p. 213: 6-7, 11-41 odd, 42-45, 47-51 odd (only part a), 54-66, 75

6. $\frac{d}{dx}\left(\frac{e^x}{e^x+1}\right)=\frac{(e^x+1)e^x-e^x(e^x)}{(e^x+1)^2}=\frac{e^{2x}+e^x-e^{2x}}{(e^x+1)^2}=\frac{e^x}{(e^x+1)^2}$, which is answer (**D**).

7. Product Rule: $f(x)=(1+2x^2)(x-x^2)\Rightarrow$

$f'(x)=(1+2x^2)(1-2x)+(x-x^2)(4x)=1-2x+2x^2-4x^3+4x^2-4x^3=1-2x+6x^2-8x^3$.

Multiplying first: $f(x)=(1+2x^2)(x-x^2)=x-x^2+2x^3-2x^4\Rightarrow f'(x)=1-2x+6x^2-8x^3$ (equivalent)

11. By the Product Rule, $g(x)=(2+2\sqrt{x})e^x\Rightarrow$

$$g'(x)=(2+2\sqrt{x})(e^x)'+e^x(2+2\sqrt{x})'=(2+2\sqrt{x})e^x+e^x\left(\frac{2}{2\sqrt{x}}\right)$$
$$=e^x\left[(2+2\sqrt{x})+\left(\frac{1}{\sqrt{x}}\right)\right]=e^x\left(2+2\sqrt{x}+\frac{1}{\sqrt{x}}\right)$$

13. By the Quotient Rule, $y=\frac{e^x}{1-e^x}\Rightarrow\frac{(1-e^x)e^x-e^x(e^x)}{(1-e^x)^2}=\frac{e^x-e^{2x}+e^{2x}}{(1-e^x)^2}=\frac{e^x}{(1-e^x)^2}$.

The notations $\overset{\text{PR}}{\Rightarrow}$ and $\overset{\text{QR}}{\Rightarrow}$ indicate the use of the Product and Quotient Rules, respectively.

15. $G(x)=\frac{x^2-2}{2x+1}\overset{\text{QR}}{\Rightarrow}G'(x)=\frac{(2x+1)(2x)-(x^2-2)(2)}{(2x+1)^2}=\frac{4x^2+2x-2x^2+4}{(2x+1)^2}=\frac{2x^2+2x+4}{(2x+1)^2}$

17. $J(v)=(v^3-2v)(v^{-4}+v^{-2})\overset{\text{PR}}{\Rightarrow}$

$$J'(v)=(v^3-2v)(-4v^{-4}-2v^{-3})+(v^{-4}+v^{-2})(3v^2-2)$$
$$=-4v^{-2}-2v^0+8v^{-4}+4v^{-2}+3v^{-2}-2v^{-4}+3v^0-2v^{-2}=1+v^{-2}+6v^{-4}$$

19. $f(z)=(1-e^z)(z+e^z)\overset{\text{PR}}{\Rightarrow}$

$$f'(z)=(1-e^z)(1+e^z)+(z+e^z)(-e^z)=1-(-e^z)^2-ze^z-(e^z)^2=1-ze^z-2e^{2z}$$

21. $y=\frac{\sqrt{x}}{2+x}\overset{\text{QR}}{\Rightarrow}y'=\frac{(2+x)\left(\frac{1}{2\sqrt{x}}\right)-\sqrt{x}(1)}{(2+x)^2}=\frac{\frac{1}{\sqrt{x}}+\frac{\sqrt{x}}{2}-\sqrt{x}}{(2+x)^2}=\frac{\frac{2+x-2x}{2\sqrt{x}}}{(2+x)^2}=\frac{2-x}{2\sqrt{x}(2+x)^2}$

23. $y=\frac{1}{t^3+2t^2-1}\overset{\text{QR}}{\Rightarrow}y'=\frac{(t^3+2t^2-1)(0)-1(3t^2+4t)}{(t^3+2t^2-1)^2}=-\frac{3t^2+4t}{(t^3+2t^2-1)^2}$

25. $h(r)=\frac{ae^r}{b+e^r}\overset{\text{QR}}{\Rightarrow}h'(r)=\frac{(b+e^r)ae^r-(ae^r)(e^r)}{(b+e^r)^2}=\frac{abe^r+ae^{2r}-ae^{2r}}{(b+e^r)^2}=\frac{abe^r}{(b+e^r)^2}$

27. $y=\left(z^2+e^z\right)\sqrt{z} \overset{\text{PR}}{\Rightarrow} y'=\left(z^2+e^z\right)\dfrac{1}{2\sqrt{z}}+\sqrt{z}\left(2z+e^z\right)$

$$=\frac{z^2}{2\sqrt{z}}+\frac{e^z}{2\sqrt{z}}+2z\sqrt{z}+\sqrt{z}e^z=\frac{z^2+e^z+4z^2+2ze^z}{2\sqrt{z}}=\frac{5z^2+e^z+2ze^z}{2\sqrt{z}}$$

29. $V(t)=\dfrac{4+t}{te^t} \overset{\text{QR}}{\Rightarrow} V'(t)=\dfrac{te^t(1)-(4+t)\left(te^t+e^t(1)\right)}{te^t}$

$$=\frac{te^t-4te^t-4e^t-t^2e^t-te^t}{t^2e^{2t}}=\frac{-4te^t-4e^t-t^2e^t}{t^2e^{2t}}=\frac{-e^t\left(t^2+4t+4\right)}{t^2e^{2t}}=\frac{(t+2)^2}{t^2e^t}$$

31. $F(t)=\dfrac{At}{Bt^2+Ct^3}=\dfrac{A}{Bt+Ct^2} \overset{\text{QR}}{\Rightarrow} F'(t)=\dfrac{\left(Bt+Ct^2\right)(0)-A(B+2Ct)}{\left(Bt+Ct^2\right)^2}=\dfrac{-A(B+2Ct)}{t^2(B+Ct)^2}=-\dfrac{A(B+2Ct)}{t^2(B+Ct)^2}$

33. $f(x)=\dfrac{ax+b}{cx+d} \overset{\text{QR}}{\Rightarrow} f'(x)=\dfrac{(cx+d)a-(ax+b)c}{(cx+d)^2}=\dfrac{acx+ad-acx-bc}{(cx+d)^2}=\dfrac{ad-bc}{(cx+d)^2}$

35. $f(x)=\sqrt{x}e^x \overset{\text{PR}}{\Rightarrow} f'(x)=\sqrt{x}e^x+e^x\left(\dfrac{1}{2\sqrt{x}}\right)=\left(\sqrt{x}+\dfrac{1}{2\sqrt{x}}\right)e^x=\dfrac{2x+1}{2\sqrt{x}}e^x.$

Using the Product Rule and $f'(x)=\left(x^{1/2}+\frac{1}{2}x^{-1/2}\right)e^x$, we get

$$f''(x)=\left(x^{1/2}+\tfrac{1}{2}x^{-1/2}\right)e^x+e^x\left(\tfrac{1}{2}x^{-1/2}-\tfrac{1}{4}x^{-3/2}\right)=\left(x^{1/2}+x^{-1/2}-\tfrac{1}{4}x^{-3/2}\right)e^x=\frac{4x^2+4x-1}{4x^{3/2}}e^x$$

37. $f(x)=\dfrac{x}{x^2-1} \overset{\text{QR}}{\Rightarrow} f'(x)=\dfrac{\left(x^2-1\right)(1)-x(2x)}{\left(x^2-1\right)^2}=\dfrac{x^2-1-2x^2}{\left(x^2-1\right)^2}=\dfrac{-x^2-1}{\left(x^2-1\right)^2} \Rightarrow$

$$f''(x)=\frac{\left(x^2-1\right)^2(-2x)-\left(-x^2-1\right)\left(x^4-2x^2+1\right)'}{\left(x^2-1\right)^4}=\frac{\left(x^2-1\right)^2(-2x)+\left(x^2+1\right)\left(4x^3-4x\right)}{\left(x^2-1\right)^4}$$

$$=\frac{\left(x^2-1\right)^2(-2x)+\left(x^2+1\right)(4x)\left(x^2-1\right)}{\left(x^2-1\right)^4}=\frac{\left(x^2-1\right)\left[\left(x^2-1\right)(-2x)+\left(x^2+1\right)(4x)\right]}{\left(x^2-1\right)^4}$$

$$=\frac{-2x^3+2x+4x^3+4x}{\left(x^2-1\right)^3}=\frac{2x^3+6x}{\left(x^2-1\right)^3}$$

39. $y=\dfrac{1+x}{1+e^x} \overset{\text{QR}}{\Rightarrow} y'=\dfrac{\left(1+e^x\right)(1)-(1+x)e^x}{\left(1+e^x\right)^2}=\dfrac{1+e^x-e^x-xe^x}{\left(1+e^x\right)^2}=\dfrac{1-xe^x}{\left(1+e^x\right)^2}.$

At $\left(0,\frac{1}{2}\right)$, $y'=\dfrac{1}{(1+1)^2}=\dfrac{1}{4}$, and an equation of the tangent line is $y=\frac{1}{4}(x-0)+\frac{1}{2}$ or $y=\frac{1}{4}x+\frac{1}{2}$.

41. $y=\dfrac{2x}{x^2+1} \overset{\text{QR}}{\Rightarrow} y'=\dfrac{\left(x^2+1\right)(2)-2x(2x)}{\left(x^2+1\right)^2}=\dfrac{2-2x^2}{\left(x^2+1\right)^2}$. At $(1,1)$, $y'=0$, and an equation of the tangent line is $y=0(x-1)+1$ or $y=1$. The slope of the normal line is undefined, so an equation of the normal line is $x=1$.

42. $f(x)=xe^x \Rightarrow f'(x)=xe^x+e^x(1)=e^x(x+1)$. The tangent line is horizontal when $f'(x)=0$ which happens when (**B**) $x=-1$.

43. $f(x)=\dfrac{4x}{1+x^2} \Rightarrow f'(x)=\dfrac{(1+x^2)4-(4x)(2x)}{(1+x^2)^2}=\dfrac{4+4x^2-8x^2}{(1+x^2)^2}=\dfrac{4(1-x^2)}{(1+x^2)^2}$. The tangent line is horizontal when $f'(x)=0$ which happens when $x=1,-1$. This is choice (**B**).

44. $s(t)=\dfrac{t}{t^2+5} \Rightarrow s'(t)=\dfrac{(t^2+5)\cdot 1-(t)(2t)}{(t^2+5)^2}=\dfrac{t^2+5-2t^2}{(t^2+5)^2}=\dfrac{5-t^2}{(t^2+5)^2}$. The particle is at rest when its velocity, $s'(t)$ is zero. This happens when $t=\sqrt{5}$, and at that time, the particle's position is $s(\sqrt{5})=\dfrac{\sqrt{5}}{(\sqrt{5})^2+5}=\dfrac{\sqrt{5}}{10}$.

45. $f(x)=\dfrac{x^2}{4x+1} \Rightarrow f'(x)=\dfrac{(4x+1)2x-x^2(4)}{(4x+1)^2}=\dfrac{8x^2+2x-4x^2}{(4x+1)^2}=\dfrac{4x^2+2x}{(4x+1)^2}=\dfrac{2x(2x+1)}{(4x-1)^2}=0 \Leftrightarrow$ $x=0,-\frac{1}{2}$. Therefore, f has a horizontal tangent line when (**B**), $x=0,-\frac{1}{2}$.

47. (a) $y=f(x)=\dfrac{x}{1+x^2} \Rightarrow$

$f'(x)=\dfrac{(1+x^2)(1)-x(2x)}{(1+x^2)^2}=\dfrac{1-x^2}{(1+x^2)^2}$. So the slope of the tangent line at the point $y=(3,0.3)$ is $f'(3)=-\frac{8}{100}$ and the equation of that tangent line is $y=-0.08(x-3)+0.3$ or $y=-0.08x+0.54$.

49. (a) $f(x)=\dfrac{e^x}{2x^2+x+1} \Rightarrow$

$$f'(x)=\frac{(2x^2+x+1)e^x-e^x(4x+1)}{(2x^2+x+1)^2}=\frac{e^x(2x^2+x+1-4x-1)}{(2x^2+x+1)^2}=\frac{e^x(2x^2-3x)}{(2x^2+x+1)^2}$$

51.

$f(x)=(x^2-1)e^x \Rightarrow f'(x)=(x^2-1)e^x+e^x(2x)=e^x(x^2+2x-1) \Rightarrow$

$f''(x)=e^x(2x+2)+(x^2+2x-1)e^x=e^x(x^2+4x-1)$

54. We are given that $f(5)=1,\ f'(5)=6,\ g(5)=-3$, and $g'(5)=2$.

(a) $(fg)'(5)=f(5)g'(5)+g(5)f'(5)=(1)(2)+(-3)(6)=2-18=-16$

(b) $\left(\dfrac{f}{g}\right)'(5)=\dfrac{g(5)f'(5)-f(5)g'(5)}{[g(5)]^2}=\dfrac{(-3)(6)-(1)(2)}{(-3)^2}=\dfrac{-20}{9}$

(c) $\left(\dfrac{g}{f}\right)'(5)=\dfrac{f(5)g'(5)-g(5)f'(5)}{[f(5)]^2}=\dfrac{(1)(2)-(-3)(6)}{(1)^2}=20$

55. We are given that $f(4)=2,\ g(4)=5,\ f'(4)=6,$ and $g'(4)=-3.$

(a) $h(x)=3f(x)+8g(x) \Rightarrow h'(4)=3f'(4)+8g'(4)=3(6)+8(-3)=18-24=-6$

(b) $h(x)=f(x)g(x) \Rightarrow h'(4)=f(4)g'(4)+g(4)f'(4)=2(-3)+5(6)=-6+30=24$

(c) $h(x)=\dfrac{f(x)}{g(x)} \Rightarrow \quad h'(4)=\dfrac{g(4)f'(4)-f(4)g'(4)}{[g(4)]^2}=\dfrac{5(6)-2(-3)}{5^2}=\dfrac{30+6}{25}=\dfrac{36}{25}$

(d) $h(x)=\dfrac{g(x)}{f(x)+g(x)} \Rightarrow h'(4)=\dfrac{[f(4)+g(4)]g'(4)-g(4)[f'(4)+g'(4)]}{[f(4x)+g(4)]^2}$

$$=\frac{(2+5)(-3)-5(6+(-3))}{(2+5)^2}=\frac{-21-15}{49}=-\frac{36}{49}$$

56. (a) Using the Product Rule, $\dfrac{d}{dx}\left[\sqrt{x}\cdot f(x)\right]=\sqrt{x}\cdot f'(x)+f(x)\dfrac{1}{2\sqrt{x}} \Rightarrow$

$\dfrac{d}{dx}\left[\sqrt{x}\cdot f(x)\right]_{x=4}=\sqrt{4}\cdot f'(4)+f(4)\dfrac{1}{2\sqrt{4}}=2\cdot(-1)+16\cdot\dfrac{1}{2\sqrt{4}}=-2+4=2.$

(b) Using the Quotient Rule, $\dfrac{d}{dx}\left[\dfrac{g(x)}{f(x)}\right]=\dfrac{f(x)g'(x)-g(x)f'(x)}{[f(x)]^2} \Rightarrow$

$\dfrac{d}{dx}\left[\dfrac{g(x)}{f(x)}\right]_{x=2}=\dfrac{f(2)g'(2)-g(2)f'(2)}{[f(2)]^2}=\dfrac{3\cdot(1)-(16)\cdot(-5)}{3^2}=\dfrac{3+80}{9}=\dfrac{83}{9}$

57. (a) $h'(x)=2+g'(x) \Rightarrow h'(1)=2+g'(1)=2+1=3$

(b) $q'(x)=\dfrac{f(x)g'(x)-g(x)f'(x)}{[f(x)]^2} \Rightarrow$

$q'(1)=\dfrac{f(1)g'(1)-g(1)f'(1)}{[f(1)]^2}=\dfrac{(-3)(1)-(-8)(5)}{(-3)^2}=\dfrac{-3+40}{9}=\dfrac{37}{9}$

58. $\dfrac{d}{dx}\left[\dfrac{h(x)}{x}\right]=\dfrac{xh'(x)-h(x)\cdot 1}{x^2} \Rightarrow$

$\dfrac{d}{dx}\left[\dfrac{h(x)}{x}\right]_{x=2}=\dfrac{2\cdot h'(2)-h(2)\cdot 1}{2^2}=\dfrac{2\cdot(-2)-7\cdot 1}{4}=\dfrac{-4-7}{4}=-\dfrac{11}{4}$

59. $f(x)=e^x g(x) \Rightarrow f'(x)=e^x g'(x)+g(x)e^x=e^x\left[g'(x)+g(x)\right].$

$f'(0)=e^0\left[g'(0)+g(0)\right]=1(5+2)=7$

60. $\dfrac{d}{dx}\left[\dfrac{h(x)}{x}\right]=\dfrac{xh'(x)-h(x)\cdot 1}{x^2} \Rightarrow \dfrac{d}{dx}\left[\dfrac{h(x)}{x}\right]_{x=2}=\dfrac{2h'(2)-h(2)}{2^2}=\dfrac{2(-3)-4}{4}=\dfrac{-10}{4}=-2.5$

61. $g(x)=xf(x) \Rightarrow g'(x)=xf'(x)+f(x)\cdot 1.$ Now $g(3)=3f(3)=3\cdot 4=12$ and $g'(3)=3f'(3)+f(3)=3(-2)+4=-2.$ Thus, an equation of the tangent line to the graph of g at the point where $x=3$ is $y=-2(x-3)+12,$ or $y=-2x+18.$

62. $f'(x)=x^2 f(x) \Rightarrow f''(x)=x^2 f'(x)+f(x)\cdot 2x.$ Now $f'(2)=2^2 f(2)=4(10)=40,$ so $f''(2)=2^2\cdot(40)+10(4)=200.$

63. (a) From the graphs of f and g, we obtain the following values: $f(1)=2$ since the point $(1,2)$ is on the graph of f; $g(1)=1$ since the point $(1,1)$ is on the graph of g; $f'(1)=2$ since the slope of the line segment between $(0,0)$ and $(2,4)$ is $\frac{4-0}{2-0}=2$; $g'(1)=-1$ since the slope of the line segment between $(-2,4)$ and $(0,0)$ is $\frac{0-4}{2-(-2)}=-1$. Now $u(x)=f(x)g(x)$, so

$u'(1)=f(1)g'(1)+g(1)f'(1)=2\cdot(-1)+1\cdot 2=0.$

(b) $v(x)=f(x)/g(x)$ so $v'(5)=\dfrac{g(5)f'(5)-f(5)g'(5)}{[g(5)]^2}=\dfrac{2\cdot(-\frac{1}{3})-3\cdot\frac{2}{3}}{2^2}=\dfrac{-\frac{8}{3}}{4}=-\dfrac{2}{3}.$

64. (a) $P(x)=F(x)G(x)$, so $P'(2)=F(2)G'(2)+G(2)F'(2)=3\cdot\frac{2}{4}+2\cdot 0=\frac{3}{2}.$

(b) $Q(x)=F(x)/G(x)$, so $Q'(7)=\dfrac{G(7)F'(7)-F(7)G'(7)}{[G(7)]^2}=\dfrac{1\cdot\frac{1}{4}-5\cdot(-\frac{2}{3})}{1}=\dfrac{1}{4}+\dfrac{10}{3}=\dfrac{43}{12}.$

65. (a) $y=xg(x)\Rightarrow y'=xg'(x)+g(x)\cdot 1=xg'(x)+g(x)$

(b) $y=\dfrac{x}{g(x)}\Rightarrow y'=\dfrac{g(x)\cdot 1-xg'(x)}{[g(x)]^2}=\dfrac{g(x)-xg'(x)}{[g(x)]^2}$

(c) $y=\dfrac{g(x)}{x}\Rightarrow y'=\dfrac{xg'(x)-g(x)\cdot 1}{x^2}=\dfrac{xg'(x)-g(x)}{[g(x)]^2}$

66. (a) $y=x^2f(x)\Rightarrow y'=x^2f'(x)+f(x)(2x)$

(b) $y=\dfrac{f(x)}{x^2}\Rightarrow y'=\dfrac{x^2f'(x)-f(x)(2x)}{(x^2)^2}=\dfrac{xf'(x)-2f(x)}{x^3}$

(c) $y=\dfrac{x^2}{f(x)}\Rightarrow y'=\dfrac{f(x)(2x)-x^2f'(x)}{[f(x)]^2}$

(d) $y=\dfrac{1+xf(x)}{\sqrt{x}}\Rightarrow y'=\dfrac{\sqrt{x}\left[xf'(x)+f(x)\right]-\left[1+xf(x)\right]\frac{1}{2\sqrt{x}}}{\left[\sqrt{x}\right]^2}$

$$=\frac{x^{3/2}f'(x)+x^{1/2}f(x)-\frac{1}{2}x^{-1/2}-\frac{1}{2}x^{1/2}f(x)}{x}\cdot\frac{2x^{1/2}}{2x^{1/2}}=\frac{xf(x)+2x^2f'(x)-1}{2x^{3/2}}$$

75. (a) $f(20)=10{,}000$ means that when the price of the fabric is \$20/yard, 10,000 yards will be sold. $f'(20)=-350$ means that as the price of the fabric increases past \$20/yard, the amount of fabric which will be sold is decreasing at a rate of 350 yards per (dollar per yard).

(b) $R(p)=pf(p)\Rightarrow R'(p)=pf'(p)+f(p)\cdot 1\Rightarrow$

$R'(20)=20f'(20)+f(20)=20(-350)+10{,}000=3000.$

This means that as the price of the fabric increases past \$20/yard, the total revenue is increasing at \$3000/(\$/yard). Note that the Product Rule indicates that we will lose \$7000/(\$/yard) due to selling less fabric, but this loss is more than made up for by the additional revenue due to the increase in price.