6.4

p. 515: 11-19, 21-34

11. $s(t) = te^{-t} \Rightarrow v(t) = s'(t) = t(-e^{-t}) + e^{-t} \cdot 1 = e^{-t}(1-t) \Rightarrow \; v(0) = e^{0}(1-0) = 1.$

12. The particle's position at time 2 is $s(0) + \int_0^2 v(t)\,dt = 5 + \int_0^2 \left(6t^2 + 2t\right) dt = 5 + \left[2t^3 + t^2\right]_0^2$

$= 5 + \left(16 + 4\right) - 0 = 25$

13. $s(t) = 4t^3 - t^2 - 2t + 1 \Rightarrow v(t) = s'(t) = 12t^2 - 2t - 2.$

$v(t) = 0 \Leftrightarrow 12t^2 - 2t - 2 = 0 \Leftrightarrow 2(3t+1)(2t-1) = 0 \Leftrightarrow t = \frac{1}{2}$, since $t \ge 0$.

$a(t) = v'(t) = 24t - 2 \Rightarrow a\left(\frac{1}{2}\right) = 24\left(\frac{1}{2}\right) - 2 = 10.$

14. $s(t) = 4t^3 + 8t^2 - 16t + 5 \Rightarrow v(t) = s'(t) = 12t^2 + 16t - 16$, and $v(0.5) = 12(0.5)^2 + 16(0.5) - 16 = -5 < 0.$
$a(t) = v'(t) = 24t + 16 \Rightarrow a(0.5) = 24(0.5) + 16 = 28 > 0.$ Since the velocity and acceleration have opposite signs at $t = 5$, the particle's speed is decreasing.

15. $a(t) = v'(t) = t\sqrt{t^3+1} \Rightarrow v(4) = v(0) + \int_0^2 t\sqrt{t^3+1}\,dt \overset{\text{CAS}}{\approx} 4 + 3.916 = 7.916$

16. $v(t) = s'(t) = e^{t^2/6} \Rightarrow s(4) = s(1) + \int_1^4 e^{t^2/6}\,dt \overset{\text{CAS}}{\approx} 4 + 4.450 = 8.850.$

17. For $0 < t < 8$, $v(t) = 0$ when $t = 2$, and 6. $v(t) < 0$ for $2 < t < 6$, and $v(t) > 0$ for $0 < t < 2$ and $6 < t < 8$. Thus $v(t)$ reaches a local maximum at $t = 2$, and this would be when the particle is farthest to the right.

18. $s(t) = 2\pi t + \cos(2\pi t) \Rightarrow v(t) = s'(t) = 2\pi - 2\pi\sin(2\pi t) \Rightarrow a(t) = v'(t) = -4\pi^2\cos(2\pi t)$ To find the maximum velocity, we first find any critical points in the interval: $0 = v'(t) = -4\pi^2\cos(2\pi t)$ $\Leftrightarrow \cos(2\pi t) = 0 \Leftrightarrow 2\pi t = \frac{1}{2}\pi,\; 2\pi t = \frac{3}{2}\pi$, or $2\pi t = \frac{5}{2}\pi.$ $2\pi t = \frac{1}{2}\pi \Rightarrow t = \frac{1}{4},\; 2\pi t = \frac{3}{2}\pi \Rightarrow t = \frac{3}{4}$, and $2\pi t = \frac{5}{2}\pi \Rightarrow t = \frac{5}{4}.$ Now evaluate the velocity at the endpoints and critical points:

t	**velocity at time t**
0	2π
0.25	$2\pi - 2\pi\sin\left(\frac{\pi}{2}\right) \approx 0$
0.75	$2\pi - 2\pi\sin\left(\frac{3\pi}{2}\right) = 4\pi$
1.25	$2\pi - 2\pi\sin\left(\frac{5\pi}{2}\right) \approx 0$
1.5	$2\pi - 2\pi\sin\left(3\pi\right) = 2\pi$

The velocity is maximized at time $t = \frac{3}{4}$ when the velocity is $2\pi - 2\pi\sin\left(\frac{3\pi}{2}\right) \approx 4\pi.$

19. $s(t) = 25 + te^{-t/20} \Rightarrow v(t) = s'(t) = t\left(-\frac{1}{20}e^{-t/20}\right) + e^{-t/20} = e^{-t/20}\left(1 - \frac{t}{20}\right).$ Then
$a(t) = v'(t) = e^{-t/20}\left(-\frac{1}{20}\right) + \left(1 - \frac{t}{20}\right)e^{-t/20}\left(-\frac{1}{20}\right) = -\frac{1}{20}e^{-t/20}\left(1 + 1 - \frac{t}{20}\right) = -\frac{1}{20}e^{-t/20}\left(2 - \frac{t}{20}\right).$
$a(t) = 0 \Leftrightarrow 2 = \frac{t}{20} \Leftrightarrow 40 = t.$ $a(t) > 0 \Leftrightarrow t > 40$, and $a(t) < 0 \Leftrightarrow 0 < t < 40.$ Thus minimum velocity of the particle is $v(40) = e^{-40/20}\left(1 - \frac{40}{20}\right) = e^{-2}(-1) = -e^{-2}$ when $t = 40$.

21. $v(t) = \int a(t)\,dt = \int 12\cos(3t)\,dt = 4\sin(3t) + C.$ Then $v(0) = 2 = 4\sin(0) + C \Rightarrow C = 2.$ Thus, $v(t) = 4\sin(3t) + 2.$ The particle is at rest when $v(t) = 0 \Rightarrow 4\sin(3t) = -2 \Rightarrow \sin(3t) = -\frac{1}{2} \Rightarrow$ $3t = \frac{7\pi}{6} + 2\pi n$, or $3t = \frac{11\pi}{6} + 2\pi n$, n any integer. So in the interval $\left[0, \pi\right]$ the particle is at rest when $t = \frac{7\pi}{18}$ and $t = \frac{11\pi}{18}$.

22. $s(t)=t^3-2t^2-4t+5 \Rightarrow v(t)=3t^2-4t-4=(3t+2)(t-2)$. On $[0,5]$ $v(t)=0 \Leftrightarrow t=2$, and $v(t)<0 \Leftrightarrow 0<t<2$. S0 $|v(t)|=\begin{cases}-3t^2+4t+4 & \text{if } 0\le t\le 2\\ 3t^2-4t-4 & \text{if } 2<t\le 5\end{cases}$. Thus, the total distance traveled by the particle during the first 5 seconds is

$$\int_0^5|v(t)|\,dt=\int_0^2(-3t^2+4t+4)\,dt+\int_2^5(3t^2-4t-4)\,dt=\left[-t^3+2t^2+4t\right]_0^2+\left[t^3-2t^2-4t\right]_2^5$$
$$=(-8+8+8)-0+(125-50-20)-(-8-8-8)=8+63=71$$

Using technology, we find $\int_0^5\left|3t^2-4t-4\right|dt=71$.

23. (a) $s(t)=s(3)+\int_3^t v(x)\,dx=0+\int_3^t\left(3x^2-6x\right)dx=\left[x^3-3x^2\right]_3^t=t^3-3t^2-(27-27)=t^3-3t^2$.

(b) $s(t)=\int v(t)\,dt=\int\left(3t^2-6t\right)dt=t^3-3t^2+C$. $s(3)=0\Rightarrow(3)^3-3\cdot(3)^2+C=0\Rightarrow C=0$. So $s(t)=t^3-3t^2$.

(c) The two methods do yield the same position function (as they should).

24. The first particle's position at time t is $14+\int_2^t(8x+4)\,dx=14+\left[4x^2+4x\right]_2^t$
$=14+\left(4t^2+4t\right)-(16+8)=4t^2+4t-10$. The position of the second particle at time t is
$23+\int_2^t(4x+5)\,dx=23+\left[2x^2+5x\right]_2^t=23+\left(2t^2+5t\right)-(8+10)=2t^2+5t+5$. The particles are at the same position when $4t^2+4t-10=2t^2+5t+5\Rightarrow 2t^2-t-15=(2t+5)(t-3)=0$. Since $t\ge 0$, the particles have the same position at time $t=3$.

25. $a(t)=\dfrac{3t+1}{\sqrt{t^2+t+1}}\Rightarrow v(6)=v(0)+\displaystyle\int_0^6\frac{3t+1}{\sqrt{t^2+t+1}}dt\overset{\text{Calculator}}{=}7+10.537=18.537$, option (C).

26. (a) $s(6)=s(0)+\int_0^6\left(e^{t\sin t}-1\right)dt\overset{\text{Calculator}}{=}3+4.575=7.575$.

(b) $v(t)=e^{t\sin t}-1=0\Leftrightarrow e^{t\sin t}=1\Leftrightarrow t\sin t=\ln 1=0\Leftrightarrow t=0,\pi,2\pi$. $v(t)>0\Leftrightarrow 0<t<\pi$ and $2\pi<t<7$, so the particle changes direction twice (from right to left at $t=\pi$ and from left to right at $t=2\pi$).

(c) Using technology, $|v(t)|=\left|e^{t\sin t}-1\right|=0.5\Leftrightarrow t=a\approx 3.006308976$ and $t=b\approx 3.350007340$.

(d) Average acceleration $=\dfrac{1}{3-1}\displaystyle\int_1^3 a(t)\,dt=\frac{v(3)-v(1)}{3-1}=\frac{e^{3\sin 3}-1-\left(e^{\sin 1}-1\right)}{2}\approx-0.396$.

(e) Using part (b), the particle is moving to the right when $0<t<\pi$. After this, the particle starts moving to the left, so largest k with the total distance traveled $=\int_0^k v(t)\,dt$ is $k=\pi$.

(f) $s(t)=s(0)+\int_0^t\left(e^{x\sin x}-1\right)dx$; $s(6)=s(0)+\int_0^6\left(e^{t\sin t}-1\right)dt\approx\sum_{i=1}^{3}v(t_i)\Delta x$, where $\Delta x=\frac{6-0}{3}=2$. Thus,
$s(6)\approx 3+2\left[v(2)+v(4)+v(6)\right]=3+2\left[(e^{2\sin 2}-1)+(e^{4\sin 4}-1)+(e^{6\sin 6}-1)\right]\approx 9.797$.

(g) $a(t)=v'(t)=e^{t\sin t}\cdot\left(t\cos t+\sin t\right)\Rightarrow a(6)=e^{6\sin 6}(6\cos 6+\sin 6)\approx 1.0252>0$ but $v(6)=e^{6\sin 6}-1\approx-0.813<0$, so the particle is slowing down.

(h) Assuming that the distance function is defined and continuous on $[0,k]$ and differentiable on $(0,k)$, by the Mean Value Theorem, there exists a value c, $0 \le c \le k$, such that

$v(c) = s'(c) = \dfrac{s(k)-s(0)}{k-0} = \dfrac{s(k)-3}{k}$.

27. Using technology, we find that $v(t) > 0$ for $2 < t < a \approx 3.544907702$, and $v(t) < 0$ for $a < t < 5$. Thus, in the interval $[2,5]$, the particle is farthest to the right at time $t = a$. The position of the particle at this time is $s(2) + \int_2^a v(t)\,dt \overset{\text{CAS}}{\approx} 6 + 3.256 = 9.256$, choice (**A**).

28. (a) Total distance traveled $= \int_0^{10} \left|\sin\left(\frac{3\pi}{4}\right)\right| dt$.

(b) The particle is moving to the left when the velocity is negative. In the given interval, $\sin\left(\frac{\pi}{4}t\right) < 0 \Leftrightarrow 4 < t < 8$.

(c) The position of the particle at time $t = 3$ is $s(0) + \int_0^3 \sin\left(\frac{\pi}{4}t\right) dt = 3 + \left[-\frac{4}{\pi}\cos\left(\frac{\pi}{4}t\right)\right]_0^3$

$= 3 + \left[-\frac{4}{\pi}\cos\left(\frac{3\pi}{4}\right) - \left(-\frac{4}{\pi}\right)\right] = 3 + \frac{4}{\pi}\left(\frac{\sqrt{2}}{2} + 1\right)$.

(d) $a(t) = v'(t) = \frac{\pi}{4}\cos\left(\frac{\pi}{4}t\right) \Rightarrow a(3) = \frac{\pi}{4}\cos\left(\frac{3\pi}{4}\right) = -\frac{\pi}{4}\cdot\frac{\sqrt{2}}{2} = -\frac{\pi\sqrt{2}}{8}$. At the same time, $v(3) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2} > 0$. Because the velocity and acceleration have opposite signs at time $t = 3$, the particle is slowing down.

29. (a) The position of the particle at time $t = 2$ is

$s(0) + \int_0^2 \left(0.5te^{-t/2} - 1\right) dt \overset{\text{CAS}}{=} 4 + \left[-\left(te^{-t/2} + 2e^{-t/2} + t\right)\right]_0^2 = 4 - \left[\left(2e^{-1} + 2e^{-1} + 2\right) - \left(0 + 2e^0 + 0\right)\right] = 4 - 4/e$.

(b) For $0 < t < 2$, $v(t) < 0$ so the total distance traveled by the particle is

$-\int_0^2 \left(0.5te^{-t/2} - 1\right) dt = \left[te^{-t/2} + 2e^{-t/2} + t\right]_0^2 = \left[\left(2e^{-1} + 2e^{-1} + 2\right) - \left(0 + 2e^0 + 0\right)\right] = 4/e$.

(c) $a(t) = v'(t) = 0.5t\left(e^{-t/2}\right)\left(-\frac{1}{2}\right) + e^{-t/2}(0.5) = 0.5e^{-t/2}\left(1 - \frac{t}{2}\right)$. $a(3) = 0.5e^{-3/2}\left(1 - \frac{3}{2}\right)\frac{1}{4} = -e^{-3/2} < 0$.

$v(3) = 0.5(3)e^{-3/2} - 1 < 0$. Since the acceleration and velocity are both negative, at time $t = 3$, the particle is speeding up.

30. (a) $s(t) = te^{-t/5} \Rightarrow v(t) = t\left(-\frac{1}{5}e^{-t/5}\right) + e^{-t/5} = e^{-t/5}(1 - \frac{1}{5}t)$, and $a(t) = e^{-t/5}\left(-\frac{1}{5}\right) + \left(1 - \frac{1}{5}t\right)\left(-\frac{1}{5}\right)e^{-t/5}$

$= -\frac{1}{5}e^{-t/5}\left(2 - \frac{1}{5}t\right)$. $v(7) = e^{-7/5}\left(1 - \frac{7}{5}\right) < 0$ and $a(7) = -\frac{1}{5}e^{-7/5}\left(2 - \frac{7}{5}\right) < 0$, so the speed of the particle is increasing at time $t = 7$.

(b) $a(t) = v'(t) = -\frac{1}{5}e^{-t/5}\left(2 - \frac{1}{5}t\right) = 0 \Leftrightarrow \left(2 - \frac{1}{5}t\right) = 0 \Leftrightarrow 2 = \frac{1}{5}t \Leftrightarrow 10 = t$. This is the only critical point. Then $v(0) = e^0 = 1$, $v(10) = -e^{-2} \approx -0.135$, and $v(50) = -9e^{-10} \approx -0.000409$. Thus the minimal velocity is $v(10) = -e^{-2} \approx -0.135$.

31. Using technology we see that in the interval $[2,7]$, $v(t) < 0$ for $a < t < b$, where $a \approx 3.544907702$, and $b \approx 6.139960248$. Thus the particle reaches its leftmost position at $t = b$ and the total distance it travels to this position is

$$\int_2^b |v(t)|\,dt = \int_2^a \left(e^{t/2}\cos\left(\tfrac{t^2}{8}\right)\right) dt - \int_a^b \left(e^{t/2}\cos\left(\tfrac{t^2}{8}\right)\right) dt \overset{\text{CAS}}{\approx} 22.753.$$

32. (a) The position of the bus is given by $s_B(t) = 200 + \int_0^t h(x)\,dx = 200 + \int_0^t \left[4e^{-0.001x} + 0.1x + 4\right]dx$ and

the position of the car is given by $s_C(t) = \int_0^t c(x)\,dx = \int_0^t \left[30\tan^{-1}\left(\frac{x}{50}\right)\right]dx.$

At time $t = 150,$ the distance between the two vehicles is

$$|s_B(t) - s_C(t)| = \left|200 + \int_0^t \left[4e^{-0.001x} + 0.1x + 4\right]dx - \int_0^t \left[30\tan^{-1}\left(\frac{x}{50}\right)\right]dx\right| \approx 1411.599 \text{ meters.}$$

(b) The car is farthest behind when the bus is farthest ahead, that is, when

$D(t) = s_B(t) - s_C(t) = 200 + \int_0^t \left[4e^{-0.001x} + 0.1x + 4\right]dx - \int_0^t \left[30\tan^{-1}\left(\frac{x}{50}\right)\right]dx$ is maximized. We find

the critical numbers of $D(t)$ by solving $D'(t) = h(t) - c(t) = 4e^{-0.001t} + 0.1t + 4 - 30\tan^{-1}\left(\frac{t}{50}\right) = 0.$ The

only critical value in $[0, 300]$ is $t = 16.550$ seconds. Since $D'(t) > 0$ for $t < 16.550$ and $D'(t) < 0$ for $t > 16.550,$ the absolute maximum occurs at $t = 16.550$ seconds, and this is when the car is farthest behind the bus.

33. (a) Average velocity $= \frac{1}{4-2}\int_2^4 \left[4\sqrt{t}\cos\left(e^{t/3}\right)\right]dt \overset{\text{Calc}}{=} -5.652.$

(b) The particle is moving to the right when its velocity is positive. Because $\sqrt{t} \ge 0,$ the velocity is positive when $\cos\left(e^{t/3}\right) > 0 \Leftrightarrow 0 < e^{t/3} < \pi/2$ and $3\pi/2 < e^{t/3} < 5 \Rightarrow 0 < t < 3\ln\left(\frac{\pi}{2}\right)$ and $3\ln\left(\frac{3\pi}{2}\right) < t < 5.$

(c) $a(t) = v'(t) = -4\sqrt{t}\sin\left(e^{t/3}\right)\cdot e^{t/3}\cdot\frac{1}{3} + 4\cos\left(e^{t/3}\right)\frac{1}{2\sqrt{t}}.$ When $t = 3,$ $v(t) = 4\sqrt{3}\cos(e) > 0$ and $a(3) = -\frac{4\sqrt{3}e}{3}\sin(e) + \frac{2}{\sqrt{3}}\cos(e) < 0,$ so the speed is decreasing.

(d) $s(t) = s(0) + \int_0^t v(x)\,dx = 5 + \int_0^t 4\sqrt{x}\cos\left(e^{x/3}\right)dx.$ From part (b), we know we need to consider the position of the particle at points $t = 0, t = 3\ln\left(\frac{\pi}{2}\right),$ and $t = 5.$ $s(0) = 5, s\left(3\ln\left(\frac{\pi}{2}\right)\right) \approx 6.0299,$ and $s(5) \approx -7.403.$ So the absolute maximum of the velocity occurs when $t = 3\ln\left(\frac{\pi}{2}\right),$ and at that time, the particle is farthest to the right.

34. $v(t) = 4t^3 - 4t = 4(t-1)(t+1).$ In the interval $[0, 2],$ $v(t) > 0 \Leftrightarrow 1 < t < 2.$ So

$$|v(t)| = \begin{cases} -4t^3 + 4t & \text{if } 0 \le t \le 1 \\ 4t^3 - 4t & \text{if } 1 < t \le 2 \end{cases} \Rightarrow$$

$$\text{average speed} = \frac{1}{2-0}\left[\int_0^1 (-4t^3 + 4t)\,dt + \int_1^2 (4t^3 - 4t)\,dt\right] = \tfrac{1}{2}\left(\left[-t^4 + 2t^2\right]_0^1 + \left[t^4 - 2t^2\right]_1^2\right)$$
$$= \tfrac{1}{2}\left[(-1+2) - 0 + (16-8) - (1-2)\right] = \tfrac{10}{2} = 5, \text{ option } (\mathbf{A}).$$