<u>3.6</u>

p. 234: 7-8, 17-41 odd (skip 35), 61, 63, 68-78, 82-85, 87

7. $\frac{d}{dx}e^{x\sin x} = e^{x\sin x}(x\sin x)' = e^{x\sin x}(x(\cos x) + \sin x(1)) = e^{x\sin x}(x\cos x + \sin x)$, option (**B**).

8. $f(x) = \sqrt{3x^2+1} = (3x^2+1)^{1/2} \Rightarrow f'(x) = \frac{1}{2}(3x^2+1)^{-1/2}(3\cdot 2x) = \frac{3x}{\sqrt{3x^2+1}}$. Therefore, the slope of the tangent line at the point where $x = 4$ is $f'(4) = \frac{3\cdot 4}{\sqrt{3\cdot 4^2+1}} = \frac{12}{\sqrt{49}} = \frac{12}{7}$. This is choice (**C**).

17. $F(x) = \left(1+x+x^2\right)^{99} \Rightarrow F'(x) = 99\left(1+x+x^2\right)^{98}\cdot\frac{d}{dx}\left(1+x+x^2\right) = 99\left(1+x+x^2\right)^{98}(1+2x)$.

19. $f(x) = \frac{1}{\sqrt[3]{x^2-1}} = (x^2-1)^{-1/3} \Rightarrow f'(x) = -\frac{1}{3}(x^2-1)^{-4/3}(2x) = \frac{-2x}{3(x^2-1)^{4/3}}$

21. $g(\theta) = \cos^2(\theta) \Rightarrow f'(\theta) = 2\cos(\theta)\cdot(-\sin\theta) = -2\sin\theta\cos\theta = -\sin 2\theta$

23. $f(t) = t\sin\pi t \Rightarrow f'(t) = t(\cos\pi t)\cdot\pi + \sin\pi t\cdot 1 = \pi t\cos\pi t + \sin\pi t$

25. $g(x) = e^{x^2-x} \Rightarrow g'(x) = e^{x^2-x}(2x-1)$

27. $g(x) = \left(x^2+1\right)^3\left(x^2+2\right)^6 \Rightarrow$

$$g'(x) = \left(x^2+1\right)^3\cdot 6\left(x^2+2\right)^5\cdot 2x + 3\left(x^2+1\right)^2\cdot 2x\cdot\left(x^2+2\right)^6$$
$$= 6x\left(x^2+1\right)^2\left(x^2+2\right)^5\left[2\left(x^2+1\right)+\left(x^2+2\right)\right] = 6x\left(x^2+1\right)^2\left(x^2+2\right)^5\left(3x^2+4\right)$$

29. $F(t) = (3t-1)^4(2t+1)^{-3} \Rightarrow$

$$F'(t) = (3t-1)^4(-3)(2t+1)^{-4}(2) + (2t+1)^{-3}\cdot 4(3t-1)^3(3)$$
$$= 6(3t-1)^3(2t+1)^{-4}[-(3t-1)+2(2t+1)] = 6(3t-1)^3(2t+1)^{-4}(t+3)$$

31. $y = \left(x+\frac{1}{x}\right)^5 \Rightarrow y' = 5\left(x+\frac{1}{x}\right)^4\frac{d}{dx}\left(x+\frac{1}{x}\right) = 5\left(x+\frac{1}{x}\right)^4\left(1-\frac{1}{x^2}\right)$

Another form of the answer is $y' = \frac{5(x^2+1)^4(x^2-1)}{x^6}$.

33. $f(t) = 2^{t^3} \Rightarrow f'(t) = 2^{t^3}(\ln 2)\frac{d}{dt}(t^3) = 3(\ln 2)t^2 2^{t^3}$

37. $f(z) = e^{z/(z-1)} \Rightarrow f'(t) = e^{z/(z-1)}\frac{d}{dz}\left(\frac{z}{z-1}\right) = e^{z/(z-1)}\frac{(z-1)(1)-z(1)}{(z-1)^2} = \frac{-e^{z/(z-1)}}{(z-1)^2}$

39. $J(\theta) = \tan^2(n\theta) = [\tan(n\theta)]^2 \Rightarrow$

$$J'(\theta) = 2\tan(n\theta)\frac{d}{d\theta}\tan(n\theta) = 2\tan(n\theta)\sec^2(n\theta)\cdot n = 2\tan(n\theta)\sec^2(n\theta)$$

41. $F(t) = \frac{t^2}{\sqrt{t^3+1}} \Rightarrow \quad F'(t) = \frac{(t^3+1)^{1/2}(2t) - t^2\cdot\frac{1}{2}(t^3+1)^{-1/2}(3t^2)}{\left(\sqrt{t^3+1}\right)^2}$

$$= \frac{t(t^3+1)^{-1/2}[2(t^3+1)-\frac{3}{2}t^3]}{(t^3+1)} = \frac{t\left(\frac{1}{2}t^2+2\right)}{(t^3+1)^{3/2}} = \frac{t(t^3+4)}{2(t^3+1)^{3/2}}$$

61. $y=\sqrt{1+x^3}=(1+x^3)^{1/2} \Rightarrow y'=\frac{1}{2}(1+x^3)^{-1/2}\cdot 3x^2=\dfrac{3x^2}{2(1+x^3)^{1/2}}$. At $(2,3)$, $y'=\dfrac{3\cdot 4}{2\sqrt{9}}=2$, and an equation of the tangent line is $y=2(x-2)+3$ or $y=2x-1$.

63. $y=xe^{-x^2} \Rightarrow y'=xe^{-x^2}(-2x)+e^{-x^2}(1)=e^{-x^2}(-2x^2+1)$. At $(0,0)$, $y'=e^0(1)=1$, and an equation of the tangent line is $y=1(x-0)+0$ or $y=x$.

68. For the tangent line to be horizontal $f'(x)=0$. $f(x)=2\sin x+\sin^2 x \Rightarrow$ $f'(x)2\cos x+2\sin x\cos x=0 \Leftrightarrow 2\cos x(1+\sin x)=0 \Leftrightarrow \cos x=0$, or $\sin x=-1$, so $x=\frac{\pi}{2}+2n\pi$ or $\frac{3\pi}{2}+2n\pi$, where n is any integer. Now $f\left(\frac{\pi}{2}\right)=3$ and $f\left(\frac{3\pi}{2}\right)=-1$, so the points on the curve with a horizontal tangent are $\left(\frac{\pi}{2}+2n\pi,3\right)$ and $\left(\frac{3\pi}{2}+2n\pi,-1\right)$, where n is any integer.

69. $y=\sqrt{1+2x} \Rightarrow y'=\frac{1}{2}(1+2x)^{-1/2}\cdot 2=\dfrac{1}{\sqrt{1+2x}}$. The line $6x+2y=1$ $\left(\text{or } y=-3x+\frac{1}{2}\right)$ has slope –3, so the tangent line perpendicular to it must have slope $\frac{1}{3}$. Thus $\dfrac{1}{3}=\dfrac{1}{\sqrt{1+2x}} \Leftrightarrow$ $\dfrac{1}{3}=\dfrac{1}{\sqrt{1+2x}} \Leftrightarrow \sqrt{1+2x}=3 \Rightarrow 1+2x=9 \Leftrightarrow 2x=8 \Leftrightarrow x=4$. When $x=4$, $y=\sqrt{1+2(4)}=3$, so the point is $(4,3)$.

70. $F(x)=f(g(x)) \Rightarrow F'(x)=f'(g(x))\cdot g'(x)$, so $F'(5)=f'(g(5))\cdot g'(5)=f'(-2)\cdot 6=4\cdot 6=24$.

71. $h(x)=\sqrt{4+3f(x)} \Rightarrow h'(x)=\frac{1}{2}(4+3f(x))^{-1/2}\cdot 3f'(x)$, so $h'(1)=\frac{1}{2}(4+3f(1))^{-1/2}\cdot 3f'(1)=\frac{1}{2}(4+3\cdot 7)^{-1/2}\cdot 3\cdot 4=\dfrac{6}{\sqrt{25}}=\dfrac{6}{5}$.

72. (a) $h(x)=f(g(x)) \Rightarrow h'(x)=f'(g(x))\cdot g'(x)$, so $h'(1)=f'(g(1))\cdot g'(1)=f'(2)\cdot 6=5\cdot 6=30$.

(b) $H(x)=g(f(x)) \Rightarrow H'(x)=g'(f(x))\cdot f'(x)$, so $H'(1)=g'(f(1))\cdot f'(1)=g'(3)\cdot 4=9\cdot 4=36$.

73. (a) $F(x)=f(f(x)) \Rightarrow F'(x)=f'(f(x))\cdot f'(x)$, so $F'(2)=f'(f(2))\cdot f'(2)=f'(1)\cdot 5=4\cdot 5=20$.

(b) $G(x)=g(g(x)) \Rightarrow G'(x)=g'(g(x))\cdot g'(x)$, so $G'(3)=g'(g(3))\cdot g'(3)=g'(2)\cdot 9=7\cdot 9=63$.

74. (a) $f(x)=p\left(\sqrt{q(x)}\right) \Rightarrow f'(x)=p'\left(\sqrt{q(x)}\right)\cdot\frac{1}{2}(q(x))^{-1/2}\cdot q'(x) \Rightarrow$

$f'(5)=p'\left(\sqrt{q(5)}\right)\cdot\frac{1}{2}(q(5))^{-1/2}\cdot q'(5)=p'\left(\sqrt{16}\right)\cdot\dfrac{1}{2\sqrt{16}}\cdot 7=p'(4)\cdot\dfrac{7}{8}=8\cdot\dfrac{7}{8}=7$

(b) $h(x)=\dfrac{q(x)}{x} \Rightarrow h'(x)=\dfrac{xq'(x)-q(x)}{x^2} \Rightarrow h'(4)=\dfrac{4\cdot q'(4)-q(4)}{4^2}=\dfrac{4\cdot 2-4}{16}=\dfrac{4}{16}=\dfrac{1}{4}$

75. (a) $h(x)=\dfrac{f(x)}{g(x)} \Rightarrow h'(x)=\dfrac{g(x)\cdot f'(x)-f(x)g'(x)}{(g(x))^2} \Rightarrow$

$h'(2)=\dfrac{g(2)\cdot f'(2)-f(2)g'(2)}{(g(2))^2}=\dfrac{5(1)-(-3)(-2)}{5^2}=\dfrac{5-6}{25}=-\dfrac{1}{25}$

(b) $h(x)=f(g(x)) \Rightarrow h'(x)=f'(g(x))\cdot g'(x) \Rightarrow h'(2)=f'(g(2))\cdot g'(2)=f'(5)\cdot(5)=7\cdot=35$

(c) $h(x)=\sqrt{f(x)} \Rightarrow h'(x)=\dfrac{1}{2\sqrt{f(x)}}f'(x) \Rightarrow h'(5)=\dfrac{f'(5)}{2\sqrt{f(5)}}=\dfrac{7}{2\sqrt{4}}=\dfrac{7}{2\cdot 2}=\dfrac{7}{4}$

76. $D(x)=\dfrac{[f(x)]^2}{x} \Rightarrow D'(x)=\dfrac{x\cdot[2f(x)f'(x)]-[f(x)]^2\cdot 1}{x^2} \Rightarrow$

$D'(4)=\dfrac{4\cdot[2f(4)f'(4)]-[f(4)]^2}{4^2}=\dfrac{4\cdot[2\cdot 6(1/6)]-6^2}{16}=\dfrac{8-36}{16}=-\dfrac{28}{16}=-\dfrac{7}{4}$. This is option (**B**).

77. The particle changes direction from left to right when the velocity, $v(t)$, changes from negative to positive. $s(t)=\cos t-\cos^2 t \Rightarrow v(t)=s'(t)=(-\sin t)-2\cos t(-\sin t)=\sin t(2\cos t-1)$. $v(t)=0 \Rightarrow \sin t(2\cos t-1)=0 \Rightarrow \sin t=0 \Rightarrow t=0$, or $t=\pi$, or $(2\cos t-1)=0 \Rightarrow t=\frac{\pi}{3}$. The velocity is negative for $\frac{\pi}{3}<t<\pi$, so the particle changes direction from left to right when (A) $t=\pi$.

78. The particle is at rest when its velocity, $v(t)=\dfrac{3\sin\left(\frac{\pi t}{2}\right)}{t}$ is 0. That occurs when $3\sin\left(\dfrac{\pi t}{2}\right)=0 \Rightarrow$

$\sin\left(\dfrac{\pi t}{2}\right)=0 \Rightarrow \dfrac{\pi t}{2}=k\pi \Rightarrow t=2k$, k an integer. Thus the particle is at rest for (**C**) $t=4$.

82. $f(x)=\sqrt{6x^2+3} \Rightarrow f'(x)=\dfrac{1}{2\sqrt{(6x^2+3)}}\cdot 12x$. If the point of tangency is $P(a, f(a))$, then the slope of the tangent line at P is $f'(x)=\dfrac{6a}{\sqrt{(6a^2+3)}}=2$, since the slope of the tangent line $y=2x+k$ is 2.

Now $\dfrac{6a}{\sqrt{(6a^2+3)}}=2 \Rightarrow 36a^2=2^2(6a^2+3) \Rightarrow 9a^2=6a^2+3 \Rightarrow 3a^2=3 \Rightarrow a^2=1 \Rightarrow a=\pm 1$.

Therefore, the y-coordinate of P is $f(\pm 1)=\sqrt{6(1)^2+3}=\sqrt{9}=3$, which is option (**B**).

83. $f(x)=\sin\left(\pi\sqrt{x^2+3}\right) \Rightarrow f'(x)=\cos\left(\pi\sqrt{x^2+3}\right)\pi\dfrac{2x}{2\sqrt{x^2+3}}=\cos\left(\pi\sqrt{x^2+3}\right)\dfrac{\pi x}{\sqrt{x^2+3}}$. The slope of the tangent line at the point where $x=1$ is $f'(1)=\cos\left(\pi\sqrt{1^2+3}\right)\dfrac{\pi(1)}{\sqrt{1^2+3}}=\cos(2\pi)\cdot\dfrac{\pi}{2}=\dfrac{\pi}{2}$, which is option (**C**).

84. $y=g(\sec^2 x) \Rightarrow \dfrac{dy}{dx}=g'(\sec^2 x)\cdot 2\sec x\cdot\sec\tan x=2\sqrt{2\sec^4 x+1}\sec^2 x\tan x \Rightarrow$

$y'\left(\frac{\pi}{4}\right)=2\sqrt{2\sec^4\left(\frac{\pi}{4}\right)+1}\sec^2\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right)=\left[2\sqrt{2\left(\sqrt{2}\right)^4+1}\right]\cdot\left(\sqrt{2}\right)^2(1)=2\sqrt{8+1}\cdot 2=4\sqrt{9}=4\cdot 3=12$.

This is option (**D**).

85. $h(x)=x^2 f(x)g(x) \Rightarrow h'(x)=x^2[f(x)g(x)]'+2x\cdot f(x)g(x)$

$=x^2[f(x)g'(x)+f'(x)g(x)]+2x\cdot f(x)g(x) \Rightarrow$

$h'(1)=1^2[f(1)g'(1)+f'(1)g(1)]+2(1)\cdot f(1)g(1)=2(0)+1(-1)+2(2)(0)=-1$.

87. (a) $u(x) = f(g(x)) \Rightarrow u'(1) = f'(g(1)) \cdot g(1) = f'(3) \cdot 3$. But $f'(3) =$ the slope of the graph of f at $x = 3$, which is $\frac{3-4}{6-2} = -\frac{1}{4}$. Therefore, $u'(1) = -\frac{1}{4} \cdot 3 = -\frac{3}{4}$.

(b) $v(x) = g(f(x)) \Rightarrow v'(1) = g'(f(1)) \cdot f'(1) = g'(2) \cdot f'(1)$. But $g'(2)$ does not exist because the graph of g has a cusp at $x = 2$. Therefore $v'(1)$ does not exist.

(c) $w(x) = g(g(x)) \Rightarrow w'(1) = g'(g(1)) \cdot g'(1) = g'(3) \cdot g'(1)$. Now $g'(1) = \frac{0-3}{2-1} = -3$, and $g'(3) = \frac{2-0}{5-2} = \frac{2}{3}$. So $w'(1) = g'(3) \cdot g'(1) = \frac{2}{3}(-3) = -2$.