3 + t^2 - t \Rightarrow v(t) = s'(t) = 4t^3 - 6t^2 + 2t - 1 \Rightarrow a(t) = v'(t) = 12t^2 - 12t + 2$

(b) $a(1) = 12(1)^2 - 12(1) + 2 = 2 \text{ m/s}^2$

(c)



63. If the particle's position is given by $s(t) = t^3 - 6t^2 + 9t + 5$ then the velocity of the particle is given by $v(t) = s'(t) = 3t^2 - 12t + 9$ and the acceleration is given by $a(t) = v'(t) = 6t - 12$. Then acceleration is zero when $t = 2$, and at this time, the velocity is (**D**) –3 in/sec.

64. If the particle's position is given by $s(t) = 2t^3 + t^2 - 4t + 3$ then the velocity of the particle is given by $v(t) = s'(t) = 6t^2 + 2t - 4 = 2(3t-2)(t+1)$ and the acceleration is given by $a(t) = v'(t) = 12t + 2$. Then velocity is zero when $t = 2/3$, and at this time, the acceleration is (**B**) 10 cm/sec^2.

65. If $f(x) = 3e^x - 1$, then $f'(x) = 3e^x$ and $f'(0) = 3e^0 = 3$. So the slope of the tangent line at the point where $x = 0$ is 3, and the point on the graph has coordinates $\left(0, 3e^0 - 1\right) = (0, 2)$. So the equation of the tangent line at that point is $y = 3\left(x - 0\right) + 2$ or (**C**) $y = 3x + 2$.

70. The curve $y = 2x^3 + 3x^2 - 12x + 1$ has a horizontal tangent when $y' = 6x^2 + 6x - 12 = 0 \Leftrightarrow$ $6\left(x^2 + x - 2\right) = 0 \Leftrightarrow 6(x+2)(x-1) = 0 \Leftrightarrow x = -2$ or $x = 1$.
The points on the curve are $(-2, 21)$ and $(1, -6)$.

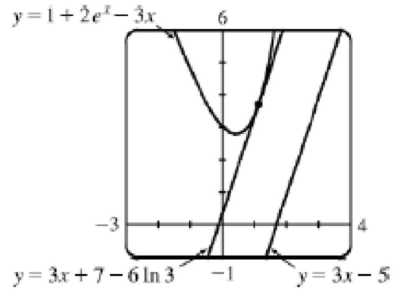
71. $f(x) = e^x - 2x \Rightarrow f'(x) = e^x - 2$. $f'(x) = 0 \Rightarrow e^x = 2 \Rightarrow x = \ln 2$, so f has a horizontal tangent when $x = \ln 2$.

72. $y = 2e^x + 3x + 5x^3 \Rightarrow y' = 2e^x + 3 + 15x^2$. Since $2e^x > 0$ and $15x^2 \geq 0$ we must have $y' > 0 + 3 + 0 = 3$, so no tangent line can have slope 2.

73. $y = x^4 + 1 \Rightarrow y' = 4x^3$. The slope of the line $32x - y = 15$(or $y = 32x - 15$) is 32, so the slope of any line parallel top it is also 32. Thus, $y' = 32 \Leftrightarrow 4x^3 = 32 \Leftrightarrow x^3 = 8 \Leftrightarrow x = 2$, which is the x-coordinate of the point on the curve at which the slope is 32. The y-coordinate is $2^4 + 1 = 17$, so an equation of the tangent line is $y = 32(x-2) + 17$ or $y = 32x - 47$.

74. The slope of the line $3x - y = 15$(or $y = 3x - 15$) is 3, so the slope of both tangent lines to the curve is 3. $y = x^3 - 3x^2 + 3x - 3 \Rightarrow y' = 3x^2 - 6x + 3 = 3\left(x^2 - 2x + 1\right) = 3\left(x-1\right)^2$. Thus, $3\left(x-1\right)^2 = 3 \Rightarrow \left(x-1\right)^2 = 1 \Rightarrow x - 1 = \pm 1 \Rightarrow x = 0$ or 2, which are the x-coordinates at which the tangent lines have slope 3. The points on the curve are and so the tangent line equations are $y = 3(x-0) - 3$ or $y = 3x - 3$ and $y = 3(x-2) - 1$ or $y = 3x - 7$.

75. The slope of $y = 1 + 2e^x - 3x$ is given by $m = y' = 2e^x - 3$.
The slope of $3x - y = 5 \iff y = 3x - 5$ is 3.
$m = 3 \Rightarrow 2e^x - 3 = 3 \Rightarrow e^x = 3 \Rightarrow x = \ln 3$.
This occurs at the point $(\ln 3, 7 - 3\ln 3) \approx (1.0986, 3.704)$.

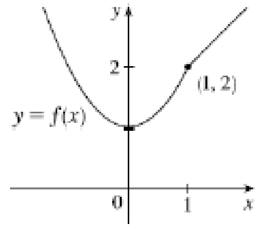


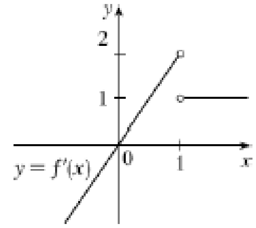
76. The slope of $y = \sqrt{x}$ is given by $y' = \frac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$. The slope of $2x + y = 1$ (or $y = -2x + 1$) is –2, so the desired normal line must have slope –2, and hence, the tangent line to the curve must have slope $\frac{1}{2}$. This occurs if $\dfrac{1}{2\sqrt{x}} = \dfrac{1}{2} \Rightarrow \sqrt{x} = 1 \Rightarrow x = 1$.
When $x = 1$, $y = \sqrt{1} = 1$, and an equation of the normal line is $y = -2(x-1) + 1$ or $y = -2x + 3$.

86. $f(x) = \dfrac{p}{\sqrt{x}} + q\sqrt{x} = px^{-1/2} + qx^{1/2} \Rightarrow f'(x) = -\dfrac{1}{2}px^{-3/2} + \dfrac{1}{2}qx^{-1/2} = -\dfrac{p}{2\sqrt{x^3}} + \dfrac{q}{2\sqrt{x}}$. Then
$f'(4) = -\dfrac{p}{2\sqrt{4^3}} + \dfrac{q}{2\sqrt{4}} = \dfrac{q}{4} - \dfrac{p}{16}$ which must be zero since the tangent line is horizontal at this point.
In addition, $y = f(4) = 12 = \dfrac{p}{\sqrt{4}} + q\sqrt{4} = \dfrac{p}{2} + 2q$. Solving both equations simultaneously, we find $p = 12$, and $q = 3$.

87. $f(x) = ax - \dfrac{b}{x} = ax - bx^{-1} \Rightarrow f'(x) = a - b(-1)x^{-2} = a + bx^{-2}$. When $x = 1$, $f(1) = a(1) - \dfrac{b}{(1)} = a - b$.
The slope of the line $y = 5x + 6$ is 5 so we must have $f'(1) = a + \dfrac{b}{1^2} = a + b = 5$ **(1),** and the point $(1, f(1)) = (1, a - b)$ must also be on this line. Thus, $a - b = 5(1) + 6 = 11 \Rightarrow a = b + 11$.
Now substituting into (**1**) we find, $5 = a + b = (11 + b) + b = 11 + 2b \Rightarrow -6 = 2b \Rightarrow b = -3$, and

91. $f(x)=\begin{cases} x^2+1 & \text{if } x<1 \\ x+1 & \text{if } x\geq 1 \end{cases}$





Calculate the left- and right-hand derivatives:

$$f_-'(1)=\lim_{h\to 0^-}\frac{f(1+h)-f(1)}{h}$$

$$=\lim_{h\to 0^-}\frac{\left[(1+h)^2+1\right]-(1+1)}{h}=\lim_{h\to 0^-}\frac{h^2+2h}{h}=\lim_{h\to 0^-}(h+2)=2 \quad \text{and}$$

$$f_+'(1)=\lim_{h\to 0^+}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0^+}\frac{\left[(1+h)+1\right]-(1+1)}{h}=\lim_{h\to 0^+}\frac{h}{h}=\lim_{h\to 0^+}1=1.$$

Since the left and right limits are different, $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$ does not exist, that is, $f'(1)$ does not exist. Therefore, f is not differentiable at 1.

92. $g(x)=\begin{cases} 2x & \text{if } x\leq 0 \\ 2x-x^2 & \text{if } 0<x<2 \\ 2-x & \text{if } x\geq 2 \end{cases}$

Investigate the left- and right-hand derivatives at $x=0$ and $x=2$:

$$g_-'(0)=\lim_{h\to 0^-}\frac{g(0+h)-g(0)}{h}=\lim_{h\to 0^-}\frac{2h-2(0)}{h}=2 \quad \text{and}$$

$$g_+'(0)=\lim_{h\to 0^+}\frac{g(0+h)-g(0)}{h}=\lim_{h\to 0^+}\frac{\left(2h-h^2\right)-2(0)}{h}=\lim_{h\to 0^+}(2-h)=2,$$

so g is differentiable at $x=0$.

$$g_-'(2)=\lim_{h\to 0^-}\frac{g(2+h)-g(2)}{h}=\lim_{h\to 0^-}\frac{2(2+h)-(2+h)^2-(2-2)}{h}=\lim_{h\to 0^-}\frac{-2h-h^2}{h}=\lim_{h\to 0^-}(-2-h)=-2$$

$$g_+'(2)=\lim_{h\to 0^+}\frac{g(2+h)-g(2)}{h}=\lim_{h\to 0^+}\frac{\left[2-(2+h)\right]-(2-2)}{h}=\lim_{h\to 0^+}\frac{-h}{h}=\lim_{h\to 0^+}(-1)=-2,$$ so g is not differentiable at $x=2$. Thus a formula for g' is

$$g'(x)=\begin{cases} -2 & \text{if } x\leq 0 \\ 2-2x & \text{if } 0<x<2 \\ -1 & \text{if } x>2 \end{cases}$$