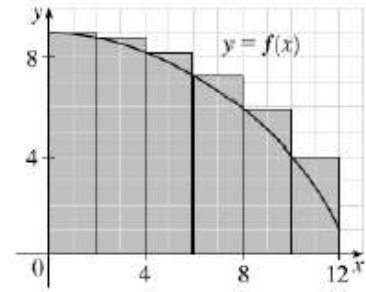


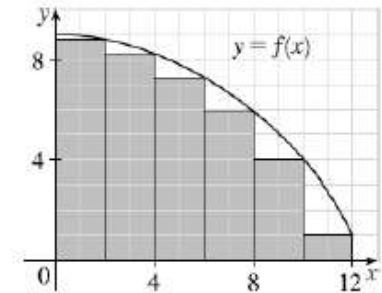
p. 411: 8-9, 11-12, 15-16, 21, 23, 32-33, 41-42

$$\begin{aligned}
 8. \quad (a) \quad (i) \quad L_6 &= \sum_{i=1}^6 f(x_{i-1})\Delta x \quad \left[ \Delta x = \frac{12-0}{6} = 2 \right] \\
 &= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)] \\
 &= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)] \\
 &\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1) = 2(43.3) = 86.6
 \end{aligned}$$

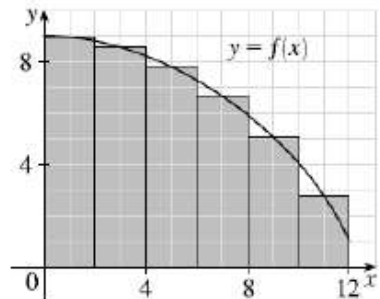


$$\begin{aligned}
 (ii) \quad R_6 &= L_6 + 2 \cdot (12) - 2 \cdot (0) \\
 &\approx 86.6 + 2(1) - 2(9) = 70.6
 \end{aligned}$$

[Add the area of the rightmost lower rectangle and subtract the area of the leftmost upper rectangle.]



$$\begin{aligned}
 (ii) \quad M_6 &= \sum_{i=1}^6 f(x_{i-1})\Delta x \\
 &= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)] \\
 &\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8) \\
 &= 2(39.7) = 79.4
 \end{aligned}$$



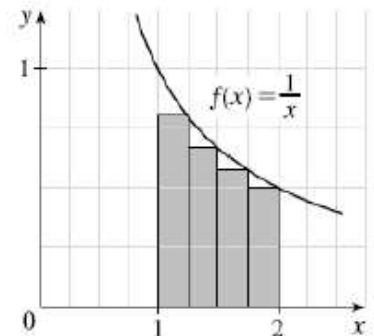
(b) Since  $f$  is *decreasing*, we obtain an *overestimate* by using *left* endpoints; that is,  $L_6$ .

(c) Since  $f$  is *decreasing*, we obtain an *underestimate* by using *right* endpoints; that is,  $R_6$ .

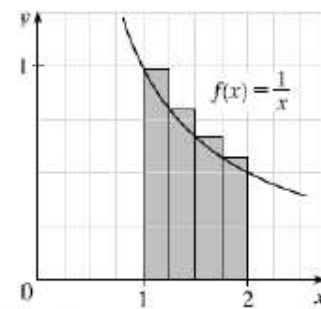
(d)  $M_6$  gives the best estimate because the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in  $L_6$  and  $R_6$ .

$$\begin{aligned}
 9. \quad (a) \quad R_4 &= \sum_{i=1}^4 f(x_i)\Delta x \quad \left[ \Delta x = \frac{2-1}{4} = \frac{1}{4} \right] = \left[ \sum_{i=1}^4 f(x_i) \right] \Delta x \\
 &= [f(x_1) + f(x_2) + f(x_3) + f(x_4)]\Delta x \\
 &= \left[ \frac{1}{5/4} + \frac{1}{6/4} + \frac{1}{7/4} + \frac{1}{8/4} \right] \frac{1}{4} = \left[ \frac{4}{5} + \frac{2}{3} + \frac{4}{7} + \frac{1}{2} \right] \frac{1}{4} \approx 0.6345
 \end{aligned}$$

Since  $f$  is *decreasing* on  $[1, 2]$ , an *underestimate* is obtained by using the *right* endpoint approximation,  $R_4$ .



$$\begin{aligned}
 \text{(b) } L_4 &= \sum_{i=1}^4 f(x_{i-1})\Delta x = \left[ \sum_{i=1}^4 f(x_{i-1}) \right] \Delta x \\
 &= [f(x_0) + f(x_1) + f(x_2) + f(x_3)]\Delta x \\
 &= \left[ \frac{1}{1} + \frac{1}{5/4} + \frac{1}{6/4} + \frac{1}{7/4} \right] \frac{1}{4} = \left[ 1 + \frac{4}{5} + \frac{2}{3} + \frac{4}{7} \right] \frac{1}{4} \approx 0.7595
 \end{aligned}$$



$L_4$  is an overestimate. Alternatively, we could just add the area of the leftmost upper rectangle and subtract the area of the rightmost lower rectangle, that is

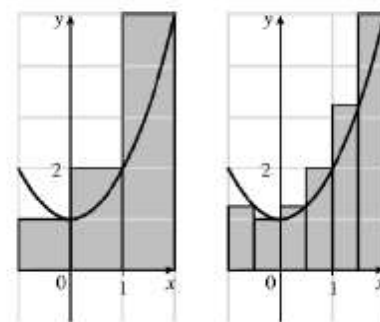
$$L_4 = R_4 + f(1) \cdot \frac{1}{4} - f(2) \cdot \frac{1}{4}.$$

11. (a)  $f(x) = 1 + x^2$  and  $\Delta x = \frac{2 - (-1)}{3} = 1 \Rightarrow$

$$R_3 = 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot 5 = 8.$$

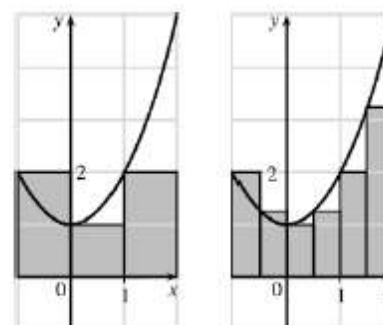
$$\Delta x = \frac{2 - (-1)}{6} = 0.5 \Rightarrow$$

$$\begin{aligned}
 R_6 &= 0.5 \cdot [f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)] \\
 &= 0.5 \cdot (1.25 + 1 + 1.25 + 2 + 3.25 + 5) \\
 &= 0.5(13.75) = 6.875
 \end{aligned}$$



(b)  $L_3 = 1 \cdot f(-1) + 1 \cdot f(0) + 1 \cdot f(1) = 1 \cdot 2 + 1 \cdot 1 + 1 \cdot 2 = 5$

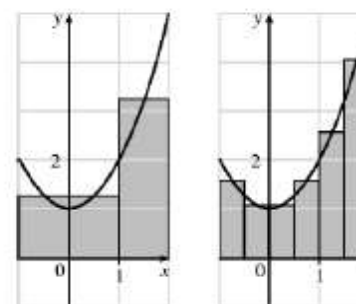
$$\begin{aligned}
 L_6 &= 0.5 \cdot [f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)] \\
 &= 0.5 \cdot (2 + 1.25 + 1 + 1.25 + 2 + 3.25) \\
 &= 0.5(10.75) = 5.375
 \end{aligned}$$



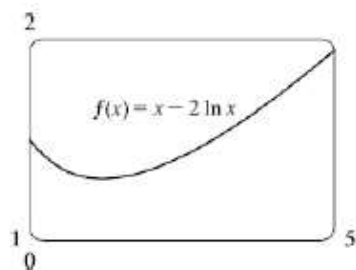
(c)  $M_3 = 1 \cdot f(-0.5) + 1 \cdot f(0.5) + 1 \cdot f(1.5)$   
 $= 1 \cdot 1.25 + 1 \cdot 1.25 + 1 \cdot 3.25 = 5.75$

$$\begin{aligned}
 M_6 &= 0.5 \cdot [f(-0.75) + f(-0.25) + f(0.25) \\
 &\quad + f(0.75) + f(1.25) + f(1.75)] \\
 &= 0.5 \cdot (1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625) \\
 &= 0.5(11.875) = 5.9375
 \end{aligned}$$

(d)  $M_6$  appears to be the best estimate.

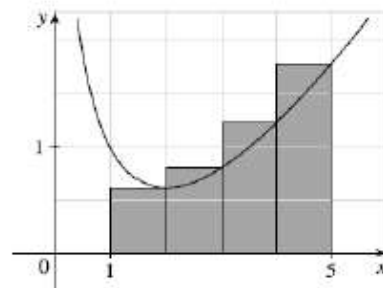


12. (a)

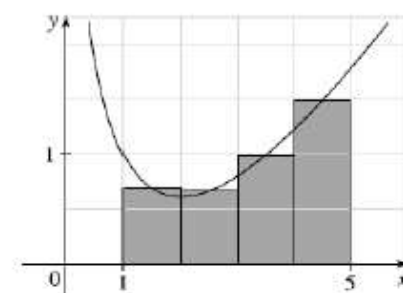


(b)  $f(x) = x - 2 \ln x$  and  $\Delta x = \frac{5-1}{4} = 1 \Rightarrow$

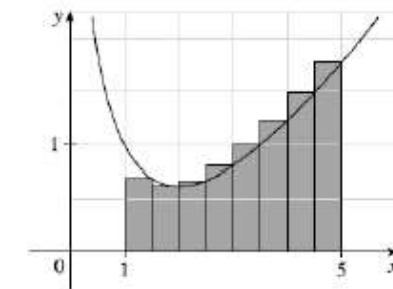
$$\begin{aligned} \text{(i) } R_4 &= 1 \cdot f(2) + 1 \cdot f(3) + 1 \cdot f(4) + 1 \cdot f(5) \\ &= (2 - 2 \ln 2) + (3 - 2 \ln 3) + (4 - 2 \ln 4) + (5 - 2 \ln 5) \\ &\approx 4.425 \end{aligned}$$



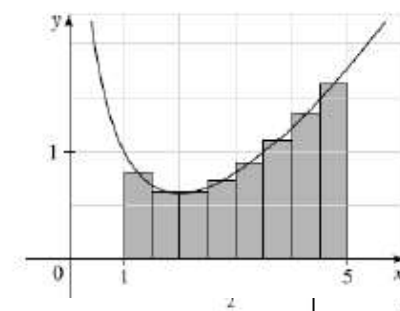
$$\begin{aligned} \text{(ii) } M_4 &= 1 \cdot f(1.5) + 1 \cdot f(2.5) + 1 \cdot f(3.5) + 1 \cdot f(4.5) \\ &= (1.5 - 2 \ln 1.5) + (2.5 - 2 \ln 2.5) \\ &\quad + (3.5 - 2 \ln 3.5) + (4.5 - 2 \ln 4.5) \\ &\approx 3.843 \end{aligned}$$



$$\begin{aligned} \text{(c) (i) } R_8 &= \frac{1}{2} [f(1.5) + f(2) + \dots + f(5)] \\ &= \frac{1}{2} [(1.5 - 2 \ln 1.5) + (2 - 2 \ln 2) + \dots + (5 - 2 \ln 5)] \\ &\approx 4.134 \end{aligned}$$



$$\begin{aligned} \text{(ii) } M_8 &= \frac{1}{2} [f(1.25) + f(1.75) + \dots + f(4.75)] \\ &= \frac{1}{2} [(1.25 - 2 \ln 1.25) + (1.75 - 2 \ln 1.75) + \dots \\ &\quad + (4.75 - 2 \ln 4.75)] \\ &\approx 3.889 \end{aligned}$$



15. If  $f$  is nonnegative and continuous on  $[0, 2]$ , then the right

Riemann sum  $R_4$  is represented by (D)  $f(0.5) \cdot \frac{1}{2} + f(2) \cdot \frac{1}{2} + f(1.5) \cdot \frac{1}{2} + f(2) \cdot \frac{1}{2}$

16. If the area under the graph of  $y = \sin x$  on the interval  $[0, \pi]$  is estimated by  $L_4$ , then that estimate is

$$L_4 = \left[ \sin(0) + \sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{4}\right) + \sin\left(\frac{3\pi}{4}\right) \right] \frac{\pi}{4} = \frac{\pi(1 + \sqrt{2})}{4}, \text{ which is option (B).}$$

$$21. \text{ Area of } R \approx T_3 = \frac{1}{2}(f(6) + f(12)) \cdot 6 + \frac{1}{2}(f(12) + f(20)) \cdot 8 + \frac{1}{2}(f(20) + f(24)) \cdot 4 \\ = 3(5 + 7) + 4(7 + 4) + 2(4 + 7) = 3(12) + 4(11) + 2(11) = 36 + 44 + 22 = 102$$

23. We are given subintervals so we will have 3 rectangles of width 12, and 3 midpoints. Then  $M_3 = 12 \cdot [f(8) + f(20) + f(32)] = 12 \cdot (9 + 20 + 10) = 12 \cdot 39 = 468$ , option (B).

32. (B)

33. If  $f$  is decreasing, continuous, positive and concave down on  $[a, b]$ , then (B)  $L_n \geq M_n \geq R_n \geq R_n$ .

41. If  $f(x) = x^2$  on  $[6, 4]$  using a regular partition with 100 subintervals, then  $\Delta x = \frac{6-4}{100} = \frac{2}{100} = \frac{1}{50}$ . Then

$$R_{100} = L_{100} - f(x_0) \cdot \Delta x + f(x_n) \cdot \Delta x = L_{100} - 4^2 \cdot \frac{1}{50} + 6^2 \cdot \frac{1}{50} = 0.4.$$

42. The trapezoidal sum associated with the given function and table is

$$61 = \frac{1}{2}(3[f(2) + f(5)] + 3[f(5) + f(8)] + 2[f(8) + f(10)]) = \frac{1}{2}(3[7 + 9] + 3[9 + k] + 2[k + 11]) \\ = \frac{1}{2}(48 + 27 + 22 + 5k) = 97 + 5k. \text{ So } 122 = 97 + 5k \Rightarrow 25 = 5k \Rightarrow k = 5. \text{ This is option (B).}$$