p. 455: 58-66, 75-81 odd, 82

58. If $w'(t)$ is the rate of change of weight in pounds per year, then $w(t)$ represents the weight in pounds of the child at age $t$. We know from the Net Change Theorem that $\int_5^{10} w'(t)\,dt = w(10) - w(5)$, so the integral represents the increase in the child's weight (in pounds) between the ages of 5 and 10.

59. If $r'(t)$ is the rate at which water flows over a dam in cubic feet per second, then $r(t)$ represents the amount of water held by the dam at time $t$ (seconds). Then by the Net Change Theorem, $\int_0^{60} r'(t)\,dt$ is the number of cubic feet of water that flowed over the dam in the first 60 seconds (1 minute).

60. If $r(t)$ is the rate at which water flows into a tank and $d(t)$ is the rate at which water flows out of the tank at time $t$, both measured in cubic feet per hour, then by the Net Change Theorem, $\int_3^6 [r(t) - d(t)]\,dt$ is net change in the amount of water (in cubic feet) that is in the tank from the third hour to the sixth hour.

61. Since $r(t)$ is the rate at which oil leaks, we can write $r(t) = -V'(t)$ where $V(t)$ is the volume of oil at time $t$. [Note that the minus sign is needed because $V$ is decreasing, so $V'(t)$ is negative, but $r(t)$ is positive.] Thus, by the Net Change Theorem,
$\int_0^{120} r(t)dt = -\int_0^{120} V'(t)\,dt = -[V(120) - V(0)] = V(0) - V(120)$, which is the number of gallons of oil that leaked from the tank in the first two hours (120 minutes).

62. By the Net Change Theorem, $\int_0^{15} n'(t)\,dt = n(15) - n(0) = n(15) - 100$ represents the increase in the bee population in 15 weeks. So $100 + \int_0^{15} n'(t)\,dt = n(15)$ represents the total bee population after 15 weeks.

63. By the Net Change Theorem, $\int_{1000}^{5000} R'(x)\,dx = R(5000) - R(1000)$, so it represents the increase in revenue when production is increased from 1000 units to 5000 units.

64. The slope of the trail is the rate of change of the elevation $E$, so $f(x) = E'(x)$. By the Net Change Theorem, $\int_3^5 f(x)\,dx = \int_3^5 E'(x)\,dx = E(4) - E(3)$ is the change in the elevation $E$, between $x = 3$ miles and $x = 5$ miles from the start of the trail.

65. In general, the unit of measurement for $\int_a^b f(x)\,dx$ is the product of the unit for $f(x)$ and the unit for $x$. Since $f(x)$ is measured in newtons and $x$ is measured in meters, the units for $\int_0^{100} f(x)\,dx$ are newton-meters (or joules) (A newton-meter is abbreviated N·m.)

66. The units for $a(x)$ are pounds per foot and the units for $x$ are feet, so the units for $da/dx$ are pounds per foot per foot, denoted (lb/ft)/ft. The unit of measurement for $\int_2^8 a(x)\,dx$ is the product of pounds per foot and feet; that is, pounds.

75. By the Net Change Theorem, the amount of water that flows from the tank during the first 10 minutes is $\int_0^{10} r(t)\,dt = \int_0^{10} (200 - 4t)\,dt = 1800$ liters.

77. (a) By the Net Change Theorem, the total amount spewed into the atmosphere is

$Q(6)-Q(0)=\int_0^6 r(t)\,dt=Q(6)$ since $Q(0)=0$. The rate $r(t)$ is positive so $Q$ is an increasing function.

Thus, an upper estimate for $Q(6)$ is $R_6$ and a lower estimate for $Q(6)$ is $L_6$. $\Delta t=\dfrac{b-a}{n}=\dfrac{6-0}{6}=1$.

$$R_6=\sum_{i=1}^{6} r(t_i)\Delta t=10+24+36+46+54+60=230 \text{ tonnes.}$$

$$L_6=\sum_{i=1}^{6} r(t_{i-1})\Delta t=R_6+r(0)-r(6)=230+2-60=172 \text{ tonnes.}$$

(b) $\Delta t=\dfrac{b-a}{n}=\dfrac{6-0}{3}=2$. $Q(6)\approx M_3=2[r(1)+r(3)+r(5)]=2(10+36+54)=2(100)=200$ tonnes.

79. By the Net Change Theorem, the amount of water after four days is

$$25{,}000+\int_0^4 r(t)\,dt\approx 25{,}000+M_4=25{,}000+\tfrac{4-0}{4}[r(0.5)+r(1.5)+r(2.5)+r(3.5)]$$
$$\approx 25{,}000+[1500+1770+740+(-690)]=28{,}320 \text{ liters}$$

81. Use the midpoint of each of four 2-day intervals. Let $t=0$ correspond to July 18 and note that the inflow rate, $r(t)$, is in $\text{ft}^3/\text{s}$.

Amount of water $=\int_0^8 r(t)\,dt\approx[r(1)+r(3)+r(5)+r(7)]\dfrac{8-0}{4}\approx[6401+4249+3821+2628](2)$
$=34{,}198.$

Now multiply the number of seconds in a day, $24\cdot 60^2$, to get $2{,}954{,}707{,}200\ \text{ft}^3$.

82. Let $P(t)$ denote the bacteria population at time $t$ (in hours). By the Net Change Theorem,

$$P(1)-P(0)=\int_0^1 P'(t)\,dt=\int_0^1 (1000\cdot 2^t)\,dt=1443.$$

Thus the population after one hours is 4000 + 1443 = 5443.

p. 506: 17-27

17. $R(t)=Ae^{kt}$, $R(0)=1.5\Rightarrow A=1.5$. $R(7)=2.3=1.5e^{7k}\Rightarrow \ln\left(\frac{2.3}{1.5}\right)=7k\Rightarrow \frac{1}{7}\ln\left(\frac{2.3}{1.5}\right)=k$. The total increase in the bacteria population for $2\le t\le 5$ is $\int_2^5 R(t)\,dt=\int_2^5 1.5e^{kt}\,dt=5580$ bacteria.

18. $\int_0^{30} r'(t)\,dt\approx\sum_{i=1}^{3} r'(\overline{t}_i)\,\Delta t$, where $\Delta t=\frac{30-0}{3}=0$.

$$\int_0^{12} r'(t)\,dt\approx 10[r'(5)+r'(15)+r'(25)]=0[5.6+4.0+2.7]=10(12.3)=123\text{cm}$$

$\int_0^{30} r'(t)\,dt$ tells us that the radius of the balloon increased by 123 cm during the time from 0 to 30 minutes.

19. (a) $\frac{1}{8}\int_0^8 T(t)\,dt$ is the average value of the temperature of the entire 8-inch rod, in degrees Fahrenheit.

(b) $\frac{1}{8}\int_0^8 T(t)\,dt \approx \frac{1}{8}\left[\left(\frac{T(2)+T(0)}{2}\right)(2)+\left(\frac{T(5)+T(2)}{2}\right)(3)+\left(\frac{T(7)+T(5)}{2}\right)(2)+\left(\frac{T(8)+T(7)}{2}\right)(1)\right]$

$$=\frac{1}{8}\left[(92+78)+3\left(\frac{66+78}{2}\right)+(54+66)+\left(\frac{52+54}{2}\right)\right]=\frac{1}{8}(415)=51.875$$

(c) $\int_0^8 T'(t)\,dt = T(8)-T(0)=52-92=-40.$ The temperature of the rod has decreased by 40 degrees Fahrenheit from the top of the 8-inch rod to the bottom.

20. (a) $E(4)=\frac{10}{2+\ln(8)}\approx 2.451$ $L(4)=3+10\cos\left(\frac{16}{40}\right)\approx 12.211.$ Since $L(4)>E(4),$ the amount of gasoline is decreasing.

(b) The net change in the number of gallons of gasoline in the tank is given by the difference between the amount that enters and the amount that leaves during the interval $[4,6]$; that is,

$\int_4^6 [E(t)-L(t)]\,dt \overset{\text{CAS}}{\approx} -17.186.$ Thus, the net change is a loss of 17.186 gallons of gas in the tank.

(c) Let $G_{\text{ave}}$ be the average amount of gasoline in the tank. Then

$$G_{\text{ave}}=\frac{1}{6-4}\int_4^6 (225+E(t)-L(t))\,dt \overset{\text{CAS}}{\approx} 216.407 \text{ gallons per hour.}$$

(d) There are $G(7)=225+\int_0^7 [E(t)-L(t)]\,dt \approx 161.499$ gallons of gasoline in the tank at the 7th hour.

(e) The number of gallons of gasoline in the tank at time $t=x$ is $G(x)=225+\int_0^x [E(t)-L(t)]\,dt.$

(f) We find the critical points of $G$: $G'(x)=E(x)-L(x),$ and $G'(x)=0 \Leftrightarrow x=a\approx 8.120.$ Evaluating $G$ at the critical point and endpoint we find $G(0)=225,\ G(a)\approx 159.113,$ and $G(10)\approx 166.147.$ Thus the absolute minimum of $G$ for $0\le t\le 10$ is $\approx 159.113$ gallons at approximately $t=8.120$ hours.

(g) $225+\int_0^k [E(t)-L(t)]\,dt = 215$

21. (a) $\int_0^{10} T'(t)\,dt = T(10)-T(0)=38-61=-23.$ The temperature of the coffee has cooled by 23 degrees Celsius over the time interval from 0 to 10 minutes.

(b) $\frac{1}{10}\int_0^{10} T(t)\,dt$ represents the average temperature of the coffee over the time interval from 0 to 10 minutes. $\frac{1}{10}\int_0^{10} T(t)\,dt \approx \frac{1}{10}\left[3\cdot\frac{T(3)+T(0)}{2}+2\cdot\frac{T(5)+T(3)}{2}+1\cdot\frac{T(6)+T(5)}{2}+2\cdot\frac{T(8)+T(6)}{2}+2\cdot\frac{T(10)+T(8)}{2}\right]$

$$=0.1\left[3\cdot\left(\tfrac{61+59}{2}\right)+(59+52)+\tfrac{(52+48)}{2}+(48+41)+(41+38)\right]=0.1(509)=50.9^\circ\text{C}$$

(c) $T'(4)\approx \frac{T(5)-T(3)}{5-3}=\frac{52-59}{2}=-3.5^\circ\text{C/min}.$

(d) $C(t)=C(10)+\int_{10}^{12} C'(t)\,dt = 38+\int_{10}^{12} -2\cos(0.1t)\,dt = 36.189^\circ\text{C}.$

22. (a) $\int_0^5 M(t)\,dt = \int_0^5 3\pi \sin\left(\frac{\pi t}{12}\right) dt = 26.683$ inches of snow.

(b) $A(t) = 60 + \int_0^t [S(x) - M(x)]\,dx = 60 + \int_0^t \left[0.14x^3 - 0.16x^2 + 0.54x - 0.1 - 3\pi \sin\left(\frac{\pi x}{12}\right)\right] dx.$

(c) $A'(5) = S(5) - M(5) = 6.996$ inches per hour.

(d) $A'(t) = S(t) - M(t) = 0.14x^3 - 0.16x^2 + 0.54x - 0.1 - 3\pi \sin\left(\frac{\pi x}{12}\right).$

$A'(t) = 0 \Leftrightarrow S(t) - M(t) = 0.14x^3 - 0.16x^2 + 0.54x - 0.1 = 3\pi \sin\left(\frac{\pi x}{12}\right) \Leftrightarrow t \approx 3.936.$

$A(0) = 60, A(3.936) \approx 68.543, A(5) \approx 65.224.$ Therefore, the minimum snow depth occurs at time $t = 0$ and then the depth is 60 inches.

23. The temperature at 8 AM will be $T(0) + \int_0^8 R(t)\,dt = 22 + \pi \int_0^8 \sin\left(\tfrac{\pi}{12} t\right) dt \approx 40\,^\circ$F, which is choice (**C**).

24. (a) $\int_0^5 r(t)\,dt$ is the total change in the temperature from midnight to 5:00 AM.

(b) $r(5) = 2\sin\left(\frac{5\pi}{12}\right) \approx 1.932$ degrees Celsius per hour.

(c) The average temperature of the room between midnight at 5:00 AM is

$T_{\text{ave}} = \frac{1}{5-0}\int_0^5 (22 + r(t))\,dt \approx 23.132^\circ$C.

(d) The temperature of the room at 5:00 AM is $T(5) = 22 + \int_0^5 \left(2\sin\frac{\pi t}{12}\right) dt \approx 27.662^\circ$C.

(e) Between midnight at 5:00 AM, the change in the room's temperature is

$T(5) - T(0) = 27.662 - 22 = 5.662^\circ C.$

25. (a) The total amount of water flooded into the basement

$\int_3^5 w(t)\,dt = \int_3^5 3\sqrt{t} + 4\,dt \approx 19.968$ gallons.

(b) Let $A(t) = 12 + \int_0^t [w(x) - p(x)]\,dx.$ Then $A(3)$ is the total amount of water in the basement at time $t = 3$ hours, and $A'(3) = w(3) - p(3) \approx 1.817.$ The amount of water in the basement is increasing at a rate of 1.817 gallons per hour then.

(c) The total amount of water in the basement at time $t, 0 \le t \le 8$ is $A(t) = 12 + \int_0^t [w(x) - p(x)]\,dx.$

$A'(t) = w(t) - p(t) = 0 \Leftrightarrow t = 3.400.$ Then $A(0) = 12, A(3.4) \approx 22.167,$ and $A(8) < 0,$ so the maximum amount of water in the basement was 22.167 gallons, when $t = 3.4$ hours.

26. $f\ (x) = 4x \quad \Rightarrow f(x) = 4\ln|x| + C.$ $f\left(\sqrt{e}\right) = 4\ln\sqrt{e} + C = 4\ln\left(e \quad\right) + C = 4 \cdot \tfrac{1}{2} + C = 2 + C,$ and

$f\left(\sqrt{e}\right) = 5 \Rightarrow C = 3.$ Thus $f(e) = 4\ln e + 3 = 4 + 3 = 7,$ which is option (**A**).

27. $f(5) = f(1) + \int_1^5 f'(x)\,dx = 2 + \int_1^5 \sqrt{x^3 + 6}\,dx \overset{\text{CAS}}{\approx} 2 + 24.6718 = 26.672,$ which is option (**D**).