<u>3.7 and 3.8</u>

p. 248: 41-51 odd

41. $y=\tan^{-1}(x^2) \Rightarrow y'=\frac{1}{1+(x^2)^2}\cdot\frac{d}{dx}(x^2)=\frac{1}{1+x^4}\cdot 2x=\frac{2x}{1+x^4}$

43. $g(x)=\arccos\sqrt{x} \Rightarrow g'(x)=-\frac{1}{\sqrt{1-(\sqrt{x})^2}}\frac{d}{dx}\sqrt{x}=-\frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}=-\frac{1}{2\sqrt{x}\sqrt{1-x}}$

45. $y=\tan^{-1}\left(x-\sqrt{x^2+1}\right) \Rightarrow$

$$y'=\frac{1}{1+\left(x-\sqrt{x^2+1}\right)^2}\left(1-\frac{x}{\sqrt{x^2+1}}\right)=\frac{1}{1+x^2-2x\sqrt{x^2+1}+x^2+1}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\right)$$

$$=\frac{\sqrt{x^2+1}-x}{2\left(1+x^2-x\sqrt{x^2+1}\right)\sqrt{x^2+1}}=\frac{\sqrt{x^2+1}-x}{2\left[\sqrt{x^2+1}(1+x^2)-x(x^2+1)\right]}=\frac{\sqrt{x^2+1}-x}{2\left[(1+x^2)\left(\sqrt{x^2+1}-x\right)\right]}$$

$$=\frac{1}{2(1+x^2)}$$

47. $R(t)=\arcsin(1/t) \Rightarrow R'(t)=\frac{1}{\sqrt{1-(1/t)^2}}\frac{d}{dt}\frac{1}{t}=\frac{1}{\sqrt{1-(1/t)^2}}\left(-\frac{1}{t^2}\right)=-\frac{1}{\sqrt{1-(1/t)^2}}\frac{1}{\sqrt{t^4}}=-\frac{1}{\sqrt{t^4-t^2}}$

$$=-\frac{1}{\sqrt{t^2(t^2-1)}}=-\frac{1}{|t|\sqrt{t^2-1}}$$

49. $y=\cos^{-1}(\sin^{-1}t) \Rightarrow y'=-\frac{1}{\sqrt{1-(\sin^{-1}t)^2}}\cdot\frac{d}{dt}(\sin^{-1}t)=-\frac{1}{\sqrt{1-(\sin^{-1}t)^2}}\cdot\frac{1}{\sqrt{1-t^2}}$

51. $y=\arctan\sqrt{\frac{1-x}{1+x}}=\arctan\left(\frac{1-x}{1+x}\right)^{1/2} \Rightarrow$

$$y'=\frac{1}{1+\left(\sqrt{\frac{1-x}{1+x}}\right)^2}\cdot\frac{d}{dx}\left(\frac{1-x}{1+x}\right)^{1/2}=\frac{1}{1+\frac{1-x}{1+x}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)^{-1/2}\cdot\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}$$

$$=\frac{1}{\frac{1+x}{1+x}+\frac{1-x}{1+x}}\cdot\frac{1}{2}\left(\frac{1-x}{1+x}\right)\cdot\frac{-2}{(1+x)^2}=\frac{1+x}{2}\cdot\frac{1}{2}\cdot\frac{(1+x)^{1/2}}{(1-x)^{1/2}}\cdot\frac{-2}{(1+x)^2}$$

$$=\frac{-1}{2(1-x)^{1/2}(1+x)^{1/2}}=\frac{-1}{2\sqrt{1-x^2}}$$

p. 259: 7-31 odd, 44, 56-59, 62-66, 68-71

7. $f(x)=\frac{\ln x}{x^2} \Rightarrow f'(x)=\frac{x^2\cdot\frac{1}{x}-\ln x(2x)}{\left(x^2\right)^2}=\frac{x-2x\ln x}{x^4}=\frac{x(1-2\ln x)}{x^4}=\frac{1-2\ln x}{x^3}$

9. $f(x)=\ln(\sin^2 x)=\ln(\sin x)^2=2\ln|\sin x| \Rightarrow f'(x)=2\cdot\frac{1}{\sin x}\cdot\cos x=2\cot x$

11. $y=\frac{1}{\ln x}=(\ln x)^{-1} \Rightarrow y'=-1(\ln x)^{-2}\cdot\frac{1}{x}=-\frac{1}{x(\ln x)^2}$

13. $f(x)=\log_{10}\sqrt{x} \Rightarrow f'(x)=\frac{1}{\sqrt{x}\ln 10}\cdot\frac{d}{dx}\sqrt{x}=\frac{1}{\sqrt{x}\ln 10}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2(\ln 10)x}$

Or: $f(x)=\log_{10}\sqrt{x}=\log_{10}x^{1/2}=\frac{1}{2}\log_{10}x \Rightarrow f'(x)=\frac{1}{2}\cdot\frac{1}{x\ln 10}=\frac{1}{2(\ln 10)x}$

15. $g(t)=\sqrt{1+\ln t} \Rightarrow g'(t)=\frac{1}{2}(1+\ln t)^{-1/2}\frac{d}{dt}(1+\ln t)=\frac{1}{2\sqrt{1+\ln t}}\cdot\frac{1}{t}=\frac{1}{2t\sqrt{1+\ln t}}$

17. $h(x)=\ln\left(x+\sqrt{x^2-1}\right) \Rightarrow h'(x)=\frac{1}{x+\sqrt{x^2-1}}\left(1+\frac{x}{\sqrt{x^2-1}}\right)=\frac{1}{x+\sqrt{x^2-1}}\cdot\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}=\frac{1}{\sqrt{x^2-1}}$

19. $P(v)=\frac{\ln v}{1-v} \Rightarrow P'(v)=\frac{(1-v)(1/v)-(\ln v)(-1)}{(1-v)^2}\cdot\frac{v}{v}=\frac{1-v+v\ln v}{v(1-v)^2}$

21. $y=\ln|1+t-t^3| \Rightarrow y'=\frac{1}{1+t-t^3}\cdot\frac{d}{dt}(1+t-t^3)=\frac{1-3t^2}{1+t-t^3}$

23. $y=\ln(\csc x-\cot x) \Rightarrow$

$$y'=\frac{1}{\csc x-\cot x}\cdot\frac{d}{dx}(\csc x-\cot x)=\frac{1}{\csc x-\cot x}(-\csc x\cot x+\csc^2 x)=\frac{\csc x(\csc x-\cot x)}{\csc x-\cot x}=\csc x$$

25. $H(z)=\ln\sqrt{\frac{a^2-z^2}{a^2+z^2}}=\ln\left(\frac{a^2-z^2}{a^2+z^2}\right)^{1/2}=\frac{1}{2}\ln\left(\frac{a^2-z^2}{a^2+z^2}\right)=\frac{1}{2}\ln(a^2-z^2)-\frac{1}{2}\ln(a^2+z^2) \Rightarrow$

$$H'(z)=\frac{1}{2}\cdot\frac{1}{a^2-z^2}\cdot(-2z)-\frac{1}{2}\cdot\frac{1}{a^2+z^2}\cdot(2z)=\frac{z}{z^2-a^2}-\frac{z}{z^2+a^2}=\frac{z(z^2+a^2)-z(z^2-a^2)}{(z^2-a^2)(z^2+a^2)}$$

$$=\frac{z^3+za^2-z^3+za^2)}{(z^2-a^2)(z^2+a^2)}=\frac{2a^2z}{z^4-a^4}$$

27. $y=\log_2(x\log_5 x) \Rightarrow$

$$y'=\frac{1}{(x\log_5 x)(\ln 2)}\frac{d}{dx}(x\log_5 x)=\frac{1}{(x\log_5 x)(\ln 2)}\left(x\cdot\frac{1}{x\ln 5}+\log_5 x\right)=\frac{1}{(x\log_5 x)(\ln 2)(\ln 5)}+\frac{1}{x(\ln 2)}$$

Note that $\log_5 x(\ln 5)=\frac{\ln x}{\ln 5}(\ln 5)=\ln x$ by the Change of Base theorem.

Thus, $y'=\frac{1}{x\ln x\ln 2}+\frac{1}{x(\ln 2)}=\frac{1+\ln x}{x\ln x\ln 2}$

29. $f(x)=3^x x^3 \Rightarrow \ln(f(x))=\ln\left(3^x x^3\right)=\ln 3^x+\ln x^3=x\ln 3+3\ln x \Rightarrow$

$$\frac{f'(x)}{f(x)}=\ln 3+3\cdot\frac{1}{x} \Rightarrow f'(x)=f(x)\left(\ln 3+\frac{3}{x}\right)=3^x x^3\left(\ln 3+\frac{3}{x}\right)=3^x x^2\left(x\ln 3+3\right)$$

31. $g(x)=e^{2x}\ln\sqrt{x^2+4}=e^{2x}\cdot\frac{1}{2}\ln(x^2+4) \Rightarrow \quad g'(x)=e^{2x}\cdot\frac{1}{2}\ln(x^2+4)$

$$=\frac{1}{2}e^{2x}\frac{2x}{x^2+4}+\frac{1}{2}\ln(x^2+4)e^{2x}\cdot 2=\frac{xe^{2x}}{x^2+4}+\ln(x^2+4)e^{2x}=e^{2x}\left(\frac{x}{x^2+4}+\ln(x^2+4)\right)$$

44. If $f(x)=\log_3(x^2+1)$, then $f'(x)=\dfrac{2x}{\ln 3(x^2+1)}$, option (**C**).

56. $f(x)=\ln(x+\ln x) \Rightarrow f'(x)=\dfrac{1}{x+\ln x}\dfrac{d}{dx}(x+\ln x)=\dfrac{1}{x+\ln x}\left(1+\dfrac{1}{x}\right)$

Substitute 1 for x to get $f'(1)=\dfrac{1}{1+\ln 1}\left(1+\dfrac{1}{1}\right)=\dfrac{1}{1+0}(1+1)=1\cdot 2=2.$

57. $f(x)=\cos(\ln x^2) \Rightarrow f'(x)=-\sin(\ln x^2)\dfrac{d}{dx}\ln x^2=-\sin(\ln x^2)\dfrac{1}{x^2}2x=-\dfrac{2\sin(\ln x^2)}{x}$

Substitute 1 for x to get $f'(1)=-\dfrac{2\sin(\ln 1^2)}{1}=-2\sin 0=0.$

58. $y=\ln(x^2-3x+1) \Rightarrow y'=\dfrac{1}{x^2-3x+1}\cdot(2x-3) \Rightarrow y'(3)=\frac{1}{1}\cdot 3=3,$ so an equation of the tangent line at $(3,0)$ is $y=3(x-3)+0$ or $y=3x-9$.

59. $y=x^2\ln x \Rightarrow y'=x^2\cdot\dfrac{1}{x}+(\ln x)(2x) \Rightarrow y'(1)=1+0=1,$ so an equation of the tangent line at $(3,0)$ is $y=1(x-1)+0$ or $y=x-1$.

62. $y=\dfrac{\ln x}{x} \Rightarrow y'=\dfrac{x(1/x)-\ln x}{x^2}=\dfrac{1-\ln x}{x^2}$

$y'(1)=\dfrac{1-0}{1^2}=1$ and $y'(e)=\dfrac{1-1}{e^2}=0 \Rightarrow$ equations of tangent lines are

$y=1(x-1)+0$ or $y=x-1,$ and $y=0(x-e)+1/e$ or $y=1/e.$

63. $f(x)=\dfrac{1}{x\ln x} \Rightarrow f'(x)=\dfrac{x\ln x\cdot(0)-1\cdot\left[x\dfrac{1}{x}+\ln x\cdot(1)\right]}{(x\ln x)^2}=-\dfrac{1+\ln x}{(x\ln x)^2}.$

The tangent line is horizontal when $f'(x)=-\dfrac{1+\ln x}{(x\ln x)^2}=0 \Rightarrow (1+\ln x)=0 \Rightarrow -1=\ln x \Rightarrow x=\dfrac{1}{e}.$

$$g(x)=\frac{(\ln x)^2}{x} \Rightarrow g'(x)=\frac{x\cdot\left(2\ln x\dfrac{1}{x}\right)-(\ln x)^2\cdot 1}{x^2}=\frac{2\ln x-(\ln x)^2}{x^2}=\frac{\ln x(2-\ln x)}{x^2}$$

The tangent line to the graph of g is horizontal when

$g'(x)=0 \Rightarrow \dfrac{\ln x(2-\ln x)}{x^2}=0 \Rightarrow \ln x(2-\ln x)=0 \Rightarrow x=e$ or $x=e^2$. When $x=e,$ $g(e)=0,$ but this is not one of the given choices. So the y-coordinate of a point on the graph of g for which there is a horizontal tangent line is $g(e^2)=\dfrac{4}{e^2}$, (**A**).

64. $y=\dfrac{\ln x}{x}\Rightarrow y'=\dfrac{x(1/x)-\ln x}{x^2}=\dfrac{1-\ln x}{x^2}\Rightarrow$

the tangent line is horizontal when $1-\ln x=0\Rightarrow 1=\ln x\Rightarrow x=e$. (**A**).

65. $f(x)=cx+\ln(\cos x)\Rightarrow f'(x)=c+-\dfrac{\sin x}{\cos x}=c-\tan x$. Then $6=f'(\frac{\pi}{4})=c-\tan\frac{\pi}{4}=c-1\Rightarrow 7=c$.

66. $f(x)=\log_b(3x^2)\Rightarrow f'(x)=\dfrac{6x}{3x^2}\cdot\dfrac{1}{\ln b}=\dfrac{2}{x\ln b}$. Then $3=f'(1)=\dfrac{2}{\ln b}\Rightarrow \ln b=\dfrac{2}{3}\Rightarrow b=e^{2/3}$.

68. $s(t)=te^{-t^2}\Rightarrow v(t)=s'(t)=t\left(e^{-t^2}\right)\cdot(-2t)+e^{-t^2}=e^{-t^2}(1-2t^2)$. The particle moving to the right from position 0, and then comes to rest when $v(t)=e^{-t^2}(1-2t^2)=0$. The particle moves to the left when $(1-2t^2)<0\Rightarrow 1<2t^2\Rightarrow \dfrac{1}{2}<t^2\Rightarrow t>\dfrac{\sqrt{2}}{2}$ [since $t\ge 0$]. At this time, the position of the particle is $s\left(\dfrac{\sqrt{2}}{2}\right)=\dfrac{\sqrt{2}}{2}e^{-1/2}=\dfrac{1}{\sqrt{2e}}$. This is option (**C**).

69. $s(t)=\ln(1+t^2)\Rightarrow v(t)=s'(t)=\dfrac{2t}{1+t^2}\Rightarrow a(t)=v'(t)=\dfrac{(1+t^2)(2)-2t(2t)}{\left(1+t^2\right)^2}=\dfrac{2-2t^2}{\left(1+t^2\right)^2}=\dfrac{2(1-t^2)}{\left(1+t^2\right)^2}$

The acceleration is zero when $\dfrac{2(1-t^2)}{\left(1+t^2\right)^2}=0\Rightarrow(1-t^2)=0\Rightarrow t=1$ [since $t\ge 0$]. At this time, the particle's position is $s(1)=\ln(1+1^2)=\ln 2$, option (**C**).

70. $s(t)=\dfrac{1}{t}+\ln t\Rightarrow v(t)=s'(t)=-\dfrac{1}{t^2}+\dfrac{1}{t}$. The acceleration is $a(t)=v'(t)=\dfrac{2}{t^3}-\dfrac{1}{t^2}$ The acceleration is first equal to zero when $\dfrac{2}{t^3}-\dfrac{1}{t^2}=0\Rightarrow\dfrac{2-t}{t^3}=0\Rightarrow 2-t=0\Rightarrow t=2$. This is option (**A**).

71. $y=\ln(x^2+y^2)\Rightarrow y'=\dfrac{1}{x^2+y^2}\dfrac{d}{dx}(x^2+y^2)\Rightarrow y'=\dfrac{2x+2yy'}{x^2+y^2}\Rightarrow x^2y'+y^2y'=2x+2yy'\Rightarrow$

$x^2y'+y^2y'-2yy'=2x\Rightarrow(x^2+y^2-2y)y'=2x\Rightarrow y'=\dfrac{2x}{x^2+y^2-2y}$