p. 223: 5, 9-25 odd, 45-51 odd, 59-61, 62, 67-68

5. $\frac{d}{dx}(e^x \sin x + e^x \cos x) = \frac{d}{dx}[e^x(\sin x + \cos x)] = e^x(\cos x - \sin x) + (\sin x + \cos x)e^x$
$= e^x[2\cos x](\cos x - \sin x + \sin x + \cos x) = e^x(2\cos x) = 2e^x \cos x$, (**B**).

9. $f(x) = x\cos x + 2\tan x \Rightarrow f'(x) = x(-\sin x) + (\cos x)(1) + 2\sec^2 x = \cos x - x\sin x + 2\sec^2 x$

11. $y = 2\sec x - \csc x \Rightarrow y' = 2(\sec x \tan x) - (-\csc x \cot x) = 2\sec x \tan x + \csc x \cot x$

13. $g(x) = e^x(\tan x - x) \Rightarrow g'(x) = e^x(\sec^2 x - 1) + (\tan x - x)e^x = e^x(\sec^2 x - 1 + \tan x - x)$

15. $f(t) = \frac{\cot t}{e^t} \Rightarrow f'(t) = \frac{e^t(-\csc^2 t) - (\cot t)e^t}{(e^t)^2} = \frac{e^t(-\csc^2 t - \cot t)}{(e^t)^2} = \frac{-\csc^2 t - \cot t}{e^t}$

17. $y = \sin\theta\cos\theta \Rightarrow y' = \sin\theta(-\sin\theta) + \cos\theta(\cos\theta) = \cos^2\theta - \sin^2\theta = \cos 2\theta$

19. $y = \frac{\cos x}{1 - \sin x} \Rightarrow$

$$y' = \frac{(1-\sin x)(-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2} = \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} = \frac{-\sin x + 1}{(1-\sin x)^2} = \frac{1}{1-\sin x}$$

21. $y = \frac{\sin t}{1+\tan t} \Rightarrow y' = \frac{(1+\tan t)\cos t - (\sin t)\sec^2 t}{(1+\tan t)^2} = \frac{\cos t + \sin t - \frac{\sin t}{\cos^2 t}}{(1+\tan t)^2} = \frac{\cos t + \sin t - \tan t \sec t}{(1+\tan t)^2}$

23. Using 3.4.78(a), $f(t) = te^t \cot t \Rightarrow$
$f'(t) = 1e^t \cot t + te^t \cot t + te^t(-\csc^2 t) = e^t(\cot t + t\cot t - t\csc^2 t)$

25. $f(x) = 2x\cos x - 3\sin x \Rightarrow f'(x) = 2x(-\sin x) + 2\cos x - 3\cos x = -2x\sin x - \cos x$

45. $y = \sin x + \cos x \Rightarrow y' = \cos x - \sin x$, so $y'(0) = \cos 0 - \sin 0 = 1 - 0 = 1$. An equation of the tangent line to the curve $y = \sin x + \cos x$ at the point $(0,1)$ is $y = 1(x-0) + 1$ or $y = x + 1$.

47. $y = \cos x - \sin x \Rightarrow y' = -\sin x - \cos x$, so $y'(\pi) = -\sin\pi - \cos\pi = 0 - (-1) = 1$. An equation of the tangent line to the curve $y = \cos x - \sin x$ at the point $(\pi, -1)$ is $y = 1(x - \pi) - 1$ or $y = x - \pi - 1$.

49. $y = 2x\sin x \Rightarrow y' = 2x\cos x + 2\sin x$, so $y'(\frac{\pi}{4}) = 2\frac{\pi}{4}\cos\frac{\pi}{4} + 2\sin\frac{\pi}{4} = 2\cdot\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} + 2\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}(\pi + 1)$. An equation of the tangent line to the curve $2x\sin x$ at the point $\left(\frac{\pi}{4}, \frac{\sqrt{2}\pi}{4}\right)$ is $y = \frac{\sqrt{2}}{4}(\pi+1)(x - \frac{\pi}{4}) + \frac{\sqrt{2}\pi}{4}$ or
$y = \frac{\sqrt{2}}{4}(\pi + 4)x - \frac{\sqrt{2}}{16}\pi^2$.

51. (a) $y = 3x + 6\cos x \Rightarrow y' = 3 - 6\sin x$. At $\left(\frac{\pi}{3}, \pi + 3\right)$,
$y' = 3 - 6\sin\frac{\pi}{3} = 3 - 6\frac{\sqrt{3}}{2} = 3 - 3\sqrt{3}$, and an equation of the tangent line is $y = \left(3 - 3\sqrt{3}\right)\left(x - \frac{\pi}{3}\right) + \pi + 3$, or
$y = \left(3 - 3\sqrt{3}\right)x + 3 + \pi\sqrt{3}$.

59. (a) $g(x)=f(x)\sin x \Rightarrow g'(x)=f(x)\cos x+\sin x\cdot f'(x)$, so

$g'(\frac{\pi}{3})=f(\frac{\pi}{3})\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cdot f'(\frac{\pi}{3})=4\cdot\frac{1}{2}+\frac{\sqrt{3}}{2}\cdot(-2)=2-\sqrt{3}$

(b) $h(x)=\dfrac{\cos x}{f(x)} \Rightarrow h'(x)=\dfrac{f(x)\cdot(-\sin x)-\cos x\cdot f'(x)}{[f(x)]^2}$, so

$$h'(\tfrac{\pi}{3})=\frac{f(\frac{\pi}{3})\cdot(-\sin\frac{\pi}{3})-\cos\frac{\pi}{3}\cdot f'(\frac{\pi}{3})}{[f(\frac{\pi}{3})]^2}=\frac{4\left(-\frac{\sqrt{3}}{2}\right)-\frac{1}{2}(-2)}{4^2}=\frac{-2\sqrt{3}+1}{1616}=\frac{1-2\sqrt{3}}{}$$

60. $f(x)=x+2\sin x$ has a horizontal tangent when $f'(x)=0 \Leftrightarrow 1+2\cos x=0 \Leftrightarrow \cos x=-\frac{1}{2} \Leftrightarrow$ $x=\frac{2}{3}\pi+2\pi n$ or $\frac{4}{3}\pi+2\pi n$, where n is an integer. Note that $\frac{4}{3}$ and $\frac{2}{3}$ are $\pm\frac{\pi}{3}$ units from π. This allows us to write the solutions in the more compact, equivalent form $(2n+1)\pi\pm\frac{\pi}{3}$, n an integer.

61. $f(x)=e^x\cos x$ has a horizontal tangent when $f'(x)=0$.

$f'(x)=e^x(-\sin x)+(\cos x)e^x=e^x(\cos x-\sin x)$.

$f'(x)=0 \Leftrightarrow \cos x-\sin x=0 \Leftrightarrow \cos x=\sin x \Leftrightarrow \tan x=1 \Leftrightarrow x=\frac{\pi}{4}+n\pi$, n an integer.

62. $y=6\cos x \Rightarrow y'=-6\sin x \Rightarrow$ $y\left(\frac{\pi}{6}\right)=6\frac{\sqrt{3}}{2}$, and $y'\left(\frac{\pi}{6}\right)=-6\sin\left(\frac{\pi}{6}\right)=-3$. The equation of the tangent line is $y=-3\left(x-\frac{\pi}{6}\right)+3\sqrt{3}=-3x+\frac{\pi}{2}+3\sqrt{3}$. This line has y-intercept $\left(0,\frac{\pi}{2}+3\sqrt{3}\right)$ and x-intercept $\left(\frac{\pi}{6}+\sqrt{3},0\right)$. Thus the area of the triangle is $A=\frac{1}{2}\left(\frac{\pi}{6}+\sqrt{3}\right)\left(\frac{\pi}{2}+3\sqrt{3}\right)=\frac{1}{2}\left(\frac{\pi^2}{12}+\pi\sqrt{3}+9\right)$

67. If the particle's position is $s(t)=4\cos t\sin t-4\sin t$, then the particle is at rest when its velocity, $v(t)=s'(t)=4\cos t(\cos t)+4\sin t(-\sin t)-4\cos t=4\cos^2 t-4\sin^2 t-4\cos t=4(\cos^2 t-\sin^2 t-\cos t)$ is zero. This occurs when $0=4(\cos^2 t-\sin^2 t-\cos t)\Rightarrow 0=\cos^2 t-\sin^2 t-\cos t\Rightarrow$

$0=(1-\sin^2 t)-\sin^2 t-\cos t\Rightarrow 0=(1-2\sin^2 t)-\cos t\Rightarrow 0=2\cos^2 t-\cos t-1\Rightarrow$

$0=(2\cos t+1)(\cos t-1)\Rightarrow(\cos t-1)=0$, or $(2\cos t+1)=0$. But since $t\geq 0$, we must have $\cos t=-\frac{1}{2}\Rightarrow t=\frac{2\pi}{3}$ (**B**) or $t=\frac{4\pi}{3}$ (**C**)

68. The particle is at rest when its velocity is zero. If the position is $s(t)=\dfrac{\cos t}{2+\sin t}$, then its velocity is

$v(t)=s'(t)=\dfrac{(2+\sin t)(-\sin t)-\cos t(\cos t)}{(2+\sin t)^2}=\dfrac{-2\sin t-(\sin^2 t+\cos^2 t)}{(2+\sin t)^2}=-\dfrac{1+2\sin t}{(2+\sin t)^2}$. So the particle is at rest when $\dfrac{1+2\sin t}{(2+\sin t)^2}=0\Rightarrow 1+2\sin t=0\Rightarrow 2\sin t=-1\Rightarrow \sin t=\dfrac{11\pi}{6}$. This is option (**D**).

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96. $y=x^4+ax^3+bx^2+cx+d \Rightarrow y(0)=d$. Since the tangent line $y=2x+1$ is equal to 1 at $x=0$, we must have $d=1$. $y'=4x^3+3ax^2+2bx+d \Rightarrow y'(0)=c$. Since the slope of the tangent line $y=2x+1$ at $x=0$ is 2, we must have $c=2$. Now $y(1)=1+a+b+c+d=a+b+4$ and the tangent line $y=2-3x$ at $x=1$ has y-coordinate -1, so $a+b+4=-1$ or $a+b=-5$ (**1**).
Also $y'(1)=4+3a+2b+6=3a+2b+6$ and the slope of the tangent line $y=2-3x$ at $x=1$ is -3, so $3a+2b+6=-3$ or $3a+2b=-9$ (**2**). Adding -2 times (**1**) to (**2**) gives us $a=1$ and $b=-6$. The curve has equation $y=x^4+x^3-6x^2+2x+1$.

97. $y = f(x) = ax^2 \Rightarrow f'(x) = 2ax$. So the slope of the tangent to the parabola at $x = 2$ is $m = 2a(2) = 4a$. The slope of the given line, $2x + y = b \Rightarrow y = -2x + b$, is seen to be -2, so we must have $4a = -2 \Rightarrow a = -\frac{1}{2}$. So when $x = 2$, the point in question has y-coordinate $-\frac{1}{2} \cdot 2^2 = -2$. Now we simply require that the given line, whose equation is $2x + y = b$, pass through the point $(2, -2)$: $2(2) + (-2) = b \Rightarrow b = 2$. So we must have $a = -\frac{1}{2}$ and $b = 2$.

98. The slope of the curve $y = c\sqrt{x}$ is $y' = \dfrac{c}{2\sqrt{x}}$ and the slope of the tangent line $y = \frac{3}{2}x + 6$ is $\frac{3}{2}$. These must be equal at the point of tangency $\left(a, c\sqrt{a}\right)$, so $\dfrac{c}{2\sqrt{a}} = \dfrac{3}{2} \Rightarrow c = 3\sqrt{a}$.

The y-coordinates must be equal at $x = a$, so $c\sqrt{a} = \frac{3}{2}a + 6 \Rightarrow \left(3\sqrt{a}\right)\sqrt{a} = \frac{3}{2}a + 6 \Rightarrow$ $3a = \frac{3}{2}a + 6 \Rightarrow \frac{3}{2}a = 6 \Rightarrow a = 4$. Since $c = 3\sqrt{a}$, we have $c = 3\sqrt{4} = 6$.

99. The line $y = 2x + 3$ has slope 2. The parabola $y = cx^2 \Rightarrow y' = 2cx$ has slope $2ca$ at $x = a$. Equating slopes gives us $2ca = 2$ or $ca = 1$. Equating y-coordinates at $x = a$ gives us $ca^2 = 2a + 3 \Leftrightarrow (ca)a = 2a + 3 \Leftrightarrow 1a = 2a + 3 \Leftrightarrow a = -3$. Thus $c = \dfrac{1}{a} = -\dfrac{1}{3}$.

100. $f(x) = \dfrac{6}{\sqrt{x}} = 6x^{-1/2} \Rightarrow f'(x) = 6\left(-\frac{1}{2}\right)x^{-3/2} = -3x^{-3/2}$.

The instantaneous rate of change at c is $f'(c) = -3c^{-3/2} = -\dfrac{3}{\sqrt{c^3}}$.

The average rate of change over the interval $[1, 4]$ is $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{\frac{6}{\sqrt{4}} - \frac{6}{\sqrt{1}}}{3} = \dfrac{3 - 6}{3} = -\dfrac{3}{3} = -1$.

They are equal when $f'(c) = -\dfrac{3}{\sqrt{c^3}} = -1 \Rightarrow \dfrac{3}{\sqrt{c^3}} = 1 \Rightarrow 3 = \sqrt{c^3} \Rightarrow c = 3^{2/3} \approx 2.080$.

101. If $P=\left(a,\sqrt{a}\right)$ is a point of intersection of the graphs of *f* and the tangent line, then the slope of the tangent line must be $m=\frac{1}{2\sqrt{a}}$, and an equation of the tangent line (which goes through the point $(8,3)$ is $y=\frac{1}{2\sqrt{a}}(x-8)+3$ or $y=\frac{1}{2\sqrt{a}}x+3-\frac{4}{\sqrt{a}}$. Since the tangent line also goes through *P*, we can write its equation as $y=\frac{1}{2\sqrt{a}}(x-a)+\sqrt{a}$ or $y=\frac{1}{2\sqrt{a}}x+\sqrt{a}-\frac{\sqrt{a}}{2}$. Equating the *y*-intercepts of both equations we find $3-\frac{4}{\sqrt{a}}=\sqrt{a}-\frac{\sqrt{a}}{2}=\frac{\sqrt{a}}{2} \Rightarrow 3=\frac{\sqrt{a}}{2}+\frac{4}{\sqrt{a}}=\frac{a+8}{2\sqrt{a}}.$

Solving, we find $a=4$ or $a=16$. Therefore, the possible slopes of such tangent lines are $m=\frac{1}{2\sqrt{4}}=\frac{1}{4}$, and $m=\frac{1}{2\sqrt{16}}=\frac{1}{8}$.

Alternative method: Set the slope of the line through *P* and $(8,3)$ equal to $\frac{1}{2\sqrt{a}}$ i.e. $\left(\text{set } \frac{\sqrt{a}-3}{a-8}=\frac{1}{2\sqrt{a}}\right)$, and solve for *a*.

102. A line that goes through the origin has equation $y=mx$. Let $P=\left(a,a^2+5\right)$ be the point where the tangent line intersection the graph of *f*. Then $f(x)=x^2+5 \Rightarrow f'(x)=2x$, the slope of the tangent line is $f'(a)=2a$, and the equation of the tangent line is $y=2ax$. At the point of tangency we have $y=a^2+5=2a(a)=2a^2 \Rightarrow 5=a^2 \Rightarrow \sqrt{5}=a$. Therefore the slope of the tangent line is $2\sqrt{5}$.